NUMERICAL STUDY AND ANALYSIS OF FUNCTIONALLY GRADED DOUBLE CURVED PANELS UNDER FREE VIBRATION

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ABSTRACT

The primary objective of this study is to investigate the linear and free vibration analysis of functionally graded double-curved shells, using higher-order shear deformation theory using Matlab numerical programming. The mechanical properties of the plate are supposed to vary continually in the direction of thickness and the volume fractions of the constituents are obtained using two types of four-parameter power-law distribution. A continuous isoparametric lagrangian model is used to develop fundamental equations for FGM shells. Convergence tests carried out, followed by an extensive literature validation to highlight the accuracy and efficiency of the current FG model. Two types of FGM combinations namely Al / Al₂O₃, Al / ZrO₂, with boundary conditions, geometric parameters, and the consequence of the four parameters are considered for the present study. Finally, some numerical problems were addressed with different volume fractions and boundary conditions to investigate the frequency response of double-curved shells.

KEYWORDS: Functionally graded materials, HSDT, Curved shells & Four parameter power law

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1. INTRODUCTION

A group of scientist from Japan first observed the importance of functionally graded materials (FGMs) for a thermal barrier application. FGMs are a blend of two constituents, such as ceramics and metals to resist high-temperature and toughness. FGMs show a smooth and constant difference in material properties through-thickness. FGMs arrangement discards stress concentrations and reduces delamination effect in layered composites. FGMs are quickly extending for engineering applications, which is fundamental for engineers. Several kinds of research produced different hypotheses for analyzing plates and shells structures.

Pandya and Kant [1] developed FE based composite plates under transverse load based on HOSDT. Reddy [2] presented theoretical formulations based on TOSDT and Finite element analysis accounts geometric nonlinearity and thermo coupling and Navier rectangular plates. The author found the gradient composition plays an important role in estimating the performance of the plate under shear. Reddy and Cheng [3] presented vibration analysis of FGM shallow shell as a polygonal form quiescent on Winkler Pasternak elastic foundation and the formulation based on CPT, FSDT and HSDT and Mori Tanaka observed frequencies are extracted different flexural, stretching, thickness shear mode conditions. Batra and Aimmamee [4] analysed free vibration behaviour and stress distribution in a thick isotropic and homogeneous plate. Kim [5] developed temperature dependent FG plates vibration analysis using Rayleigh-Ritz procedure. Temperature and heat flow thermal conditions observed. Kandasamy and Singh [6] investigated the vibration of cylindrical deep shells with skewed open circular. Tornabene and Viola [7] investigated the dynamic behaviour of parabolic shell structures using

Generalized Differential Quadrature.

Zhao et al. [8] presented free FG plate vibration analysis based on FOSDT is used to address the rotary inertia, transverse shear strain and frequency characteristics. Tornabene [9] investigated double curved shells using GDQ and FOSDT. Hosseini- Hashemi et al [10] worked on moderately thick plates to observe free vibration analysis. Talha and Singh [11] studied free vibration and static analysis of FG plates using HOSDT conjunction with FE. The material properties of the plate presumed to differ continuously in the direction of thickness by simple application of the power law and various parameters are observed. Liew et al [12] presented a review of the meshless method. Ruan et al [13] studied uniformly distributed FGM skew plate by the Kirchhoff thin plate theory which is subjected to tangential follower force are derived. Tornabene et al. [14] studied using the Generalized Differential Quadrature Approach used to test rectangular and annular composite laminates shells. Jha et al. [15] presented a brief review of the FG application and various studies on FGM plates.

Shariyat and Hosseini [16] investigated composite sandwich plates with viscoelastic core under biaxial loads. Gulshan Taj [17] presented the free vibration of FGM skew plates under the thermal environment condition and based on the HOSDT. Xie et al [18] presented free vibration on FGM parabolic and spherical shells using Haar wavelet discretization and FSDT of revolution with classical and elastic boundary conditions. Kar and Panda [19] investigated FG curved panel free vibration response of shear deformable under the linear, nonlinear, uniform, temperature fields at different geometric parameters are observed. Zhang et al [20] presented circular cylindrical double-shells under free and forced vibration. Wang et al [21] investigated on vibration behavior of different double curved shells based on first order deformation theory, and four parameter power law distribution.

Tornabene et al [22] studied FGMs of doubly-curved shells for vibrational behavior including porosity and formulation based on the FOSDT and Navier solution method. Rezaei et al [23] presented based on FOSDT for rectangular plates under free vibration analysis of FG materials with porosities. Moita et al [24] developed vibration response with active and passive damping multi-layer sandwich plate/shell embedded with FGM-Piezoelectric viscoelastic core, with plates using different boundary conditions. The material model formulation is based on shear deformation theories. Chakravarty and Pradhan [25] investigated FG thin plates lay on Winkler and Pasternak elastic foundation. Based on CPT and Rayleigh-Ritz method, Mouli and Koon [26] investigated on FG curved panels for vibration characteristics of rectangular and skew forms. Governing equations comprehended by Hamilton's standard and TSDT. Katariya and Panda [27][28] examined skew sandwich flat and curved panel for static analysis to evaluate the response of sandwich panel proposed mathematically higher-order kinematics midplane theory and Green-Lagrange strain displacement relation for the different configuration. Rezaiee-Pajand [29] studied FG shells under different loading conditions based on FEM Voigt's model on thermo-elastic nonlinear analysis. From the literature, extensive study available on mathematical model developments of FGM shells and analytical models with different shear deformation theories. However, very few works are observed on free vibration analysis of FGM shells. Hence in the present study, a linear vibration analysis was carried out for different mode shapes with different power law index were considered to estimate the response.

2. GEOMETRIC CONFIGURATION AND MATERIAL PROPERTIES

Figure 1: Shapes of various Shell Configurations Generated from a General Doubly-curved shell [19]

Figure 2: Variation of Volume Fraction through Thickness Direction for various values of Power Law Index.

Usually, functionally graded materials consist of a two-component mixture. In this work, the functionally graded material is presumed to be made of a combination of metal and ceramic components.

Table 1: Metal and Ceramics Material Properties.[8]

Composition	Materials	Young's Modulus (GPa)	Mass Density $m(\text{kg/m}^3)$	Poisson's Ratio
Metal	Aluminium (Al)	70	2702	0.3
Ceramic	Zirconia (ZrO_2)	151	3000	0.3
	Alumina (Al_2O_3)	380	3800	0.3

In FG shells, the physical properties vary constantly and smoothly from thickness direction (ξ) to the function of volume fractions of its components. The Young module E and mass density ρ Poisson ratio μ can be conveyed as a linear function.

$$E(\xi) = (E_c - E_m)V_c + E_m \quad (1a)$$

$$\rho(\xi) = (\rho_c - \rho_m)V_c + \rho_m \quad (1b)$$

$$\mu(\xi) = (\mu_c - \mu_m)V_c + \mu_m \quad (1c)$$

First, two separate power-law differences in the thickness-direction of the volume fraction of the constituents are considered. The influence on the mechanical behaviour of functionally graded shells is explored by the power-law exponent and one of the power-law distribution choices.

The base surface of the structure is ceramic rich for the first power-law application, while the top surface can be metal rich as in Fig.2 (a) shown. The second power law distribution 2(b) indicates that, for the subsequent distribution, the top surface of the plate ($\xi/h=0.5$) is ceramic rich, while the base surface of the plate ($\xi/h=-0.5$) is rich in metal. [9]

$$\text{FGM}_{1(i/j/k/\varphi)}: V_c = \left(1 - i \left(\frac{1}{2} + \frac{\xi}{h} \right) + j \left(\frac{1}{2} + \frac{\xi}{h} \right)^k \right)^\varphi \quad (2)$$

$$\text{FGM}_{2(i/j/k/\varphi)}: V_c = \left(1 - i \left(\frac{1}{2} - \frac{\xi}{h} \right) + j \left(\frac{1}{2} - \frac{\xi}{h} \right)^k \right)^\varphi \quad (3)$$

The parameters i, j, k dictates the material variation profile through the functionally graded shell thickness.

The volume fractions of all the constituent is given by

$$V_c + V_m = 1 \quad (4)$$

3. THEORETICAL FORMULATIONS

Figure 1, shows general doubly curved FGP of uniform thickness 'h' with a rectangular base of sides a and b . R_x and R_y are principal radii of curvature of the shell along x and y directions, respectively. The respective radii of curvature of flat, spherical, cylindrical, hyperbolic paraboloid and ellipsoid shells are expressed as $R_x=R_y=\infty$; $R_x=R_y=R$; $R_x=R$, $R_y=\infty$; $R_x=R$, $R_y=-R$; and $R_x=R$, $R_y=2R$ is shown in Figure 1(a),(c),(d), (e) and (f).

The displacement of FG curved Shell is obtained based on HSDT kinematics in the mid-plane of the Shell (at $z = 0$). Global displacements (u, v, w) are defined in the polynomial form of nine unknown parameters $(u_0, v_0, w_0, \theta_x, \theta_y, u_0^*, v_0^*, \theta_x^*, \theta_y^*)$ as in Pandya and Kant[1]

$$\left. \begin{aligned} \Pi(x, y, z) &= \Pi_0(x, y) + \xi \theta_x(x, y) + \xi^2 \Pi_0^*(x, y) + \xi^3 \theta_x^*(x, y) \\ \Theta(x, y, z) &= \Theta_0(x, y) + \xi \theta_y(x, y) + \xi^2 \Theta_0^*(x, y) + \xi^3 \theta_y^*(x, y) \\ \Omega(x, y, z) &= \Omega_0(x, y) \end{aligned} \right\} \quad (5)$$

where, $(\Pi_0, \Theta_0, \Omega_0)$ denote the mid-plane displacements along (x, y, z) coordinates, respectively. θ_y and θ_x are the shear rotations about the x - and y -axis, respectively, and $(\Pi_0^*, \Theta_0^*, \theta_x^*, \theta_y^*)$ are the higher-order terms defined in the mid-plane of the curved panel. These nine unknowns can be defined in the mid-plane, i.e., at $z = 0$ as $\Pi_0 = \Pi(x, y)$,

$$\Theta_0 = \Theta(x, y), \Omega_0 = \Omega(x, y), \theta_x = \frac{\partial \Pi}{\partial z}, \theta_y = \frac{\partial \Theta}{\partial z}, \Pi_0^* = \frac{1}{2} \left(\frac{\partial^2 \Pi}{\partial z^2} \right), \Theta_0^* = \frac{1}{2} \left(\frac{\partial^2 \Theta}{\partial z^2} \right), \theta_x^* = \frac{1}{6} \left(\frac{\partial^3 \Pi}{\partial z^3} \right) \text{ and}$$

$$\theta_y^* = \frac{1}{6} \left(\frac{\partial^3 \Theta}{\partial z^3} \right).$$

The linear strains are given by $\{\mathcal{E}\} = \begin{Bmatrix} \mathcal{E}_{xx} \\ \mathcal{E}_{yy} \\ \mathcal{E}_{xy} \\ \mathcal{E}_{xz} \\ \mathcal{E}_{yz} \end{Bmatrix} = \begin{Bmatrix} \left(\frac{\partial \Pi}{\partial x} + \frac{\Omega}{R_x} \right) \\ \left(\frac{\partial \Theta}{\partial y} + \frac{\Omega}{R_y} \right) \\ \left(\frac{\partial \Pi}{\partial y} + \frac{\partial \Theta}{\partial x} + \frac{2\Omega}{R_{xy}} \right) \\ \left(\frac{\partial \Pi}{\partial z} + \frac{\partial \Omega}{\partial x} - \frac{\Pi}{R_x} \right) \\ \left(\frac{\partial \Theta}{\partial z} + \frac{\partial \Omega}{\partial y} - \frac{\Theta}{R_y} \right) \end{Bmatrix}$ (6)

Now, replacing the displacements as in Equation (5) into the Equation (6) the terms of the strain may be defined.

$$\begin{Bmatrix} \mathcal{E}_{xx} \\ \mathcal{E}_{yy} \\ \mathcal{E}_{xy} \\ \mathcal{E}_{xz} \\ \mathcal{E}_{yz} \end{Bmatrix} = \begin{Bmatrix} \mathcal{E}_x^0 \\ \mathcal{E}_y^0 \\ \mathcal{E}_{xy}^0 \\ \mathcal{E}_{xz}^0 \\ \mathcal{E}_{yz}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x^1 \\ k_y^1 \\ k_{xy}^1 \\ k_{xz}^1 \\ k_{yz}^1 \end{Bmatrix} + z^2 \begin{Bmatrix} k_x^2 \\ k_y^2 \\ k_{xy}^2 \\ k_{xz}^2 \\ k_{yz}^2 \end{Bmatrix} + z^3 \begin{Bmatrix} k_x^3 \\ k_y^3 \\ k_{xy}^3 \\ k_{xz}^3 \\ k_{yz}^3 \end{Bmatrix}$$
 (7)

$$\varepsilon = \varepsilon^0 + \xi k^1 + \xi^2 k^2 + \xi^3 k^3$$
 (8)

Where the strain terms of the superscript 0, 1, 2, 3 are the extension, mid-plane ε^0 , k^1 , k^2 , and k^3 are the mid-plane strain, curvature, and higher-order terms [19]

The final form of equilibrium equation for the vibrated FGP can be written as below:

$$\left(\left([K_{gls}] - [K_{GM}] \right) - \omega_n^2 [M] \right) \{\Lambda\} = 0$$
 (9)

Where, ω_n is the eigenvalue (natural frequency) of the free vibrated FGP while Λ is the corresponding eigenvector (mode shapes).

The free vibration analysis of the non-dimensional frequency parameter $\bar{\omega}$ expressed as below

$$\left(\bar{\omega} = \omega(a^2 / h) \sqrt{\rho_c / E_c} \right)$$
 (10)

4. RESULTS AND DISCUSSIONS

4.1 Study of Convergence and Comparison

The first step, the number of nodes needed for the convergence analysis of the present model for the determination of the uniform mesh refining computed for the FG (Al /ZrO₂). CCCC condition and ($\varphi = 0$) ((6*6) mesh size chosen. For accuracy of the model Table, 2,3, and 4 demonstrate a comparison study and good agreement observed from available solutions based on FOSDT.

Table 2: Variation of the Frequency Parameter $\left(\bar{\omega} = \omega(a^2 / h) \sqrt{\rho_c / E_c}\right)$ with the Volume Fraction Exponent φ for plate Al/ Al₂O₃(a/h=10)

FGM TYPE	BCC	MODE	0			0.5		
			Present	Zhao et. al. [8]	%ERROR	Present	Zhao et. al. [8]	%ERROR
FGM1	SSSS	1	5.7670	5.6763	1.5979	5.1920	5.1105	1.5948
		2	13.8271	13.5370	2.1430	12.4702	12.2070	2.1561
		3	13.8283	13.5370	2.1519	12.4713	12.2070	2.1652
		4	19.4217	20.6330	-5.8707	17.8909	18.6300	-3.9673
	CCCC	1	9.9436	9.6354	3.1986	8.9816	8.7011	3.2237
		2	19.1612	18.3050	4.6774	17.3493	16.5670	4.7220
		3	19.1612	18.3050	4.6774	17.3493	16.5670	4.7220
		4	26.8640	25.5060	5.3242	24.3528	23.1110	5.3732
FGM2	SSSS	1	5.7670	5.6763	1.5979	5.1926	5.1105	1.6065
		2	13.8271	13.5370	2.1430	12.4730	12.2070	2.1791
		3	13.8283	13.5370	2.1519	12.4741	12.2070	2.1881
		4	19.4217	20.6330	-5.8707	17.8921	18.6300	-3.9608
	CCCC	1	9.9436	9.6354	3.1986	8.9825	8.7011	3.2341
		2	19.1612	18.3050	4.6774	17.3531	16.5670	4.7450
		3	19.1612	18.3050	4.6774	17.3531	16.5670	4.7450
		4	26.8640	25.5060	5.3242	24.3601	23.1110	5.4048

Table 3 Variation of the Frequency Parameter $\left(\bar{\omega} = \omega(a^2/h)\sqrt{\rho_c/E_c}\right)$ with the Volume Fraction Exponent φ (1, 2) for plate Al/ Al₂O₃ (a/h=10)

FGM TYPE	BCC	MODE	1			2		
			Present	Zhao et. al. [8]	%Error	Present	Zhao et. al. [8]	%Error
FGM1	SSSS	1	4.9476	4.8713	1.5663	4.7681	4.6977	1.4986
		2	11.8748	11.6330	2.0786	11.4140	11.1990	1.9198
		3	11.8759	11.6330	2.0880	11.4150	11.1990	1.9287
		4	17.0304	17.7480	-4.0433	16.0931	17.0630	-5.6842
	CCCC	1	8.5535	8.2925	3.1474	8.2117	7.9756	2.9603
		2	16.5110	15.7830	4.6126	15.8010	15.1470	4.3177
		3	16.5110	15.7830	4.6126	15.8010	15.1470	4.3177
		4	23.1665	22.0140	5.2353	22.1334	21.0990	4.9026
FGM2	SSSS	1	4.9485	4.8713	1.5848	4.7691	4.6977	1.5199
		2	11.8796	11.6330	2.1198	11.4193	11.1990	1.9671
		3	11.8806	11.6330	2.1284	11.4202	11.1990	1.9752
		4	17.0326	17.7480	-4.0309	16.0955	17.0630	-5.6702
	CCCC	1	8.5551	8.2925	3.1667	8.2134	7.9756	2.9816
		2	16.5173	15.7830	4.6525	15.8078	15.1470	4.3626
		3	16.5173	15.7830	4.6525	15.8078	15.1470	4.3626
		4	23.1787	22.0140	5.2907	22.1464	21.0990	4.9642

Table 4: Variation of the Frequency Parameter $\left(\bar{\omega} = \omega(a^2/h)\sqrt{\rho_c/E_c}\right)$ with the Volume Fraction Exponent φ (5, 8, 10) for plate Al/ AL₂O₃ (a/h=10)

FGM TYPE	BCC	MODE	5			8			10		
			Present	R Zhao et. al. [8]	%Error	Present	Zhao et. al. [8]	%Error	Present	Zhao et. al. [8]	%Error
FGM1	SSSS	1	4.6204	4.5549	1.4380	4.5396	4.4741	1.4640	4.4982	4.4323	1.4868
		2	11.0197	10.8280	1.7704	10.8255	10.6320	1.8200	10.7302	10.5330	1.8722
		3	11.0207	10.8280	1.7796	10.8264	10.6320	1.8284	10.7312	10.5330	1.8817
		4	15.0649	16.4620	-8.4868	14.6983	16.1570	-9.0283	14.5616	16.0080	-9.0355
	CCCC	1	7.9088	7.6959	2.7664	7.7658	7.5521	2.8297	7.6975	7.4815	2.8871
		2	15.1475	14.5630	4.0136	14.8679	14.2820	4.1024	14.7418	14.1490	4.1897
		3	15.1475	14.5630	4.0136	14.8679	14.2820	4.1024	14.7418	14.1490	4.1897
		4	21.1681	20.2470	4.5493	20.7743	19.8490	4.6617	20.6016	19.6660	4.7574
FGM2	SSSS	1	4.6210	4.5549	1.4512	4.5400	4.4741	1.4729	4.4985	4.4323	1.4936
		2	11.0227	10.8280	1.7981	10.8273	10.6320	1.8369	10.7316	10.5330	1.8855
		3	11.0237	10.8280	1.8074	10.8282	10.6320	1.8454	10.7325	10.5330	1.8940
		4	15.0662	16.4620	-8.4789	14.6990	16.1570	-9.0240	14.5621	16.0080	-9.0324
	CCCC	1	7.9098	7.6959	2.7794	7.7663	7.5521	2.8363	7.6979	7.4815	2.8925
		2	15.1513	14.5630	4.0397	14.8702	14.2820	4.1185	14.7435	14.1490	4.2017
		3	15.1513	14.5630	4.0397	14.8702	14.2820	4.1185	14.7435	14.1490	4.2017
		4	21.1754	20.2470	4.5854	20.7787	19.8490	4.6839	20.6050	19.6660	4.7747

4.2 Numerical Examples

4.2.1. Spherical Shell

Tables 5 and 6, show the non-dimensional frequency parameter variation with effect of the volume fraction index and Four parameter power law distribution FGM1 (i=1, j=0/k/ φ) for constituent materials Al/Al₂O₃, Al/ZrO₂ and the thickness ratio

(a/ h=10), aspect ratio (a/b=1), respectively. The volume fraction of the gradient was varied by changing the power law index. For the first four modes, results are computed. For the shell with different boundary condition SSSS and CCCC, the volume fraction exponent φ increases, the frequencies in all four modes decreased. This is due to the fact that plate has a smaller ceramic component with a larger volume fraction exponent which reduces stiffness. A similar trend is observed for the CCCC boundary condition. The second power law distribution FGM2 (i=1/j=0/k/ φ) shows the power law index increases the response of non-dimensional frequency decreases. Clamped boundary conditions exhibit higher frequency responses compared to the simply supported conditions in both power law distribution.

Table 5: Variation of the Frequency Parameter $\left(\bar{\omega} = \omega(a^2/h)\sqrt{\rho_c/E_c}\right)$ with the Volume Fraction Exponent φ for Spherical Al/ Al₂O₃ (a/h=10)

FGM TYPE	BCC	MODE	0	0.5	1	2	5	8	10
FGM1	SSSS	1	6.0715	5.2017	4.7090	4.2715	3.9766	3.8698	3.8133
		2	13.9418	11.9243	10.7657	9.7333	9.0341	8.7943	8.6736
		3	13.9431	11.9254	10.7666	9.7342	9.0349	8.7951	8.6744
		4	19.4217	17.3856	16.0843	14.5066	12.5325	11.7491	11.4433
	CCCC	1	10.3174	8.8690	8.0326	7.2616	6.6760	6.4676	6.3697
		2	19.2463	16.5991	15.0309	13.5444	12.3426	11.9260	11.7440
		3	19.2463	16.5991	15.0309	13.5444	12.3426	11.9260	11.7440
		4	26.9272	23.2664	21.0656	18.9390	17.1515	16.5428	16.2886
FGM2	SSSS	1	6.0715	5.1789	4.6783	4.2343	3.9388	3.8375	3.7844
		2	13.9418	11.8827	10.7135	9.6705	8.9652	8.7331	8.6181
		3	13.9431	11.8838	10.7145	9.6713	8.9660	8.7339	8.6189
		4	19.4217	17.4441	16.1715	14.6112	12.6210	11.8183	11.5032
	CCCC	1	10.3174	8.8495	8.0070	7.2297	6.6425	6.4391	6.3443
		2	19.2463	16.5453	14.9641	13.4641	12.2544	11.8476	11.6729
		3	19.2463	16.5453	14.9641	13.4641	12.2544	11.8476	11.6729
		4	26.9272	23.2125	21.0067	18.8675	17.0609	16.4587	16.2109

Table 6: Variation of the Frequency Parameter $\left(\bar{\omega} = \omega(a^2/h)\sqrt{\rho_c/E_c}\right)$ with the Volume Fraction Exponent φ for Spherical Al/ ZrO₂ FG shell (a/h=10)

FGM TYPE	BCC	MODE	0	0.5	1	2	5	8	10
FGM1	SSSS	1	5.8327	4.9677	4.4833	4.0657	3.8111	3.7221	3.6722
		2	13.8144	11.7932	10.6368	9.6138	8.9366	8.7077	8.5914
		3	13.9061	11.8657	10.6995	9.6649	8.9756	8.7451	8.6290
		4	19.4217	17.3856	16.0843	14.5066	12.5325	11.7491	11.4433
	CCCC	1	10.0947	8.6529	7.8247	7.0710	6.5199	6.3273	6.2356
		2	19.1529	16.4993	14.9311	13.4503	12.2648	11.8571	11.6790
		3	19.2417	16.5583	14.9768	13.4837	12.2907	11.8848	11.7083
		4	26.8790	23.1927	20.9839	18.8556	17.0801	16.4818	16.2326
FGM2	SSSS	1	5.8327	4.9580	4.4708	4.0506	3.7947	3.7075	3.6591
		2	13.8144	11.7688	10.6084	9.5791	8.8943	8.6688	8.5557
		3	13.9061	11.8635	10.7027	9.6691	8.9709	8.7370	8.6202
		4	19.4217	17.4279	16.1457	14.5784	12.5902	11.7929	11.4807

	CCCC	1	10.0947	8.6456	7.8164	7.0605	6.5067	6.3153	6.2247
		2	19.1529	16.4646	14.8911	13.4017	12.2058	11.8029	11.6292
		3	19.2417	16.5592	14.9867	13.4954	12.2892	11.8782	11.7005
		4	26.8790	23.1854	20.9903	18.8616	17.0608	16.4562	16.2066

4.2.2 Cylindrical Shell

Tables 7 demonstrates the variation of frequency parameter with the volume fraction exponent for the Al / Al₂O₃, and the geometric parameter thickness ratio (a / h= 10) and aspect ratio (a / b=1) of the FGM1 and FGM2 distributions, respectively. Table 8 shows changes in non-dimensional frequencies with the power law index. A similar trend follows for both FGM1 and FGM2 distributions of the power law. Combining aluminum with alumina (Al / Al₂O₃) reveals a higher frequency response compared to aluminum and zirconia (Al / ZrO₂.) of the cylindrical shell panels. The gradients of the material properties (E, P, and U) play significant role in approximating the shell structure's response.

Table 7: Variation of the Frequency Parameter $\left(\bar{\omega} = \omega(a^2/h)\sqrt{\rho_c/E_c}\right)$ with the Volume Fraction Exponent φ for Cylindrical Al/ Al₂O₃ FG Shell (a/h=10)

FGM TYPE	BCC	MODE	0	0.5	1	2	5	8	10
FGM1	SSSS	1	6.0715	5.4887	5.2332	5.0344	4.8586	4.7679	4.7232
		2	13.9418	12.5987	12.0049	11.5388	11.1279	10.9250	10.8263
		3	13.9431	12.5999	12.0060	11.5398	11.1289	10.9260	10.8273
		4	19.4217	17.8729	17.0067	16.0680	15.0457	14.6836	14.5489
	CCCC	1	10.3174	9.3409	8.8989	8.5354	8.2018	8.0475	7.9753
		2	19.2463	17.4564	16.6231	15.9103	15.2403	14.9510	14.8208
		3	19.2463	17.4564	16.6231	15.9103	15.2403	14.9510	14.8208
		4	26.9272	24.4452	23.2671	22.2337	21.2520	20.8474	20.6701
FGM2	SSSS	1	6.0715	5.4732	5.2122	5.0109	4.8399	4.7539	4.7113
		2	13.9418	12.5661	11.9614	11.4902	11.0889	10.8954	10.8012
		3	13.9431	12.5673	11.9625	11.4912	11.0899	10.8964	10.8022
		4	19.4217	17.9053	17.0504	16.1152	15.0818	14.7111	14.5725
	CCCC	1	10.3174	9.3264	8.8791	8.5131	8.1843	8.0346	7.9646
		2	19.2463	17.4139	16.5665	15.8474	15.1898	14.9126	14.7880
		3	19.2463	17.4139	16.5665	15.8474	15.1898	14.9126	14.7880
		4	26.9272	24.3944	23.2005	22.1598	21.1921	20.8014	20.6308

Table 8: Variation of the Frequency Parameter $\left(\bar{\omega} = \omega(a^2/h)\sqrt{\rho_c/E_c}\right)$ with the Volume Fraction Exponent φ for Cylindrical Al/ ZrO₂ FG Shell (a/h=10)

FGM TYPE	BCC	MODE	0	0.5	1	2	5	8	10
FGM1	SSSS	1	5.8327	5.2592	5.0131	4.8293	4.6739	4.5902	4.5477
		2	13.8144	12.4727	11.8828	11.4241	11.0251	10.8269	10.7298
		3	13.9061	12.5508	11.9538	11.4884	11.0857	10.8878	10.7912

FGM2	CCCC	4	19.4217	17.8729	17.0067	16.0680	15.0457	14.6836	14.5489
		1	10.0947	9.1277	8.6943	8.3438	8.0281	7.8802	7.8103
		2	19.1529	17.3619	16.5305	15.8227	15.1620	14.8768	14.7481
		3	19.2417	17.4315	16.5917	15.8777	15.2162	14.9329	14.8053
		4	26.8790	24.3836	23.2022	22.1697	21.1971	20.7982	20.6233
	SSSS	1	5.8327	5.2518	5.0033	4.8183	4.6650	4.5834	4.5420
		2	13.8144	12.4508	11.8537	11.3915	10.9987	10.8068	10.7128
		3	13.9061	12.5431	11.9443	11.4778	11.0762	10.8802	10.7846
		4	19.4217	17.8946	17.0349	16.0978	15.0683	14.7011	14.5641
	CCCC	1	10.0947	9.1209	8.6852	8.3335	8.0199	7.8741	7.8051
		2	19.1529	17.3313	16.4901	15.7776	15.1253	14.8488	14.7242
		3	19.2417	17.4234	16.5820	15.8668	15.2063	14.9248	14.7982
		4	26.879	24.362	23.1751	22.1394	21.1709	20.7775	20.6054

4.2.3. HyperboloidShell

Tables 9 and 10, show the variety of the non-dimensional frequency parameter with the volume fraction index for FGM constituents Al/Al₂O₃, Al/ZrO₂ hyperboloid shell, and the thickness proportion (a/h=10), (a/b=1), respectively. The first four modes are considered for the study. It is additionally intriguing to take note that the frequency responses show a decreasing pattern from ceramic rich to metal-rich FG structures. It is because; the ceramic materials are higher firmness rather than the metal surface. Also, Supporting conditions impact on the variation of non-dimensional frequencies. The second power-law circulation FGM2(i=1/j=0/k/φ) shows the power-law exponent increases the reaction of non-dimensional frequencies diminishes. FGM1 and FGM2 input parameters given similar qualities. It is clear that FGM1 power-law dissemination display higher frequency responses contrasted with the FGM2 responses.

Table 9: Variation of the Frequency Parameter $\left(\bar{\omega} = \omega(a^2/h)\sqrt{\rho_c/E_c}\right)$ with the Volume Fraction Exponent φ for Hyper Al/ Al₂O₃ FG shell(a/h=10)

FGM TYPE	BCC	MODE	0	0.5	1	2	5	8	10
FGM1	SSSS	1	5.7189	4.8556	4.3749	3.9653	3.7263	3.6451	3.5984
		2	13.8425	11.7850	10.6144	9.5796	8.8999	8.6782	8.5666
		3	13.8437	11.8120	10.6511	9.6232	8.9410	8.7119	8.5960
		4	19.4217	17.3856	16.0843	14.5066	12.5326	11.7491	11.4433
	CCCC	1	10.1690	8.7157	7.8811	7.1190	6.5573	6.3622	6.2698
		2	19.2201	16.5072	14.9159	13.4173	12.2298	11.8324	11.6605
		3	19.2201	16.5498	14.9739	13.4851	12.2925	11.8841	11.7062
		4	26.8607	23.1502	20.9334	18.8008	17.0301	16.4386	16.1930
FGM2	SSSS	1	5.7189	4.8583	4.3799	3.9713	3.7306	3.6478	3.6006
		2	13.8425	11.8045	10.6464	9.6166	8.9252	8.6950	8.5799
		3	13.8437	11.8218	10.6724	9.6504	8.9590	8.7229	8.6042
		4	19.4217	17.4092	16.1176	14.5435	12.5577	11.7659	11.4567
	CCCC	1	10.1690	8.7207	7.8903	7.1301	6.5646	6.3668	6.2734
		2	19.2201	16.5342	14.9605	13.4681	12.2626	11.8539	11.6776
		3	19.2201	16.5627	15.0022	13.5206	12.3147	11.8975	11.7162
		4	26.8607	23.1892	21.0045	18.8839	17.0815	16.4712	16.2184

Table 10: Variation of the Frequency Parameter $\left(\bar{\omega} = \omega(a^2/h)\sqrt{\rho_c/E_c}\right)$ with the Volume Fraction Exponent φ for Hyper Al/ ZrO₂ FG shell (a/h=10)

FGM TYPE	BCC	MODE	0	0.5	1	2	5	8	10
FGM1	SSSS	1	5.7189	5.1489	4.9065	4.7285	4.5818	4.5017	4.4606
		2	13.8425	12.4785	11.8795	11.4154	11.0206	10.8281	10.7338
		3	13.8437	12.4950	11.9017	11.4399	11.0397	10.8423	10.7458
		4	19.4217	17.8729	17.0067	16.0680	15.0457	14.6836	14.5489
	CCCC	1	10.1690	9.1933	8.7550	8.3991	8.0794	7.9311	7.8612
		2	19.2201	17.3930	16.5476	15.8317	15.1771	14.8996	14.7747
		3	19.2201	17.4180	16.5814	15.8690	15.2060	14.9211	14.7928
		4	26.8607	24.3515	23.1654	22.1313	21.1640	20.7697	20.5971
FGM2	SSSS	1	5.7189	5.1494	4.9074	4.7295	4.5824	4.5020	4.4608
		2	13.8425	12.4837	11.8869	11.4231	11.0257	10.8316	10.7368
		3	13.8437	12.4954	11.9038	11.4425	11.0406	10.8423	10.7455
		4	19.4217	17.8814	17.0170	16.0782	15.0529	14.6891	14.5538
	CCCC	1	10.1690	9.1943	8.7566	8.4009	8.0804	7.9317	7.8617
		2	19.2201	17.3992	16.5569	15.8413	15.1829	14.9034	14.7777
		3	19.2201	17.4194	16.5848	15.8729	15.2078	14.9219	14.7933
		4	26.8607	24.3588	23.1775	22.1442	21.1714	20.7742	20.6004

4.2.4. Elliptical Shell

Tables 11 and 12 show the variation of the non-dimensional frequency parameter with the volume fraction exponent for the Al / Al₂O₃, Al / ZrO₂ elliptical shell, and the thickness ratio (a/h=10) and (a/b=1), respectively. For the shell with SSSS, CCCC boundary condition, the volume fraction exponent φ decreases when $\varphi=0$ (ceramic) and ($\varphi=\infty$) for metal. The second power law distribution FGM2(i=1/j=0/k/ φ) shows the power law index increases the response of non-dimensional frequency decreases. Clamped boundary conditions exhibit higher frequency responses compared to the simply supported conditions.

Table 11: Variation of the Frequency Parameter $\left(\bar{\omega} = \omega(a^2/h)\sqrt{\rho_c/E_c}\right)$ with the Volume Fraction Exponent φ for Elliptical Al/ Al₂O₃ FG shell (a/h=10)

FGM TYPE	BCC	MODE	0	0.5	1	2	5	8	10
FGM1	SSSS	1	5.9374	5.0704	4.5825	4.1563	3.8844	3.7876	3.7348
		2	13.8592	11.8406	10.6838	9.6579	8.9730	8.7400	8.6220
		3	13.9274	11.8978	10.7350	9.7013	9.0071	8.7720	8.6535
		4	19.4217	17.3856	16.0843	14.5066	12.5325	11.7491	11.4433
	CCCC	1	10.1696	8.7267	7.8963	7.1371	6.5742	6.3760	6.2819
		2	19.1796	16.5304	14.9632	13.4813	12.2905	11.8796	11.7000
		3	19.2461	16.5801	15.0048	13.5148	12.3179	11.9069	11.7277
		4	26.8994	23.2257	21.0209	18.8938	17.1132	16.5101	16.2586
FGM2	SSSS	1	5.9374	5.0543	4.5610	4.1302	3.8573	3.7642	3.7139
		2	13.8592	11.8076	10.6436	9.6092	8.9175	8.6900	8.5765
		3	13.9274	11.8759	10.7105	9.6719	8.9702	8.7373	8.6214

		4	19.4217	17.4372	16.1597	14.5958	12.6064	11.8064	11.4927
	CCCC	1	10.1696	8.7134	7.8793	7.1159	6.5508	6.3557	6.2638
		2	19.1796	16.4862	14.9099	13.4169	12.2170	11.8134	11.6396
		3	19.2461	16.5535	14.9763	13.4804	12.2729	11.8642	11.6881
		4	26.8994	23.1951	20.9946	18.8610	17.0582	16.4553	16.2067

Table 12: Variation of the Frequency Parameter $\left(\bar{\omega} = \omega(a^2/h)\sqrt{\rho_c/E_c}\right)$ with the Volume Fraction Exponent φ for Elliptical Al/ ZrO₂ FG Shell (a/h=10)

FGM TYPE	BCC	MODE	0	0.5	1	2	5	8	10
FGM1	SSSS	1	5.9374	5.3599	5.1099	4.9196	4.7554	4.6685	4.6250
		2	13.8592	12.5178	11.9267	11.4657	11.0623	10.8622	10.7645
		3	13.9274	12.5779	11.9823	11.5165	11.1097	10.9093	10.8116
		4	19.4217	17.8729	17.0067	16.0680	15.0457	14.6836	14.5489
	CCCC	1	10.1696	9.2001	8.7641	8.4092	8.0873	7.9370	7.8663
		2	19.1796	17.3904	16.5589	15.8498	15.1859	14.8991	14.7699
		3	19.2461	17.4457	16.6091	15.8957	15.2302	14.9439	14.8150
		4	26.8994	24.4107	23.2312	22.1986	21.2219	20.8202	20.6442
FGM2	SSSS	1	5.9374	5.3485	5.0945	4.9024	4.7416	4.6582	4.6163
		2	13.8592	12.4906	11.8905	11.4251	11.0296	10.8374	10.7435
		3	13.9274	12.5577	11.9557	11.4868	11.0854	10.8906	10.7957
		4	19.4217	17.9012	17.0439	16.1076	15.0761	14.7071	14.5693
	CCCC	1	10.1696	9.1895	8.7496	8.3930	8.0745	7.9276	7.8583
		2	19.1796	17.3539	16.5105	15.7959	15.1424	14.8659	14.7416
		3	19.2461	17.4203	16.5758	15.8587	15.1999	14.9206	14.7950
		4	26.8994	24.3746	23.1843	22.1464	21.1788	20.7869	20.6155

5. CONCLUSIONS

Linear vibration analysis of the static evaluation of the rectangular FG double curved panel has been investigated using MATLAB16a programming configuration. Governing equation structures are derived by the process of the variation principle. The higher order mathematical model uses TSDT kinematics and a combination of the finite element form. Finally, the relevance of the current model is established by solving various types of numerical case studies for different materials, power law index, boundary conditions, and geometric configurations.

- The frequency parameters of the clamped boundary conditions produced the highest frequency response relative to other general boundary conditions.
- The two power-law dispersions on the curved shell panels demonstrate the vibration characteristics of the structure, and the vibration response of the non-dimensional frequencies is distinct.
- As the power-law index is increased, the frequency responses are following the trend in the reverse directions for all the FG shell structures.
- Shell with cylindrical structure has a maximum frequency and minimum is found for an FG plate
- Ceramic to metal-rich FG surfaces structure has a higher frequency parameter compared to metal-rich to ceramic

FG structure. The combination of material distribution parameters i/j , k and ϕ determines to characterize the volume fraction profiles. Nevertheless, with the rise in the power-law index, the frequency reactions fall apart.

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