MODELLING, ANALYSIS AND SIMULATION FOR A 6 AXIS ARM ROBOT BY PID CONTROLLER

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ABSTRACT

This paper presents a simulation analysis on kinematics and control of a robot arm using a Proportional integral derivative (PID) controller. Robotic arms are from a series of lightweight, fast, easy to program, flexible, and safe robotic arms with 6 degrees of freedom. Moreover, a kinematics modelling of the robot arm which is very important for any application has been presented. Furthermore, the robot arm is simulated and analysis using Matlab/Simulink from the mechanical model in Solid Works. Simulation results will help the designers and engineers to evaluate the chosen and designed model.

KEYWORDS: Robot Arm, Simulation, Modelling & PID Control

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1. INTRODUCTION

Robots are used in a wide range of industrial applications and robotics technology is likely to become the high technology field of the 21st century [1, 2]. Robots were initially applied to jobs that were hot, heavy and hazardous such as die casting, forging, and spot welding. 6-axis robots can be widely used in carrying, assembly, welding, spraying, winding, motor assembly and other industrial sites which alleviate the current shortage of industrial workers and can dramatically reduce the quality of workers because of differences in the impact on product quality [3].

A six revolute joint robot configuration with 6 D. O. F is generally well suited for small parts insertion and assembly, like electronic components. Although the final goal is to design and manufacture real robotics, it is very useful to perform simulations prior to investigations with real robots. Simulations are easier to setup, less expensive, faster and more convenient to use. It allows better design exploration and helps you enhance your final real robot by selecting suitable parameters for the system you want to design [4]. There are many control techniques used to control a robot arm. The most used ones are the PID control, optimal control, adaptive control and robust control. "There are many kinds of controllers that can be used to cause a designed robot arm to move along a desired trajectory" [5]. The simplest which we used in this paper to control the robot arm is the PID controller.

Ata et al. [6] show a model a two-link flexible manipulator by Bernoulli-Euler theory. A dynamic modelling technique has been addressed by Subudhi and Morris [7] developed multiple flexible link-joint systems using Euler-Lagrange formulation. Swern & Tricamo [8] designed a control algorithm that eliminates the constraint

forces supplying a position correction to each arm. A control model has also been proposed by Khemaissia [9] introduced goal-directed multi joint-arm movement using a hybrid neuro-genetic algorithm. Colbaugh et al [10] showed a strategy aims at adaptively generating the position set points for the standard robot controllers, without requiring knowledge of both manipulator and payload model parameters and dynamics. Some more researchers also have studied the dynamic formulation of flexible manipulators using either Lagrangian formulation [11-14] or Euler formulations [15]. However, going through the literature it has been observed that validation of the analytic models for the flexible manipulators is difficult and often becomes impossible. Moreover, control of such manipulators is a complex problem. Quite a few researchers have tried to control flexible manipulators using different control algorithms such as Proportional Derivative Control Law [16], Robust Control Law [17], Neuro-Fuzzy-based control law [18], etc. However, there are some inherent problems existing with those control laws. In this paper, the 6 DOF robot arm has been studied. The direct and inverse kinematics has been obtained by calculation [19]. Then the dynamic model of the system has been extracted by the Lagrange method that is very powerful in modeling the developed mechanical systems. In this paper, a PID controller has been designed for 6 DOF robotic which give the simple and fast way to control the robot arm.

2. METHODOLOGY

2.1 System Modelling

In accomplishing this paper, the scope of the work has been divided into a few parts. The first part is to design CAD model of 6 DOF robot arm. The second part is to design the PID controller for the robot arm for comparison purpose. The third part is to perform simulation using MATLAB/Simulink. The last part is to analyse the output performance of both controllers. The research methodology flow chart is shown in Figure 1.

Figure 1: Flow chart for Design of Robotic Arm

2.2 Design of Arm Robotic

First each part of robotic arm designed in solid works and these whole parts are assembled as per our requirement. The names of the parts are base, link 1, link 2, link 3 and link 4. As shown in below Figure 2.

Figure 2: Design of Base, Link 1, Link 2, Link 3, link 4 and Robotics Arm in Solid Works

2.3 Kinematic Equations of Robot Arm

Kinematics is the relationships between the positions, velocities, and accelerations of the links of a manipulator, where a manipulator is an arm. Robot kinematics are two types these are forward kinematics and inverse kinematics; here forward kinematics is used. We need to consider the transformation relations between joint n and $n+1$ that are in the two adjacent coordinate systems while defining the 4 important parameters by the D-H method

- a_i = distance from Z_i to Z_{i+1} measured along X_i
- α_i = angle from Z_i to Z_{i+1} measured along X_i
- d_i = distance from X_{i-1} to X_i measured along Z_i
- θ_i = angle from X_{i-1} to X_i measured along Z_i

Figure 3: Denavit - Hartenberg General Frame Assignment

2.3.1 Forward Transformation Matrices

Finding the link parameters and joint parameters of the robotic arm, based on that parameters calculate the transformation matrices of each joint present in the robotic arm. After getting these matrices and add these matrices based on the Denavit-Hartenberg parameters. Link transformation matrices for robotic arm

- Rotate the frame $X_{i-1}Y_{i-1}Z_{i-1}$ about the Z_{i-1} axis by an angle θ_i . After this rotation, the current X_{i-1} axis will be parallel to the X_i axis.
- Translate the frame $X_{i-1}Y_{i-1}Z_{i-1}$ along the Z_{i-1} axis by d_i units so that the current X_{i-1} axis will be aligned with the X_i axis.
- Translate the frame $X_{i-1}Y_{i-1}Z_{i-1}$ along the X_i axis by a_{i-1} units such that the current X_{i-1} axis will coincide with the X_i axis.
- Rotate the frame $X_{i-1}Y_{i-1}Z_{i-1}$ about the X_i axis by an angle α_i . After that, the current frame $X_{i-1}Y_{i-1}Z_{i-1}$ coincides with the frame $X_iY_iZ_i$.

Table 1: The Transformation Matrix

#	Θ	D	a	α
1	Θ_1	0	a_0	90°
2	Θ_2	0	a_1	0
3	Θ_3	0	a_2	90°
4	Θ_4	d_3	0	-90°
5	Θ_5	0	0	90°
6	Θ_6	d_4	0	0

Based on table 1, the transformation matrix of every joint is expressed as

$${}^0A_1 = \begin{bmatrix} C_1 & 0 & S_1 & a_0C_1 \\ S_1 & 0 & -C_1 & a_0S_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & d_3C_2 \\ S_2 & C_2 & 0 & d_3S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}_{2A3} = \begin{bmatrix} C3 & 0 & S3 & a2C3 \\ S3 & 0 & -C3 & a2S3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}_{3A4} = \begin{bmatrix} C4 & 0 & -S4 & 0 \\ S4 & 0 & C4 & 0 \\ 0 & -1 & 0 & d3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}_{4A5} = \begin{bmatrix} C5 & 0 & -S5 & 0 \\ S5 & 0 & C5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}_{5A6} = \begin{bmatrix} C6 & -S6 & 0 & 0 \\ S6 & C6 & 0 & 0 \\ 0 & 0 & 0 & d4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where:

S1: Sin Θ_1

C1: Cos Θ_1

The transformation matrix from robot base to robot actuator 0A6 is expressed as.

$${}_{0A6} = {}_{0A1} {}_{1A2} {}_{2A3} {}_{3A4} {}_{4A5} {}_{5A6} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$n_x = C1[C23(C4C5C6 - S4S6) - S23S5C6] + S1(S4C5C6 + C4S6)$$

$$n_y = S1[C23(C4C5C6 - S4S6) - S23S5C6] - C1(S4C5C6 + C4S6)$$

$$n_z = S23(C4C5C6 - S4S6) + C23S5C6$$

$$o_x = C1[-C23(C4C5S6 + S4C6) + S23S5S6] + S1(-S4C5S6 + C4C6)$$

$$o_y = -S23[-C23(C4C5S6 + S4C6) + S23S5S6] - C1(-S4C5S6 + C4C6)$$

$$o_z = -S23(C4C5S6 + S4C6) - C23S5S6$$

$$a_x = C1(C23C4S5 + S23C5) + S1S4S5$$

$$a_y = S1(C23C4S5 + S23C5) - C1S4S5$$

$$a_z = S23C4S5 - C23C5$$

$$p_x = C1[a_0 + a_1C2 + a_2C23 + d3S23 + l4(C23C4S5 + S23C5)] + d4S1S4S5$$

$$p_y = S1[a_0 + a_1C2 + a_2C23 + d3S23 + d4(C23C4S5 + S23C5)] - d4C1S4S5$$

$$p_z = a_1S2 + a_2S23 - d3C23 + d4(S23C4S5 - C23C5).$$

2.3.2 Inverse Kinematics

The joints of 6-axis robot as shown in Figure 2, the position of sixth arm terminal is and the position of fifth joint is

$${}^0A_4 = {}^0A_1 {}^1A_2 {}^2A_3 {}^3A_4 = \begin{bmatrix} n'_x & o'_x & a'_x & Q_x \\ n'_y & o'_y & a'_y & Q_y \\ n'_z & o'_z & a'_z & Q_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Based on equation (1) and (2), the relationship between P_x, P_y, P_z and Q_x, Q_y, Q_z can be expressed as:

$$\begin{cases} Q_x = P_x - d_4 a_x \\ Q_y = P_y - d_4 a_y \\ Q_z = P_z - d_4 a_z \end{cases}$$

- θ_1

$${}^0A_1^{-1} \begin{bmatrix} n'_x & o'_x & a'_x & Q_x \\ n'_y & o'_y & a'_y & Q_y \\ n'_z & o'_z & a'_z & Q_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^1A_2 {}^2A_3 {}^3A_4$$

Based on element (3,4), θ_1 can be expressed as below

$$\theta_1 = \arctan\left(\frac{Q_y}{Q_x}\right) = \arctan\left(\frac{P_y - d_4 a_y}{P_x - d_4 a_x}\right) \text{ or } \theta_1 = \pi + \arctan\left(\frac{Q_y}{Q_x}\right) = \pi + \arctan\left(\frac{P_y - d_4 a_y}{P_x - d_4 a_x}\right)$$

- θ_3

$${}^0A_2^{-1} {}^0A_1^{-1} \begin{bmatrix} n'_x & o'_x & a'_x & Q_x \\ n'_y & o'_y & a'_y & Q_y \\ n'_z & o'_z & a'_z & Q_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^2A_3 {}^3A_4$$

Based on element (1,4) and (2,4), the following equations are obtained.

$$\begin{cases} C_2 C_1 Q_x + C_2 S_1 Q_y + S_2 Q_z - C_2 a_0 = d_3 S_3 + a_2 C_3 + a_1 \\ -S_2 C_1 Q_x - S_2 S_1 Q_y + C_2 Q_z + S_2 a_0 = -d_3 C_3 + a_2 S_3 \end{cases}$$

So, θ_3 can be expressed as

$$\theta_3 = 2 \tan^{-1} \left(\frac{K_1 \pm \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_2 + K_3} \right)$$

$$\begin{cases} K_1 = 2a_1d_3 \\ K_2 = 2a_1a_3 \\ K_3 = Q_x + Q_y + Q_z - 2Q_xa_0C_1 - 2Q_ya_0S_1 + a_0^2 - a_1^2 - a_2^2 - d_3^2 \end{cases}$$

Based on equations (3), θ_2 can be expressed as below.

$$\theta_2 = a \tan 2(S_2, C_2) \begin{cases} C_2\mu_1 + S_2\nu_1 = \gamma_1 \\ C_2\mu_2 + S_2\nu_2 = \gamma_2 \end{cases}$$

$$\begin{cases} \mu_1 = C_1Q_x + S_1Q_y - a_0 \\ \nu_1 = Q_z \\ \gamma_1 = d_3S_3 + a_2C_3 + a_1 \\ \mu_2 = Q_z \\ \nu_2 = -C_1Q_x - S_1Q_y + a_0 \\ \gamma_2 = -d_3C_3 + a_2S_3 \end{cases}$$

- $\theta_5 =$

$${}^0A_1^{-1} {}^1A_2^{-1} {}^2A_3^{-1} \begin{bmatrix} n'_x & o'_x & a'_x & p'_x \\ n'_y & o'_y & a'_y & p'_y \\ n'_z & o'_z & a'_z & p'_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= {}^3A_4 {}^4A_5 {}^5A_6 \begin{bmatrix} \dots & \dots & C_4C_5 & \dots \\ \dots & \dots & S_4S_5 & \dots \\ \dots & \dots & C_5 & \dots \\ \dots & \dots & 0 & 1 \end{bmatrix}$$

Based on element (3,3), θ_5 can be expressed as $\theta_5 = \arccos(a_xC_1S_{23} + a_yS_1S_{23} - a_zC_{23})$

- θ_4

Base on element(1,3) and (2,3) of equation (4), θ_4 can be expressed as below.

$$\theta_4 = a \tan 2(S_4, C_4)$$

$$C_4 = \frac{a_xC_1C_{23} + a_yS_1C_{23} + a_zS_{23}}{S_5}$$

$$S_4 = \frac{a_xS_1 - a_yC_1}{S_5}$$

- $\theta_6 =$

$${}^3A_4^{-1} {}^2A_3^{-1} {}^1A_2^{-1} {}^0A_1^{-1} \begin{bmatrix} n'_x & o'_x & a'_x & p'_x \\ n'_y & o'_y & a'_y & p'_y \\ n'_z & o'_z & a'_z & p'_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^4A_5 {}^5A_6$$

$$\begin{bmatrix} C_5 C_6 & -C_5 S_6 & S_5 & d_4 S_5 \\ S_5 C_6 & -S_5 S_6 & -C_5 & d_4 C_5 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Base on element(2,1) and (2,2) of equation(5), θ_6 can be expressed as below.

$$\theta_6 = a \tan 2(S_6, C_6)$$

$$C_6 = -\frac{n_x c_1 C_{23} + n_y S_1 S_{23} + n_z C_{23}}{S_5}$$

$$S_6 = \frac{o_x C_1 S_{23} + o_y S_1 S_{23} + o_z C_{23}}{S_5}$$

2.4 Dynamics for Arm Robotics

The arm robotics' dynamic equations have the general form of:

$$M(\underline{q}) \ddot{\underline{q}} + C(\underline{q}, \dot{\underline{q}}) \dot{\underline{q}} + \underline{g}(\underline{q}) = \underline{u}$$

Where :

$M(\underline{q})$: is the symmetric positive definite mass inertia matrix of the system

$C(\underline{q}, \dot{\underline{q}})$: is the matrix of Coriolis and centrifugal terms

$\underline{g}(\underline{q})$: is the vector of gravity terms and $\underline{u}$ is the input vector

The inverse dynamic has the form:

$$\ddot{\underline{q}} = M^{-1}(\underline{q}) (\underline{u} - C(\underline{q}, \dot{\underline{q}}) \dot{\underline{q}} - \underline{g}(\underline{q}))$$

The matrix $M(\underline{q})$ would be simply calculated as [19]

$$M(\underline{q}) = [\sum_{i=1}^n (m_i J_{v_i}^T J_{v_i} + J_{w_i}^T R_i I_i R_i^T J_{w_i})]$$

Where:

J_{v_i}, J_{w_i} : are the linear and angular part of the Jacobian matrix J_i , respectively.

For deriving the matrix $C(\underline{q}, \dot{\underline{q}})$ it would be useful to know the passivity property of robotic manipulators which is the result of the skew-symmetry property of the matrix $M(\underline{q}) - 2C(\underline{q}, \dot{\underline{q}})$. For reaching this property the elements of the matrix C_{ij} must be derived from the elements of the inertia matrix m_{ij} via the following formula [19],

$$C_{ij} = \sum_{k=1}^n \frac{1}{2} \left(\frac{\partial m_{ij}}{\partial q_k} + \frac{\partial m_{ik}}{\partial q_j} - \frac{\partial m_{kj}}{\partial q_i} \right) \dot{q}_k$$

Finally, the elements of the gravity vector $g_i(\underline{q})$ [19],

$$g_i(\underline{q}) = \frac{\partial P}{\partial q_i}$$

Having $M(\underline{q})$, $C(\underline{q}, \dot{\underline{q}})$ and $g_i(\underline{q})$ completes the dynamical model development.

3. SIMULATION OF THE ROBOT ARM

3.1 Design Controller Circuit for Robotics Arm

By study the mathematical models of the kinematics and dynamics in the previous section have been used to develop the Matlab model for the robot here. By CAD export robotic arm converted into simulink circuit block in MATLAB/Simulink as shown in below figure.

Figure 4: Basic Simulink Model of Robotic Arm

3.1.1 Design PID Control for Robotics Arm

PID (Proportional Integrate Derivative) controller is considered the most widely used control technique in control applications. A high number of applications and control engineers had used the PID controller in daily life. On the other hand, many research papers, number of master and doctoral theses and books had highlighted this subject. PID control offers an easy method of controlling a process by varying its parameters. PID works well in industrial applications such as in slow industrial manipulators where large components of joint inertia are added by actuators. Since the invention of PID control in 1910, and Ziegler-Nichols' (ZN) tuning method in 1942 [20] and [21], PID controllers became one of the dominant and popular issues in control theory due to the simplicity of implementation, the ease of designing, and the ability to be used in a wide range of applications [22]. Moreover, they are available at low cost. Finally, it provides robust and reliable performance for most systems if the parameters are tuned properly. However, the PID controller has its own limitation; the PID performances can give only satisfactory performance if the requirement is reasonable and the process parameters variation are limited.

3.2 Circuit Model of Robotic Arm

The robotic arm a controller block required. So attach the controller blocks and reference blocks to that robotic arm by MATLAB/Simulink as shown in below figure 5.

In this circuit model contains reference angle trajectory, reference angle test vector, controller, robotic arm blocks and finally scope are presented. In this reference angle trajectory, reference angle test vector block are used to given the joint angles to the robotic arm. Controller block controls the dc motors present in the robotic arm for this purpose we use PID (Proportional Integrate Derivative) controllers. In this PID controller gives an error of ± 30 error (Joint angle error). Based on that robotic arm moves and are as shown in following below Figure 6 to Figure 9.

Figure 5: Circuit Model of Robotic Arm

Figure 6: Reference Tracking

Figure 7: Controller Effort

Figure 8: Input Disturbance Rejection

Figure 9: Output Disturbance Rejection

4. SIMULATION RESULTS AND DISCUSSIONS

Figure 10: Robotic Arm in Simulation

Comparison of the Angluer velocity: The control input, forces the robotic manipulators to track the desired trajectories. Figure 11 shows the torque performance in PID controller. According to the following graph, PID controller has steady stable torque performance in second, third and fourth joints but it has oscillation in the first joint. In the control

forces, smaller amplitude means less energy. According to Figure 11, the amplitude of the control forces in first joint of PID controller is much larger than the other joints.

Figure 11: Simulation Result Waveforms of Robotic Arm

5. CONCLUSIONS

The six-axis Robot arm in MATLAB/SIMULINK is modelled based computer program, which can be used to simulate the functions of a real robotic manipulator in terms of design parameters, movement, and control. In this paper, after extract the dynamic and kinematics formulations, these formulations are model and implemented by MATLAB/SIMULINK. After that, a SimMechanics model of robotics arm was developed with the parameter values obtained from the manufacturer material. At the end, a cross-validation of the models was performed by using the mathematical model for the control of Simmechanics model. Through simulation, the accuracy of the mathematical model was explained.

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