AN ANALYTICAL SOLUTION TO MHD FLOW AND HEAT TRANSFER OF VISCO-ELASTIC FLUID OVER AN EXPONENTIALLY STRETCHING SHEET THROUGH POROUS MEDIA WITH NON-UNIFORM HEAT SOURCE/SINK

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ABSTRACT

This paper deals with the study of boundary layer flow and heat transfer characteristics of an electrically conducting visco-elastic fluid in the porous medium over an exponentially stretching sheet with internal heat generation/absorption. The governing partial differential equations are converted into ordinary differential equations by using the suitable similarity transformations and these equations are solved analytically by using Homotopy Analysis Method (HAM). The analytical solutions are obtained in the form of infinite series and the convergence of the series solution is explicitly discussed. The effect of visco-elastic parameter, porosity parameter, Magnetic parameter, suction/blowing parameter, non-uniform heat source/sink parameter, Prandtl number, Eckert number on velocity and temperature are discussed through graphs. The numerical values of the skin friction coefficient and Nusselt number for different parameters are presented in the tabular form.

KEYWORDS: Visco-Elastic Fluid, MHD, Exponentially Stretching Sheet, Porous Medium, Non-Uniform Heat Source/Sink & HAM

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INTRODUCTION

The study of flow and heat transfer in porous medium has a wide range of applications in the industry, this leads to pay much attention for the researchers to deal with such problems. In many engineering and manufacturing processes, the boundary layer behaviour on a continuous moving solid surface has many applications. Sakiadis [1] was the first to develop the boundary layer behaviour on a continuous solid surface moving with a constant speed. This analysis is reviewed by Tsou et al. [2] to study the flow and heat transfer on a continuous moving surface both analytically and experimentally. Kumaran et al. [3] studied the boundary layer flow phenomena of a viscous fluid by assuming quadratic stretching sheet. Erickson et al. [4]. Investigated the heat and mass transfer phenomena with suction or injection over a stretching surface.

The study of MHD flow of visco-elastic fluids in the presence of magnetic field has many applications in polymer technology and metallurgy. In these processes, the rate of cooling can be controlled by drawing the continuous strips or filaments through a quiescent fluid and in the process of drawing, sometimes these strips are stretched. By drawing these strips in an electrically conducting fluid, the rate of cooling can be controlled and the final product with desired quality can be obtained. Initially, Sarpakaya [5] investigated the non-Newtonian fluid

flow in a magnetic field. Chakrabarti et al. [6] studied the flow and heat transfer of an electrically conducting fluid subject to magnetic field. Vajravelu et al.[7] analyzed the flow and heat transfer characteristics of viscous fluid over a stretching sheet with internal heat generation/absorption in the presence of a magnetic field. Besides that, Cortell [8] examined the heat transfer characteristics of an electrically conducting second grade fluid subjected to suction over a stretching sheet. Hymavathi et al.[9] studied the heat transfer of an electrically conducting visco-elastic fluid flow over an isothermal stretching sheet.

However, all the above work is carried out without considering the porous medium. The heat transfer in porous medium has generated the increasing interest in the past several decades owing to the importance of porous media in diverse fields such as chemical engineering, geophysics, etc. In view of this, R.K. Gupta et al.[10] analyzed the visco-elastic effects in non-Newtonian fluid flow through porous medium. Vajravelu [11] studied flow and heat transfer characteristics of a viscous fluid in a saturated porous medium with internal heat generation/absorption. Later Cortell [12] extended the previous work with suction/blowing. Abeletal. [13] analyzed the effect of non-uniform heat source/sink on heat transfer of MHD visco-elastic fluid flow in porous medium. Ali [14] discussed the flow and heat transfer characteristic of a continuous stretching surface with suction or injection. M. S. Abel et al. [15] discussed the heat transfer effects of second grade fluid with viscous dissipation and non-uniform heat source/sink through porous medium.

Initially, Homotopy Analysis Method (HAM) was introduced by Shijun Liao [16] which is powerful and easy-to-use technique for linear problems. Liao [17] applied Homotopy Analysis Method to the magneto hydrodynamic viscous flows over a stretching sheet. Abdallah [18] derived an analytic solution for transferring heat in a MHD viscous fluid over a stretching sheet by using the Homotopy Analysis Method.

Elbashbeshy [19] added a new dimension to the stretching surface to analyze the flow and heat transfer characteristics over an exponentially stretching continuous surface. Sujith Kumar Khan et al. [20] extended the work of [19] to viscoelastic fluid flow and heat transfer. Bidin and Nazar [21] investigated the effect of thermal radiation on flow and heat transfer over an exponentially stretching sheet. Iswar Chandra Mandaletal. [22] studied the boundary layer flow and heat transfer of an exponentially stretching sheet through porous medium with variable surface heat flux. Hymavathi Talla et.al[23] applied Homotopy Analysis Method to analyze the flow and heat transfer characteristics of a visco-elastic fluid over an exponentially stretching sheet through porous medium.

The aim of the present paper is to analyze the MHD flow and heat transfer of visco-elastic fluid over an exponentially stretching sheet through porous medium with non-uniform heat source/sink. The governing partial differential equations are transformed to set of ordinary differential equations and are solved analytically using the Homotopy analysis method. The influence of various parameters like Magnetic parameter (M), suction parameter (S), space dependent parameter (A^*), temperature dependent parameter (B^*), Prandtl number (Pr), Eckert number (Ec), etc. on the velocity and temperature profiles are interpreted through graphs.

MATHEMATICAL FORMULATION

Consider the steady, two dimensional electrically conducting visco-elastic fluid over an exponentially stretching sheet in a porous medium. The flow being confined to $y > 0$. The flow is generated due to stretching of the sheet from a slit with a force such that the velocity of the boundary sheet is an exponential order of the flow along x-axis, y-axis is normal to it. A uniform magnetic field of strength B_0 is applied normally to the stretching surface. The governing boundary layer

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\alpha_1}{\rho} \left\{ u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right\} - \frac{\nu}{k^*} u - \frac{\sigma B_0^2 u}{\rho} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\nu u^2}{k^* C_p} \right) + \frac{q'''}{\rho C_p} \quad (3)$$

Where u and v are the velocities in x and y directions respectively, T is the temperature of the fluid, ρ is the density of the fluid, k^* is the permeable electrical conductivity of porous medium, σ is electrical conductivity of the fluid, C_p is the specific heat at constant pressure, α is the thermal diffusivity, q_r is the radiative heat flux and q''' is the rate of internal heat generation/absorption coefficient which can be expressed as

$$q''' = \left(\frac{k u_w(x)}{e^{x/2l}} \right) [A^* (T_w - T_\infty) f'(\eta) + B^* (T - T_\infty)]$$

Where A^* and B^* are parameters of space and temperature dependent internal heat generation/absorption. The case $A^* > 0$ and $B^* > 0$ corresponds to internal heat generation and $A^* < 0$ and $B^* < 0$ corresponds to internal heat absorption.

The boundary conditions under the consideration is given by

$$\begin{aligned} u &= U_w(x) = U_0 e^{x/l}, \quad v = -v_0, \quad T = T_w & \text{at } y = 0 \\ u &= 0, \quad u_y = 0, \quad T = T_\infty & \text{as } y \rightarrow \infty \end{aligned} \quad (4)$$

Where U_0, l are the reference velocity and the reference length respectively. T_∞ is the temperature far away from the stretching sheet.

Equation of continuity (1) is identically satisfied if we choose the stream function ψ such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (5)$$

The momentum and energy equations can be transformed into the corresponding ordinary differential equations by introducing the similarity transformations:

$$\eta = y \sqrt{\frac{U_0}{2\nu l}} e^{x/2l} \quad (6)$$

$$\psi(x, y) = \sqrt{2\nu l U_0} f(x, \eta) e^{x/2l}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (7)$$

Where η is the similarity variable, f is the dimensionless stream function, $\theta(\eta)$ be the dimensionless temperature.

The momentum and energy equations are transformed to

$$f''' - 2f'^2 + ff'' - k_1 \left\{ 3ff''' - \frac{1}{2}ff^{iv} - \frac{3}{2}f''^2 \right\} - (M + k_2)f' = 0 \quad (8)$$

$$\theta'' + \text{Pr} f \theta' + \text{Ec} \text{Pr} f'^2 + k_2 \text{Pr} \text{Ec} f'^2 + [A^* f' + B^* \theta] = 0 \quad (9)$$

with the boundary conditions

$$\begin{aligned} f &= S, & f' &= 1, & \theta &= 1 & \text{at } \eta &= 0 \\ f' &= 0, & f'' &= 0 & \theta &= 0 & \text{as } \eta &\rightarrow \infty \end{aligned} \quad (10)$$

Where $k_1 = \frac{\alpha_1 U_w}{\rho \nu l}$ is the dimensionless visco-elastic parameter, $k_2 = \frac{2\nu l}{k^* U_w}$ is the porosity parameter,

$M = \frac{2\sigma B_0^2 l}{\rho U_0 e^{x/l}}$ is the magnetic parameter, $\text{Ec} = \frac{u_w^2}{c_p (T - T_\infty)}$ is the Eckert number, $S = \frac{\nu_0}{u_w \sqrt{2U_0 l}}$ is the suction

parameter and $\text{Pr} = \frac{\nu}{\alpha}$ is the Prandtl number.

HOMOTOPY ANALYSIS SOLUTION

In this section, we employ HAM to solve the equation (8) and (9) subjected to the boundary conditions (10). Let us choose the initial guesses f_0, θ_0 as

$$f_0(\eta) = S + 1 - e^{-\eta} \quad (11)$$

$$\theta_0(\eta) = e^{-\eta} \quad (12)$$

The linear operators are selected as

$$L(f) = f''' - f' \quad (13)$$

$$L(\theta) = \theta'' - \theta \quad (14)$$

Which has the following property

$$L_f[C_1 + C_2 e^\eta + C_3 e^{-\eta}] = 0 \quad (15)$$

$$L_\theta[C_4 e^\eta + C_5 e^{-\eta}] = 0 \quad (16)$$

Where $C_i (i = 1, 2, 3, 4, 5)$ are the arbitrary constants

If $p \in [0, 1]$ is the embedding parameter, h is the non-zero auxiliary parameter and $H(\eta)$ is the auxiliary function, then we construct the following zero-the ordinary deformation equation:

$$(1-p)L(\tilde{f}(\eta; p) - f_0(\eta)) = p\hbar_1 H_1(\eta) N_1[\tilde{f}(\eta; p)] \quad (17)$$

$$(1-p)L(\tilde{\theta}(\eta; p) - \theta_0(\eta)) = p\hbar_2 H_2(\eta) N_2[\tilde{f}(\eta; p), \tilde{\theta}(\eta; p)] \quad (18)$$

Subject to the boundary conditions

$$\tilde{f}(0; p) = 0, \quad \tilde{f}'(0; p) = 1 \quad \tilde{f}'(\infty; p) = 0$$

$$\tilde{f}''(\infty; p) = 0, \quad \tilde{\theta}(0; p) = 1, \quad \tilde{\theta}'(\infty; p) = 0 \quad (19)$$

We define non linear operator as

$$N_1(\tilde{f}(\eta; p)) = \frac{\partial^3 f}{\partial \eta^3} - 2\left(\frac{\partial f}{\partial \eta}\right)^2 + \frac{\partial^2 f}{\partial \eta^2} f - k_1 \left(3\frac{\partial f}{\partial \eta} \frac{\partial^3 f}{\partial \eta^3} - \frac{1}{2} f \frac{\partial^4 f}{\partial \eta^4} - \frac{3}{2} \left(\frac{\partial^2 f}{\partial \eta^2}\right)^2 \right) - (k_2 + M) \frac{\partial f}{\partial \eta} \quad (20)$$

$$N_2(\tilde{f}(\eta; p), \tilde{\theta}(\eta; p)) = \frac{\partial^2 \theta}{\partial \eta^2} + \text{Pr} f \frac{\partial \theta}{\partial \eta} + Ec \text{Pr} \frac{\partial^2 f}{\partial \eta^2} + k_2 Ec \text{Pr} \left(\frac{\partial f}{\partial \eta}\right)^2 + \left[A^* \frac{\partial f}{\partial \eta} + B^* \theta \right] \quad (21)$$

For $p=0$ and $p=1$, we have

$$\begin{aligned} \tilde{f}(\eta, 0) &= \tilde{f}_0(\eta), & \tilde{f}(\eta, 1) &= \tilde{f}(\eta) \\ \tilde{\theta}(\eta, p) &= \tilde{\theta}_0(\eta), & \tilde{\theta}(\eta, 1) &= \tilde{\theta}(\eta) \end{aligned} \quad (22)$$

Thus as p increases from 0 to 1, $\tilde{f}(\eta; p)$ varies from $\tilde{f}(\eta; 0)$ to $\tilde{f}(\eta)$ and $\tilde{\theta}(\eta; p)$ varies from $\tilde{\theta}(\eta; 0)$ to $\tilde{\theta}(\eta)$. Now, expanding $\tilde{f}(\eta, p)$, $\tilde{\theta}(\eta, p)$ in Taylor's series with respect to p , we have

$$\tilde{f}(\eta, p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) p^m \quad (23)$$

$$\tilde{\theta}(\eta, p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) p^m \quad (24)$$

Where

$$f_m(\eta) = \frac{1}{m!} \frac{\partial^m \tilde{f}(\eta, p)}{\partial p^m} \bigg|_{p=0} \quad (25)$$

$$\theta_m(\eta) = \frac{1}{m!} \frac{\partial^m \tilde{\theta}(\eta, p)}{\partial p^m} \bigg|_{p=0} \quad (26)$$

If the initial approximations, auxiliary linear operators and non-zero auxiliary parameters are chosen in such a way that the series (23) and (24) converges at $p=1$, and thus

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) \quad (27)$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) \quad (28)$$

Differentiating equation (17) and (18) 'm' times with respect to 'p', setting $p=0$ and finally dividing with $m!$, we get the m^{th} -order deformation equations as follows:

$$L_f[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_1 H_1(\eta) R_m^f(\eta) \quad (29)$$

$$L_\theta[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = \hbar_2 H_2(\eta) R_m^\theta(\eta) \quad (30)$$

subject to the boundary conditions

$$\begin{aligned} f_m(0) &= 0, & f'_m(0) &= 0, & f'_m(\infty) &= 0 \\ f''_m(\infty) &= 0 & \theta_m(0) &= 0, & \theta_m(\infty) &= 0 \end{aligned} \quad (31)$$

$$\begin{aligned} R_m^f(\eta) &= f'''_{m-1} + \sum_{i=0}^{m-1} f_i f''_{m-1-i} - 2 \sum_{i=0}^{m-1} f'_i f'_{m-1-i} - K_1 \left\{ 3 \sum_{i=0}^{m-1} f'_{m-1-i} f'''_i - \frac{1}{2} \sum_{i=0}^{m-1} f_{m-1-i} f_i^{iv} \right. \\ &\quad \left. - \frac{3}{2} \sum_{i=0}^{m-1} f''_{m-1-i} f_i'' \right\} - (k_2 + M) f'_{m-1} \end{aligned} \quad (32)$$

$$R_m^\theta(\eta) = \theta''_{m-1} + \text{Pr} \sum_{i=0}^{m-1} f_i \theta'_{m-1-i} + Ec \text{Pr} \sum_{i=0}^{m-1} f_i'' f''_{m-1-i} + k_2 Ec \text{Pr} \sum_{i=0}^{m-1} f'_i f'_{m-1-i} + A^* f'_{m-1} + B^* \theta_{m-1} \quad (33)$$

$$\chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1 \end{cases} \quad (34)$$

We choose the auxiliary function as

$$H_1(\eta) = 1, \quad H_2(\eta) = 1 \quad (35)$$

If we let $f_m^*(\eta)$, $\theta_m^*(\eta)$ as the special functions of m^{th} order deformation equations, then the general solutions are given by

$$f_m(\eta) = f_m^*(\eta) + C_1 + C_2 e^{-\eta} + C_3 e^{\eta} \quad (36)$$

$$\theta_m(\eta) = \theta_m^*(\eta) + C_4 e^{-\eta} + C_5 e^{\eta} \quad (37)$$

Where the integral constants $C_i (i = 1, 2, 3, 4, 5)$ are determined using the boundary conditions (31).

Now it is easy to solve the linear non-homogeneous equation (29) and (30) using MATHEMATICA software one after the other by considering $m = 1, 2, 3 \dots$

CONVERGENCE OF HAM SOLUTION

The convergence region and rate of approximation of series solutions are obtained by using the Homotopy Analysis Method. These solutions dependent on the auxiliary parameters h_1 and h_2 . To find h_1 and h_2 , h curves are plotted in the Figure 1 & 2 for 20th order of approximation. By plotting these curves it is observed that the valid regions of h_1 and h_2 are about $-1.28 \leq h \leq 0.16$, $-2.2 \leq h \leq 0.1$ **Error! Bookmark not defined.** The convare given of HAM solution for different orders of approximations is given in Table 1.

Figure 1: h-curve for $f''(0)$

Figure 2: h-curve for $\theta'(0)$

RESULTS AND DISCUSSIONS

An analysis is carried out to study the flow and heat transfer of an electrically conducting visco-elastic fluid over an exponentially stretching sheet with non-uniform heat source/sink in porous medium. The analytic solutions are obtained by using the Homotopy Analysis Method. The effect of visco elastic parameter, porosity parameter, suction parameter, non-uniform heat source/sink parameter, Prandtl number and Eckert number on flow and heat transfer is represented graphically. Table.2 enlists the comparison of $\theta'(0)$ for different values of visco-elastic parameter k_1 and are compared with the available results in the literature and are seen in good agreement.

**Table 1: Convergence of HAM Solution for Different Orders of Approximations when $k_1 = 0.1$, $k_2 = 0.1$, $Pr = 1$, $Ec = 0.1$
 $S = 0.1$ $A^* = -0.01$ $B^* = -0.01$ $M = 0.1$**

Order	$-f''(0)$	$-\theta(0)$
5	1.53031	0.26029
10	1.53083	0.51825
15	1.53092	0.54813
20	1.53093	0.539213
25	1.53093	0.539213
30	1.53093	0.539213
35	1.53093	0.539213

40	1.53093	0.539213
45	1.53093	0.539213
50	1.53093	0.539213

Table 2: Comparison of $-\theta(0)$ for different Values of k_1 when $k_2 = 1.0$, $Ec = 0 = A^* = B^* = M$, $S = 0.5$ and $Pr = 0.7$

k_1	Cortell(8)	M.Subhas Abel(13)	Present Study
0.1	3.833174	3.833333	3.833343
0.7	3.910973	3.91108	3.911099
1.7	3.937998	3.93823	3.93824

Figure 3: Velocity $f'(\eta)$ for different Values of k_1

Figure 4: Temperature $\theta(\eta)$ for different Values of k_1

Figure 3 & 4 illustrates the effect of visco-elastic parameter on velocity and temperature profiles. It is shown that the velocity decreases and temperature increases by increasing the values of visco-elastic parameter k_1 . This is due to the reason that in visco-elastic fluids the elastic property reduces the frictional force.

Figure 5: Velocity $f'(\eta)$ for different Values of k_2

Figure 6: Temperature $\theta(\eta)$ for different Values of k_2

Figure 5 & 6 illustrates the effect of porosity parameter on velocity and temperature profiles. Thus it can be referred that increase in permeability parameter k_2 decreases the boundary layer thickness and consequently brings about an increase in heat transfer rate.

Figure 7: Temperature $\theta(\eta)$ for different Values of A^*

Figure 8: Temperature $\theta(\eta)$ for different Values of B^*

The effect of space dependent parameter A^* and temperature dependent heat source/sink parameter B^* is shown in the Figure 7 and 8. It illustrates that the temperature profiles increase with increasing values of A^* and B^* which enlarges the thermal boundary thickness.

Figure 9: Velocity $f'(\eta)$ for different Values of S

Figure 10: Temperature $\theta(\eta)$ for different Values of S

Figure 9 & 10 depict the effect of suction parameter on velocity and temperature. In the boundary layer, the velocity and temperature distribution made uniform by the effect of suction. Imposition of fluid suction has a tendency to reduce both boundary layer thickness and thermal thickness. With this effect both velocity and temperature of the fluid are reduced.

Figure 11: Temperature $\theta(\eta)$ for different Values of Pr

Figure 12: Temperature $\theta(\eta)$ for different Values Ec

Figure 11 & 12 depict the effect of Prandtl number Pr and Eckert number Ec on temperature. These two plots reveal that by increasing the Prandtl number the temperature decreases, whereas it increases by increasing the Eckert number Ec . Fluids having smaller values of Prandtl number will poses thicker thermal boundary layer, so that the diffusion of heat reduces by increasing the values of Prandtl number.

Figure 13: Velocity $f'(\eta)$ for different Values of M

The effect of magnetic parameter M on velocity is shown in the Figure 13. It illustrates that the velocity decreases by increasing the magnetic parameter. With increase in values of M rate of transport decreases due to the Lorentz force which opposes the motion of fluid.

CONCLUSIONS

In the present paper, we discussed MHD visco-elastic fluid flow and heat transfer in Porous medium over an exponentially stretching sheet with non-uniform heat source/sink by using the Homotopy Analysis Method. The effects of various parameters like visco-elastic parameter, porosity parameter, magnetic parameter, space dependent and temperature dependent heat source/sink parameter, suction parameter, Prandtl number and Eckert number on flow and heat transfer are represented through graphs and the conclusions are as follows:

- The effect of space dependent parameter and temperature dependent heat source/sink parameter is to increase the thermal boundary layer thickness.
- With the effect of suction parameter there will be a loss in the fluid flow and less scope for heat transfer.
- The boundary layer thickness decreases with increasing values of the Magnetic parameter and visco-elastic parameter.
- The thermal boundary layer thickness increases with increasing values of Eckert number.

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