THE TRANSFORMATION OF THE WAVE FRONT AT THE TRANSVERSELY ELASTIC MEDIUM WITH VARIABLE PROPERTIES

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ABSTRACT

This paper solves the basis of the ray-path method, the problem about the wave propagation and transformation, which are generated by the shock pressure field in the spherical enclosure of the transversely isotropic elastic medium with the non-uniform density and non-uniform parameters. The techniques for construction of evolving system of the fronts and rays are proposed. The geometric singularity forms on the front surfaces for the different values of the both anisotropy and heterogeneity elastic medium are analyzed

The problem about propagation and transformation of discontinuous wave fronts in transversely isotropic heterogeneous elastic media is investigated.

KEYWORDS: *Discontinuous Wave, Heterogeneous Elastic Media, Interrupting Wave Front, Anisotropic Media & Shock Wave*

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1. INTRODUCTION

Through studies the tasks of the seismology, the seismography and through the analysis of the dynamic behaviour of tectonic structures under the influence of explosive disturbances there are some actual crucial tasks of mathematical simulation of the processes of the front reconstruction of shock waves at the anisotropic mediums with the variable properties. The specificities of the dynamic conditions, which arise in these cases, are related to the short duration of the high-intensity initial pressure field, usually concentrated on the initial time stage in the little area, and to the reconstruction of the pressure shock front surface in the measure of its extension. As the border of the area, designated for the determination, is evolving with the wave front movement, the traditional methods of the boundary problem solving of mathematical physics are ineffective for the analysis of such processes [1&2]. Usually, through studies of the pressure shock front the highest attention is paid to the questions of geometrical construction of surfaces of fractures of the field functions and the evaluation of these fractures, which provide the information about the pressure shock front and the impulse intensity, which is transferred at each front point. That's why, the geometrical optics methods play an important role in the assignment and the solving of such tasks [3], especially the zero-order approximation of the ray-path method, which provides the best quantitative description of the wide variety of the wave propagation effects of different physical nature [4]. The usage of the ray-path method involves the selection of the function of the wave optical path (eikonal function) and the construction of the eikonal of ray system and the pressure shock fronts with the help of the formula. This task is easy solvable for the isotropic mediums. The physical phenomenon of the dynamic conditions suddenly complicates through studies of the wave propagation of the strong explosion at the inhomogeneous elastic

anisotropic subsurface [5&6], as in these cases the field function is vector-mode; for each of the direction, there is three types of waves, which differ by the polarization; the wave speeds depend both on the wave polarization and the propagation direction; the rays, in the general case, are non-orthogonal to a surface of the wave front, and the ray speeds differ from the phase one, and between their directions there is always no one-to-one correspondence. With the advent of the medium non uniformity, the rays are bending, and on the fronts surface there may be formed the geometrical features in the form of caustics (catastrophes) [7-9].

2. TECHNIQUE OF NUMERICAL SIMULATION

Let's consider the transversely isotropic elastic medium. In consequence of the symmetry properties, the tensor $C_{ik,pq}$ components of the elastic constants of the medium can be represented as a hexastichous square structured matrix.

$$(C_{ik,pq}) = \begin{pmatrix} \lambda + 2\mu & \lambda & \lambda - l & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu - p & \lambda - l & 0 & 0 & 0 \\ \lambda - l & \lambda - l & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu - m & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu - m \end{pmatrix}, \quad (1)$$

where λ i μ – the parameters of Lamé, l , m , p – the constants, which differ the medium from the isotropic one. For the inhomogeneous medium these parameters are the coordinate x_1 , x_2 , x_3 functions.

The dynamic balance of the elastic medium in the Cartesian reference system x_1 , x_2 , x_3 is determined by the system of three differential equations

$$\sum_{k,p,q=1}^3 \lambda_{ik,pq} \frac{\partial^2 u_q}{\partial x_k \partial x_p} - \frac{\partial^2 u_i}{\partial t^2} = 0 \quad (i = 1,2,3), \quad (2)$$

where $\lambda_{ik,pq} = c_{ik,pq} / \rho$; $c_{ik,pq}$ – the parameters of the elasticity; ρ – the density; u_1, u_2, u_3 – the vector components of the elastic displacements, t – the time.

Taking into consideration anisotropic mediums the rays, in general cases, are non-orthogonal to a surface of the wave front, we will distinguish the vectors of the phase $\mathbf{v}$ and ray ξ speeds, considering that the, nt is a surface of the stationary phase $\mathbf{n} \cdot \mathbf{r} - vt = \text{const}$, the elementary area of which moves along the local unit normal element $\mathbf{n}$ at a speed of v . In this place $\mathbf{r}$ – the radius-vector of the front point.

The polarization vectors of the A wave and its phase speed v for the selected direction $\mathbf{n}$ we can build from the homogeneous system of the algebraic equations [3, 4]

$$\sum_{k,p,q=1}^3 \lambda_{ik,pq} n_k n_p A_q - v^2 A_i = 0 \quad (i = 1,2,3) \quad (3)$$

as the Eigen values and vector of matrix of its coefficients.

From the existence condition of the non-trivial solutions of this system there leaks out the formula of the third degree in respect of the phase speed square v^2

$$\left| \sum_{k,p=1}^3 \lambda_{ik,pq} n_k n_p - v^2 \delta_{iq} \right| = 0 \quad (4)$$

with the help of which for each normal direction $\mathbf{n}$ we can define and locate in descending order three speeds of the different polarized waves.

By the giving to the value v^2 at the system (3) in turns one of the determined values $v_r^2(\mathbf{n})$ ($r = 1, 2, 3$), we can define the components of the polarization vectors $\mathbf{A}^{(r)}$ of the three waves, which moves toward the considered direction with their phase speeds $\mathbf{v}_r(\mathbf{n})$.

The surface of the pressure shock front is defined by the formula [3]

$$\tau(x_1, x_2, x_3) - t = 0, \quad (5)$$

where the function τ must satisfy the differential equation at the differential derivatives of the first order

$$\sum_{i,k,p,q=1}^3 \lambda_{ik,pq} p_k p_p A_q^{(r)} A_i^{(r)} = 1, \quad (6)$$

That summarizes the eikonal equation from the geometrical optics in case of the elastic anisotropic waves.

The values p_k ($k = 1, 2, 3$) which are included in (6) present the components of the refraction vector

$$p_k \equiv \partial \tau / \partial x_k = n_k / v_r(n) \quad (k = 1, 2, 3).$$

The wave front (5) of the shock wave at the homogeneous an isotropic medium is constructed after the finding of the equation solution (6), which, with the help of the characteristic method reduces to the system of the simple differential equations

$$\begin{aligned} dx_k / d\tau &= \xi_k = \sum_{i,p,q=1}^3 \lambda_{ik,pq} p_p A_q^{(r)} A_i^{(r)}, \\ dp_k / d\tau &= 0, \quad (k = 1, 2, 3). \end{aligned} \quad (7)$$

The first group of these equations describes the wave propagation along the ray with the ray speed $\xi = \xi^{(r)}(\mathbf{n}, x_k)$. The second group of equations designates the ray straightness at the homogeneous medium.

However, if the elastic medium is non-homogeneous, the equation (7) is replaced by the more complicated formula [3, 10-12]

$$\begin{aligned} \frac{dx_m}{d\tau} &= \xi_m \equiv \sum_{i,p,q=1}^3 \lambda_{im,pq} p_p A_q^{(r)} A_i^{(r)} \quad (m = 1, 2, 3), \\ \frac{dp_m}{d\tau} &= \eta_m \equiv \frac{-1}{2} \sum_{i,k,p,q=1}^3 \frac{\partial \lambda_{ik,pq}}{\partial x_m} p_k p_p A_q^{(r)} A_i^{(r)} \end{aligned} \quad (8)$$

The presented dependences (8) permit to describe the fronts of the complex geometry, which are reconstructed during their extension. In such case the rays become curvilinear.

3. RESULTS AND DISCUSSIONS

On the basis of the proposed methods it was proved that the wave propagation tasks, which are generated by the shock pressure field at the spherical enclosure of the transversely isotropic elastic medium with the non-uniform density and the non-uniform parameters of Lamé. The results of the computational software simulation of the process of the extension of shock wave front for three different laws of parameter changing of the elastic medium are as follows:

- The density ρ changes by the vertical variable x_3 according to the following law:

$$\rho = \rho_0 + \beta \left(0,83\sqrt{1 \cdot 10^6 + (x_3 + 11250)^2} - 0,83\sqrt{1 \cdot 10^6 + (x_3 + 11000)^2} \right), x_3 \in [-15000; -7500],$$

$$\rho = \rho_0 + \beta \left(-0,83\sqrt{1 \cdot 10^6 + (x_3 + 3750)^2} + 0,83\sqrt{1 \cdot 10^6 + (x_3 + 3500)^2} \right), x_3 \in [-7500; 0],$$

Where $\rho_0 = 2760 \text{ kg} / \text{M}^3$, $E = 3 \cdot 10^{11}$, $\sigma = 0,1$, $\beta = 5$, (Figure 1b).

Figure 1

According to the appearance of the system of rays and fronts of the interrupting quasi-longitudinal qP (figure 1a) and quasi-transverse qS (figure 1c), which are extended at the medium with the considered law of the density changing (figure 1b) it may be concluded that at the points, where the density attains the largest value, there is the significant ray path bending both the quasi-longitudinal and the quasi-transverse waves. This lead to the rays superposing and to the appearance on their fronts of the geometrical features in the form of the caustics where we can consider a significant intensity increase of the stress field.

- The parameter of Lamé λ changes according to the following law:

$$\lambda = \lambda_0 + \beta \left(0,83\sqrt{1 \cdot 10^6 + (x_3 + 11250)^2} - 0,83\sqrt{1 \cdot 10^6 + (x_3 + 11000)^2} \right), x_3 \in [-15000; -7500],$$

$$\lambda = \lambda_0 + \beta \left(-0,83\sqrt{1 \cdot 10^6 + (x_3 + 3750)^2} + 0,83\sqrt{1 \cdot 10^6 + (x_3 + 3500)^2} \right), x_3 \in [-7500; 0],$$

$$\lambda_0 = 3,4 \cdot 10^{10}, E = 3 \cdot 10^{11}, \sigma = 0,1, \beta = 5 \cdot 10^8 \text{ (figure 2 b),}$$

Figure 2

In this case there was also noticed the tendency towards the ray path bending of quasi-longitudinal qP wave (figure 2a) with the increase of the parameter λ value and the formation of the negligible particularities on the front of qP wave if reached the maximum values, while the changing of this parameter had no influences on the form of the front of quasi-transverse qS-wave (figure 2b)

- The Lamé λ parameter changes according to the following law:

$$\lambda = \lambda_0 + \beta \left(-0,83\sqrt{1 \cdot 10^6 + (x_3 + 11250)^2} + 0,83\sqrt{1 \cdot 10^6 + (x_3 + 11000)^2} + 210 \right), x_3 \in [-15000; -7500],$$

$$\lambda = \lambda_0 + \beta \left(0,83\sqrt{1 \cdot 10^6 + (x_3 + 3750)^2} - 0,83\sqrt{1 \cdot 10^6 + (x_3 + 3500)^2} + 210 \right), x_3 \in [-7500; 0],$$

$$\lambda_0 = 3,4 \cdot 10^{10}, E = 3 \cdot 10^{11}, \sigma = 0,1, \beta = 1 \cdot 10^{10}. \text{ (figure 3 b).}$$

Figure 3

4. CONCLUSIONS

The Lamé λ parameter changing in one case leads to the negligible ray path bending of quasi-longitudinal qP wave in the points where the values λ are directed to the minimal. In this case the wave front extends, however the rays

are not superposing and the caustics are not forming. The quasi-transverse qS-wave, as in the case 2) does not undergo changes, suggesting that Lamé λ parameter changing does not influence on the mode of propagation of transverse waves.

The represented systems of rays and fronts permit to estimate the pattern of the voltage changing during the extension of their surface interruption. It is obvious that in the zones of rays' accumulation there is the concentration of the stress field, and in the zones of their divergence – the intensity decrease.

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REFERENCES

1. Podylchuk Iu. N., Rubtsov Iu. K. *Ray-path methods in the theory of the distribution and wave dispersion*. – K.: Scientific thought, 1988. – 220p.
2. Kubenko V. D., and Dzyuba V. V. *Diffraction of a plane acoustic wave by a rigid sphere in a cylindrical cavity: An axisymmetric problem*. // *International Applied Mechanics*. 2009. Vol. 45, № 4. P. 424–432.
3. Kravtsov Iu. A. Orlov Iu. I. *Geometric optics of inhomogeneous mediums*. – M.: Science, 1980. – 304p.
4. Petrashen G. I. *Waves distribution at the anisotropic elastic mediums*. – Leningrad: Science, 1980. – 280p.
5. Poston T., Stuart I. *Catastrophe theory and its attachments*. – M.: World, 1980. – 607p.
6. Arnold V. I. *Catastrophe theory*. – M.: Editorial of USSR, 2004. – 128p.
7. Guliaev V. I., Lugovyi P. Z., Ivanchenko G. M. Iakovenko Ie. V. *Shock diffraction on the curvilinear surface of the section of transversely isotropic elastic mediums* // *Applied mathematics and mechanics*. – 2000. – T.64, №3. – P. 394-402
8. Shukla, D. K., SINGH, M., & Jain, K. K. (2014). *Variation In Bearing Capacity Of Footing On Slopping Anisotropic Rock Mass*.
9. Arif S. J., Imdadullah., Asghar M. S. J. *Accurate measurement of velocity and acceleration of seismic vibrations near Nuclear power plants*. *Physics Procedia*. Vol.37, 2014, 43-50
10. Guliaev V. I., Lugovyi P. Z., Krytskiy V. B., Ivanchenko G. M. *Reflection and refraction of plane interrupting waves by paraboloid surfaces of the section of anisotropic elastic mediums* // *Geophysical magazine*. – 2005. – T.27, №3. – P. 418-426.
11. Bidgoli, M. N., Jing. L. *Anisotropy of strength and deformability of fractured rocks*. *Journal of rock mechanics and geotechnical engineering*, Vol. 6, 2014, 156-164
12. Guliaev V. I., Lugovyi P. Z., Ivanchenko G. M. *Shock diffraction on the ellipsoid surfaces of the section of transversely isotropic elastic mediums* // *Applied mechanics*. – 2004. – T.40, №10. – P. 98-106.
13. Musa N. W., Gulyayev V. I., Zaets Yu. A. *Transformation of seismic discontinuous waves by hyperboloid interfaces in anisotropic elastic media* // *International Journal of Scientific and Engineering Research*. 2014. Vol. 5, № 12. P. 962-977.