DESIGN AND DEVELOPMENT OF SELF ADAPTIVE DYNAMIC VIBRATION ABSORBER

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ABSTRACT

A new adaptive dynamic vibration absorber (DVA) is proposed for vibration isolation to change in amplitude of vibration in the primary system i.e., simply supported beam under harmonic excitation. Traditional means of vibration control have involved the use of passive methods. This study is different by involving an adaptive component in the design of vibration absorber. The cantilever absorber with a tip mass is designed. The vibration amplitude in primary system is suppressed by changing the natural frequency of the absorber. This is achieved by moving the mass automatically along the effective length of the cantilever rod. Finite element analysis and Euler-Bernoulli beam theory is used for evaluation of the performance of vibration isolation of the proposed absorber mounted to a primary system. Study of two different types of arrangement of the cantilevered mass absorber is carried out. It is proved theoretically that the absorber can isolate the vibration of the primary system when it subjected to a change in vibration amplitude under harmonic excitation. Both numerical and finite element analyses of the combined system using MATLAB and ANSYS respectively, have also been carried out for verification of the theoretical prediction of vibration isolation in a beam vibration.

KEYWORDS: Finite Element Analysis, Cantilever Rod, Vibration, MATLAB & ANSYS

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INTRODUCTION

Vibration Reduction Using Dynamic Vibration Absorbers

Vibrations are present in many machines and structures. In general, vibrations adversely affect the performance of a machine. Vibrations may also cause fatigue failure or even damage the machine or structure by causing excessive levels of stress. The prevention or control of the vibration of machines and structures is therefore an important design consideration. There are numerous ways of reducing and preventing vibrations, for example, by changing the stiffness of a structure, increasing damping by using materials that have high damping properties, or by using control. There are different types of control such as active, semi-active, passive and adaptive control.

The traditional Dynamic Vibration Absorber (DVA) is an auxiliary spring-mass system which tends to neutralize the vibration of a structure. ^[1, 3] The basic principle of operation is vibrating out of phase with the vibration of such structure, thereby applying a counteracting force. An important advantage of a DVA in comparison with other methods that reduce vibration is that it can also be applied to structures which are already in operation and appear to have unsatisfactory dynamic properties. DVA's are also advantageous because they are

able to reduce the vibration level of a structure at a comparatively low cost of a few additional materials. However, when applying dynamic vibration absorber to a continuous structure such as a beam, vibration can be eliminated only at the attachment point of the vibrating beam while amplification of vibration may occur in other parts of the beam. Research on suppressing vibration in a region or the whole span of a beam structure by using the dynamic vibration absorber has been reported [4, 5].

Dynamic Vibration Absorber

A dynamic vibration absorber (DVA) is a device that consists of an auxiliary mass attached to a vibrating system, in order to reduce the vibration amplitude of the system. The auxiliary mass is attached to the system by means of a spring, usually in parallel combination with a damper. Its mass is relatively small in comparison, with the mass of the vibrating system. The basic principle of operation of a DVA is vibration out-of-phase with the vibrating system, thereby applying a counteracting force, resulting in a reduction of the vibration amplitude of the vibrating system.

The Undamped Dynamic Vibration Absorber

The simplest form of a DVA is an auxiliary mass that is attached to the vibrating system with a spring only. This kind of DVA is very useful if a system is excited by a periodically varying force of constant frequency, especially when this frequency is very close to the resonance frequency of the system. Figure 1 shows the schematic representation of a system with mass m_1 and stiffness k_1 that is excited by a periodically varying force with amplitude F and angular frequency ω . An auxiliary mass m_2 is attached to the main mass m_1 by a spring with stiffness k_2 and a damper with damping constant c , which together form the DVA. In this undamped case, the damping value c is equal to zero. If the natural frequency of the DVA is chosen to be equal to the frequency of the periodically varying force (i.e.

$\omega_2 = \sqrt{\frac{k_2}{m_2}} = \omega$), the main mass m_1 will not vibrate at all.

Figure 1: Schematic Representation of a DVA Attached to a Vibration System [2]

In order to prove this statement and study the behavior of such a system in more detail, the equations of motion are written down

$$\begin{aligned} m_1 \ddot{x}_1 + (k_1 + k_2)x_1 - k_2 x_2 &= F \sin \omega t \\ m_2 \ddot{x}_2 + k_2(x_2 - x_1) &= 0 \end{aligned} \quad (1.1)$$

Since the system is undamped, the forced solution will be of the form

$$\begin{aligned}x_1 &= a_1 \sin \omega t \\x_2 &= a_2 \sin \omega t\end{aligned}\quad (1.2)$$

Substitution of equation 1.2 into equation 1.1 and dividing by $\sin \omega t$ results in the following algebraic equations

$$\begin{aligned}a_1(-m_1\omega^2 + k_1 + k_2) - k_2 a_2 &= F \\-k_2 a_1 + a_2(-m_2\omega^2 + k_2) &= 0\end{aligned}\quad (1.3)$$

Rearranging these equations and defining some new variables results in the following expressions for the vibration amplitudes

$$\begin{aligned}\frac{a_1}{x_{st}} &= \frac{1 - \frac{\omega^2}{\omega_2^2}}{(1 - \frac{\omega^2}{\omega_2^2})(1 + \frac{k_2}{k_1} - \frac{\omega^2}{\omega_1^2}) - \frac{k_2}{k_1}} \\ \frac{a_2}{x_{st}} &= \frac{1}{(1 - \frac{\omega^2}{\omega_2^2})(1 + \frac{k_2}{k_1} - \frac{\omega^2}{\omega_1^2}) - \frac{k_2}{k_1}}\end{aligned}\quad (1.4)$$

Where,

$$\omega_i = \sqrt{\frac{k_i}{m_i}} = \text{natural frequency}$$

$$x_{st} = \frac{F}{k_1} = \text{static deflection}$$

Looking at the numerator of the first equation in 1.4, it can directly be seen that the vibration amplitude of the main system a_1 is zero when the frequency of the DVA is equal to the frequency of the forcing ($\omega_2 = \omega$).

Figure 2: (a) Amplitude of Main System with and Without DVA, (b) Amplitude of DVA as Function of the Forcing Frequency. The DVA Mass is One-Tenth of the Main System Mass [2]

Now suppose that the absorber is designed to reduce the amplitude of the system when it is at resonance.

In that case $\omega_1 = \omega_2$ or equivalently $\frac{k_2}{k_1} = \frac{m_2}{m_1}$. Equation 1.4 can then be written as

$$\begin{aligned}\frac{a_1}{x_{st}} &= \frac{1 - \frac{\omega^2}{\omega_1^2}}{(1 - \frac{\omega^2}{\omega_1^2})(1 + \frac{m_2}{m_1} - \frac{\omega^2}{\omega_1^2}) - \frac{m_2}{m_1}} \\ \frac{a_2}{x_{st}} &= \frac{1}{(1 - \frac{\omega^2}{\omega_1^2})(1 + \frac{m_2}{m_1} - \frac{\omega^2}{\omega_1^2}) - \frac{m_2}{m_1}}\end{aligned}\quad (1.5)$$

In figure 2 (a) & (b) $\frac{a_1}{x_{st}}$ and $\frac{a_2}{x_{st}}$ are plotted as a function of $\frac{\omega^2}{\omega_1^2}$ for $\frac{m_2}{m_1} = 0.1$. It can be seen that at $\frac{\omega^2}{\omega_1^2} = 1$,

the amplitude of the main system a_1 is zero. Furthermore, two new resonance frequencies arise at the left and at the right of the original resonance frequency. This is due to the fact that the system has now changed into a two degree of freedom system. These two new frequencies depend on the mass ratio μ

$$\mu = \frac{m_2}{m_1} \quad (1.6)$$

In order to determine these two new frequencies, the denominators of the expressions in equation 1.5 are set equal to zero, which yields the following equation 1.7

$$\left(\frac{\omega_{new}}{\omega_1}\right)^2 = \left(1 + \frac{\mu}{2}\right) \pm \sqrt{\mu + \frac{\mu^2}{4}} \quad (1.7)$$

Figure 3 shows the above relation graphically. From the figure, it can be seen that the two resonance frequencies become farther apart with increasing mass ratio μ . A larger DVA mass can therefore, be advantageous if the frequency of the disturbing force fluctuates around the frequency for which the absorber has been tuned. The size of the DVA mass also influences the amplitude of the DVA. From the second equation of 1.4 it can be seen that if the DVA is correctly tuned to the forcing frequency (i.e. $\omega_2 = \omega$), the expression for the amplitude of the DVA reduces to

$$\frac{a_2}{x_{st}} = -\frac{k_1}{k_2} \quad (1.8)$$

Figure 3: Resonance Frequencies as a Function of the Mass Ratio μ [15]

This equation shows that the amplitude of the DVA is proportional to the reciprocal of k_2 . This means that it is proportional to the reciprocal of m_2 as well, because ω_2 do not change. So, increasing the DVA mass will lead to smaller DVA amplitudes. This can be advantageous if the DVA has to be mounted in a small space where no large amplitudes are possible

The Damped Dynamic Vibration Absorber

If the mass of a DVA is not only supported by a spring, but also by a damper with damping constant c larger than zero, it is called a damped dynamic vibration absorber. By adding this damper, the frequency range over which the response is reduced can considerably be increased. The equations of motion of such a system are as follows

$$\begin{aligned} m_1 \ddot{x}_1 + c(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2) + k_1 x_1 &= F \sin \omega t \\ m_2 \ddot{x}_2 + c(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) &= 0 \end{aligned} \quad (1.9)$$

The forced solution of this set of equations will be in the form of

$$\begin{aligned} x_1 &= a_1 \sin(\omega t - \phi_1) \\ x_2 &= a_2 \sin(\omega t - \phi_2) \end{aligned} \quad (1.10)$$

The easiest way to find the amplitudes a_1 and a_2 of these solutions is to write the solution in the complex number form

$$\begin{aligned} x_1 &= X_1 e^{i\omega t} \\ x_2 &= X_2 e^{i\omega t} \end{aligned} \quad (1.11)$$

Figure 4: Amplitude-Ratio of the Main System as a Function of the Forcing Frequency Ratio for Different Damping Ratio Values [8]

In this equation X_i is a complex constant of which the absolute value equals a_i and of which the angle equals ϕ_i .

. Substitution of these solutions into the equations of motion results in the following equations

$$\begin{aligned} -m_1\omega^2 X_1 + j\omega c(X_1 - X_2) + k_2(X_1 - X_2) + k_1 X_1 &= F \\ -m_2\omega^2 X_2 + j\omega c(X_2 - X_1) + k_2(X_2 - X_1) &= 0 \end{aligned} \quad (1.12)$$

Converting these equations into an expression for X_1 and then taking the absolute value gives the solution for the amplitude of the main system in equation 1.13.

$$\frac{a_1}{F} = \sqrt{\frac{(k_2 - m_2\omega^2)^2 + \omega^2 c^2}{[(-m_1\omega^2 + k_1)(-m_2\omega^2 + k_2) - m_2\omega^2 k_2]^2 + \omega^2 c^2 (-m_1\omega^2 + k_1 - m_2\omega^2)^2}} \quad (1.13)$$

By recalling the definitions of, μ and X_{st} and defining some new variables, the total number of variables in this solution can be decreased

$$f = \frac{\omega_2}{\omega_1} = \text{frequency ratio natural frequencies (tuning ratio)}$$

$$g = \frac{\omega}{\omega_1} = \text{forcing frequency ratio}$$

$$C = 2m_2\omega_1$$

$$\frac{a_1}{X_{st}} = \sqrt{\frac{2\frac{c}{C}g^2 + (g^2 - f^2)^2}{2\frac{c}{C}g^2 (g^2 - 1 + \mu g^2)^2 + [\mu f^2 g^2 - (g^2 - 1)(g^2 - f^2)^2]}} \quad (1.14)$$

The above equation shows the relative amplitude $\frac{a_1}{X_{st}}$ as a function of the four main variables, namely tuning

ratio f , damping ratio $\frac{c}{C}$, mass ratio μ , and forcing frequency ratio g . This solution is shown graphically in figure 4.

The figure shows the amplitude ratio $\left| \frac{a_1}{x_{st}} \right|$ as a function of the forcing frequency ratio for various values of the damping ratio.

Figure 5: Amplitude Ratio of the System as a Function of the Forcing Frequency Ratio Using the Optimum Tuning Value [22]

Now, it is to find a combination of f and $\frac{c}{C}$ for a certain chosen mass ratio μ , which results in a minimum dynamic response, i.e. a response for which the peaks are minimal throughout the frequency range. It is evident that this minimum dynamic response can be obtained by first setting the two invariant points to equal height, and second, arranging that the slope of the curve is zero at these points. In this way the two invariant points become the maxima of the curve. The first consideration can be achieved by adjusting the tuning ratio. Unfortunately, the second consideration cannot completely be achieved. By adjusting the damping ratio only the slope at one of the two invariant points can be made equal to zero. From equation 1.14 two values of g having to be found for which the equation becomes independent of damping.

$$f = \frac{1}{1 + \mu} \quad (1.15)$$

The forcing frequency ratios at which these invariant points occur are expressed by equation 1.16

$$g = \sqrt{\frac{1 \pm \sqrt{\frac{\mu}{2 + \mu}}}{1 + \mu}} \quad (1.16)$$

Next, the damping values can be determined for which the curve passes through the invariant points horizontally. This can be done by differentiating the equation 1.14 and equating it to zero for both invariant points. The two values of the damping ratio that are found in this way can be averaged in order to obtain the optimum value

$$\frac{c}{C} = \sqrt{\frac{3\mu}{8(1 + \mu)^3}} \quad (1.17)$$

Figure 5 shows the amplitude ratio as a function of the forcing frequency ratio using the optimum tuning value. Three curves are shown. Two curves have been obtained by using the values of the damping ratio that result in slope equal to zero at the left and the right invariant point respectively.

Figure 6: Maximum Amplitude Ratio of the Main System and Maximum Amplitude Ratio of the DVA Relative to the Main System as a Function of the Mass Ratio [10]

The third curve has been obtained using the averaged value of these two damping ratios shown in equation 1.17. The figure shows that the different curves do not differ very much, yet it can be seen that the curve obtained with the averaged value results in the lowest peaks.

$$\frac{a_1}{x_{11}} = \sqrt{1 + \frac{2}{\mu}} \quad (1.18)$$

From above information it can be concluded that for a certain vibrating system a DVA can be designed by choosing a mass ratio followed by a calculation of the optimum stiffness and damping of the DVA. A few aspects can be considered in order to come with a choice, some of them are analogous to the considerations made in the undamped case. First of all, a large mass ratio result in small amplitude of the main system.

DESIGN AND ANALYSIS OF DVA ON LINEAR BEAM SYSTEM

Theoretical Analysis of Vibration in Simply Supported Beam

Real- time problems are related to vibration of building structure due to earthquake, automobile, bridges due to moving loads, etc. It is very difficult to solve by considering the same. For the simplification of the problem, we will consider as beams, truss, columns, rods, wires, etc. In this study, the absorber is placed on a linear beam system i.e., simply supported rectangular cross-section beam. The resonance frequencies of the modes of the beam are adjusted by adjusting the axial position of the supports.

Figure 7: Changing the Natural Frequency of a Beam [14]

This is illustrated in figure 7 which is shown above. The natural frequencies of the beam are adjusted by moving the axial location of the supports. The purpose of constructing the test rig such that its resonance frequencies can be

adjusted is to test the effectiveness of the absorber over a range of frequencies. When the absorber is mounted on the beam, the resonance of the beam will be adjusted such that the span in which the absorber is operating will be within the same proximity as the resonant modes of the beam. This procedure will also allow for the different absorber prototypes which are constructed for different operating ranges to be tested on the same rig. table 1 provides you the dimensional parameters and material properties of steel for simply supported beam.

Table 1: Properties of Simply Supported Beam

Parameter	Value
Length, l	1.135m
Height, h	0.05m
Width, b	0.025m
Modulus of elasticity, E	207×10^9 pa
Density, ρ	7800 kg/m ³
Second moment of inertia, I	$bh^3/12$

The length of the beam was arbitrarily chosen such that the resonance frequency of the beam would be near the operating range of the absorber.

Governing Equations for Base Structure

The governing equation for the natural frequency of a simply supported beam is given below[21].

$$\omega_n = \beta_n^2 \sqrt{\frac{EI}{\rho A}} = (\beta_n l)^2 \sqrt{\frac{EI}{\rho A l^4}}$$

For the rectangular uniform beam, $E = 207 \times 10^9$ Pa, $\rho = 7800$ kg/m³.

The second moment of inertia, $I = \frac{bh^3}{12}$.

For a beam, $b = 0.025$ m, $h = 0.05$ m, $l = 1.135$ m i.e., for initial setup of the test rig.

$$I = \frac{0.025 \times 0.05^3}{12} = 2.6 \times 10^{-7}$$

The value $(\beta_n l)^2$ depends on the boundary conditions of the beam. Typical end conditions are given in table 2 [21].

Table 2: Beam Configuration

Beam Configuration	$(\beta_1 l)^2$	$(\beta_2 l)^2$	$(\beta_3 l)^2$
Simply Supported Beam	9.87	39.5	88.9
Cantilever	3.52	22	61.7
Free-Free	22.4	61.7	121
Clamped-Clamped	22.4	61.7	121
Clamped-Hinged	15.4	50	104
Hinged-Free	0	15.4	50

For a simply supported beam, $(\beta_n l)^2 = 9.87$ for the fundamental mode.

$$\omega_n = 9.87 \sqrt{\frac{207 \times 10^9 \times 2.6 \times 10^{-7}}{7800 \times 1.25 \times 10^{-3} \times 1.135^4}} = 569.24 \text{ rad/s} = 90.6 \text{ Hz}$$

First two natural frequencies of the beam are determined as 90.6 Hz and 361.4 Hz.

Modelling of Beam as a Single Degree of Freedom System

Figure 8: Schematic Representation of Beam as Spring Mass System with Point Load [14]

Usually beams are infinite degree of freedom system. For the simplification of the problem we are considering as a single degree of freedom system. The figure 8 shows the beam as a spring mass system. Consider stiffness of the beam as the spring as shown in equation 1.19 and the mass of the beam is as system mass and subjected to a harmonic force of $F_0 \cos \omega t$

Here the damping is zero. The equation of motion is

$$m_1 \ddot{x} + k_1 x = F_0 \cos \omega t \quad (1.19)$$

The homogeneous solution of this equation is given by

$$x_h(t) = C_1 \cos \omega_n t + C_2 \sin \omega_n t \quad (1.20)$$

Where $\omega_n = \sqrt{\frac{k_1}{m_1}}$ is the natural frequency of the system. Because the exciting force is $x_p(t)$ harmonized, the particular solution is also harmonic and has same frequency ω . Thus the solution is in the form which is shown below in equation 1.21

$$x_p(t) = X \cos \omega t \quad (1.21)$$

Where X is a constant that denotes the maximum amplitude of $x_p(t)$ by substituting the equation 1.21 into equation 1.19, we obtain

$$X = \frac{F_0}{k_1 - m_1 \omega^2} = \frac{\delta_{st}}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \quad (1.22)$$

Where $\delta_{st} = \frac{F_0}{k_1}$ denotes the deflection of the mass under a force F_0 and is sometimes called static deflection

because F_0 is a constant force. Thus the total solution of equation 1.21 becomes

$$x(t) = C_1 \cos \omega_n t + C_2 \sin \omega_n t + \frac{F_0}{k_1 - m_1 \omega^2} \cos \omega t \quad (1.23)$$

Using the initial condition $x(t=0) = x_0$ and $\dot{x}(t=0) = \dot{x}_0$, we find that

$$C_1 = x_0 - \frac{F_0}{k_1 - m_1 \omega^2} \quad C_2 = \frac{\dot{x}_0}{\omega_n} \quad (1.24)$$

and hence the equation 1.23 can be written as

$$x(t) = \left(x_0 - \frac{F_0}{k_1 - m_1 \omega^2} \right) \cos \omega_n t + \left(\frac{\dot{x}_0}{\omega_n} \right) \sin \omega_n t + \frac{F_0}{k_1 - m_1 \omega^2} \cos \omega t \quad (1.25)$$

The maximum amplitude X in equation 1.22 can be expressed as

$$\frac{X}{\delta_{st}} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \quad (1.26)$$

k_1 = stiffness of the simply supported beam is given by equation 1.27

$$k_1 = \frac{48 EI}{l^3} \quad (1.27)$$

m_1 = mass of the simply supported beam with a mass m in the middle of the beam.

$$m_1 = M + 0.5m \quad (1.28)$$

Where M is the mass of the beam and m is the mass which is loads on the beam.

By substituting the above values in the response equation, we can find the response of an undamped system under harmonic force.

Modeling of Beam as Continuous System under Harmonic Force

The equation of motion for an Euler- Bernoulli beam under a general forcing function can be written as [6].

$$EI \frac{\partial^4 y}{\partial x^4} + \rho A \frac{\partial^2 y}{\partial t^2} = f(x, t) \quad (1.29)$$

The boundary conditions for a simply supported beam are

$$y(0, t) = y(l, t) = 0 \quad (1.30)$$

$$\left. \frac{\partial^2 y}{\partial x^2} \right|_{x=0} = \left. \frac{\partial^2 y}{\partial x^2} \right|_{x=l} = 0 \quad (1.31)$$

The external forces considered here is a point concentrated load on the simply supported beam as shown in figure

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Figure 9: Beam as a Continuous System with a Harmonic Excitation [8]

The forced vibration solution of a beam can be determined using the mode superposition principle. For this, the deflection of the beam is assumed as

$$w(x, t) = \sum_{i=1}^n \phi_i(x) q_i(t) \quad (1.32)$$

Where $\phi_i(x)$ the i^{th} mode shape of the beam and $q_i(t)$ is generalized coordinates, or modal amplitudes, which depend on t [7]. Thus the response of the beam is given by solving the above equation [8]

$$w(x, t) = \frac{2 f_0}{\rho A l} \sum_{n=1}^{\infty} \frac{1}{\omega_n^2 - \omega^2} \sin \frac{n \pi a}{l} \sin \frac{n \pi x}{l} \sin \omega t \quad (1.33)$$

Where E young's modulus of the material, I the moment of inertia, l length of the beam and ρ is the density, Thus the response of the beam under the harmonic excitation has been solved by the above equation using MATLAB.

Finite Element Analysis of Simply Supported Beam

In order to gain an accurate prediction of the modes of the beam, a numerical analysis using finite elements was used. This analysis allows determination of the resonance frequency of each mode, which will be a function of the location of the mass along the beam. For the modeling of the beam a rectangular block is developed as per the specification mentioned in table 1, SOLID45 tetrahedral was used. The boundary conditions were then programmed by fixing one end

along the y-axis and other end along x-axis and y-axis. The final meshed model of the beam is shown in figure 10 (a) & (b) and 11 (a) & (b). The first two natural frequencies are found to be 90.365 Hz and 361.54 Hz. Mode shapes of first two natural frequencies in 3D & 2D are shown in figure 10 (a) & (b) and 11 (a) & (b).

Figure 10: Natural Frequency for First Mode Shape is 90.363Hz
(a) Meshed Model of the Beam in 3D, (b) Meshed Model of the Beam in 2D

Figure 11: Natural Frequency for Second Mode Shape is 361.54Hz
(a) Meshed Model of the Beam in 3D, (b) Meshed Model of the Beam in 2D

Design of Dynamic Vibration Absorber

A design that meets most of these requirements is shown in figure 12. The design consists of two cantilever beams with additional masses at the ends, which together can be seen as massspring systems.

Figure 12: Schematic Representation of DVA on Beam System [7]

By moving a mass along its cantilever beam, the stiffness can be tuned very precisely and consequently, it's natural frequency as well. This is achieved by using a stepper motor and a controller. The reason for using two cantilever beams instead of one is symmetry. Both cantilever beams will therefore always be tuned to the same natural frequency. The DVA is designed for single axis and multiple axis vibrations with the properties shown in table 3 and the model is shown in figure 13 (a), (b). The DVA consists of two circular rods in parallel of diameter 10mm with masses of 1 kg at both ends. The total DVA mass is therefore equal to 2 kg, which is one-sixth of the beam mass.

Figure 13: (a) Absorber for Single Axis Vibration

Figure 13: (b) Absorber for Multiple Axis Vibration

The resulting mass ratio, however, depends on the position of the DVA on the beam and the corresponding equivalent beam mass as per equation 4.16. Here one rod is threaded externally to which the mass slides and another rod is plane and used for guiding the mass.

$$m_{eq} = \frac{m_b}{2 \sin\left(\frac{n\pi x}{l}\right)} \quad (4.16)$$

By moving the masses the natural frequency of the DVA can be varied in a range from 43 Hz to 400 Hz. In order to be able to move the masses, they are attached to the box where a worm and gear mechanism is given.

Theoretical Analysis of Cantilever Absorber Using Discrete System Method

Dunkerleys equation states that for a multi degree-of-freedom system, the inverse of the fundamental frequency [18].

$$\frac{1}{\omega_1^2} = \frac{1}{\omega_{11}^2} + \frac{1}{\omega_{22}^2} + \dots + \frac{1}{\omega_{nn}^2} = \sum_{i=1}^n \frac{1}{\omega_n^2} \quad (4.17)$$

Where $\omega_{11}, \omega_{22}, \dots, \omega_{nn}$ are the corresponding natural frequencies of each component of the system shown in the equation 4.17.

Figure 14: Dimensions Used For Discrete System Analysis [22]

For the natural frequency, ω_b of a cantilevered beam of mass, m_1 ,

$$\omega_b^2 = 12.7 \frac{EI}{m_1 l^3} \quad (4.18)$$

For the absorber device, two rods are in parallel, with the mass attached at $X=L$.

$$\text{As, } k = \omega^2 m, \quad k = 12.7 \frac{EI}{l^3} \quad (4.19)$$

The total stiffness produced by the 2 rods in parallel is,

$$k_t = k_1 + k_2 \quad (4.20)$$

$$\omega_b^2 = 25.4 \frac{EI}{ml^3} \quad (4.21)$$

For the natural frequency, ω_a of a cantilevered beam of negligible mass with a concentrated mass attached at one end,

$$\omega_n^2 = \frac{3EI}{ml^3} \quad (4.22)$$

Using Dunkerleys equation,

$$\frac{1}{\omega_1^2} = \frac{1}{\omega_b^2} + \frac{1}{\omega_a^2} \quad (4.23)$$

$$= \frac{m_b l^3}{25.4EI} + \frac{m_a l^3}{3EI} \quad (4.24)$$

$$= \frac{3m_b l^3 + 25.4m_a l^3}{76.2EI} \quad (4.25)$$

$$\omega_1^2 = \frac{76.2EI}{3m_b l^3 + 25.4m_a l^3} \quad (4.26)$$

$$\omega_1 = \sqrt{\frac{76.2EI}{3m_b l^3 + 25.4m_a l^3}} \quad (4.27)$$

$$I = \pi R^4/4 = \pi (0.005)^4/4 = 4.91 \times 10^{-10} \text{ m}^4. \text{ (Second Moment of Area)}$$

$$m_b = \pi (0.005)^2 \cdot (0.105) \cdot (7800) = 0.129 \text{ kg}$$

$$m_a = \pi (0.04)^2 \cdot (0.05) \cdot 7800 = 1.96 \text{ kg}$$

Table 3: Properties of Cantilever Absorber

Parameter	Value
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Length, l	0.105m
Diameter of the beam, D	0.01m
Width of the mass, w	0.45m
Modulus of elasticity, E	207×10^9 pa
Density, ρ	7800 kg/m ³
Diameter of the absorber mass, D	0.03m

$$\omega_1 = \sqrt{\frac{76.2 \times 207 \times 10^9 \times 4.91 \times 10^{-10}}{3 \times 0.129 \times 0.105^3 + 25.4 \times 1.96 \times 0.105^3}} \quad (4.28)$$

$$= 365.168 \text{ rad/s}$$

$$f = 58.12 \text{ Hz (for dual cantilever absorber)}$$

$$f = 84.28 \text{ Hz (for multiple axis cantilever absorber)}$$

The calculated value is less than the natural frequency of the beam. The natural frequency of the absorber is changed by moving the mass along the effective length of the rod.

Modeling of Cantilever Mass Absorber as Spring Mass System

Usually cantilever beam with a tip mass are infinite degree of freedom system. For the simplification of the problem we are considering as a single degree of freedom system, i.e., spring-mass system. The figure 15 shows the schematic representation of absorber as a spring mass system.

Figure 15: Schematic Representation of Absorber as Spring Mass System [7]

Consider a cantilever beam as spring and the mass at its tip as absorber mass.

k_2 = stiffness of the cantilever beam is given by equation 4.29

$$k_2 = \frac{3EI}{l^3} \quad (4.29)$$

There are 8 cantilever beams which are connected in parallel. So $k_2 = \frac{3EI}{l^3} \times 8$

m_2 = mass of the cantilever beam is given by equation 4.30

$$m_2 = M_0 + 0.236 m \quad (4.30)$$

Where M_0 is the mass which is loaded at the tip of the cantilever and the m is the mass of the cantilever beam.

Finite Element Analysis of the Absorber

In order to gain an accurate prediction of the modes of the absorber, a numerical analysis using finite elements was used. This analysis allows determination of the resonance frequency of each mode, which will be a function of the location of the mass along the two shafts. Two main types of meshing elements in ANSYS were used to model the absorber device. For the hollow square housing in the center i.e., which attaches to the vibrating surface, shell63 was used.

Figure 16: (a) Meshed Model of Single axis Absorber

Figure 16: (b) Meshed Model of Multiple Axis Absorbers

Mathematical Modeling of System as Undamped DVA

The Figure 17 shows the schematic representation of the beam and DVA as two degree of freedom system.

Figure 17: Schematic Diagram of Beam and Cantilever Mass Absorber as Two Degree of Freedom Systems [9]

Equation of motion for the above schematic representation is given by equation 5.1

$$\begin{aligned} -k_2 a_1 + a_2 (-m_2 \omega^2 + k_2) &= 0 \\ a_1 (-m_1 \omega^2 + k_1 + k_2) - k_2 a_2 &= 0 \end{aligned} \quad (5.1)$$

$$k_1 = \text{stiffness of the simply supported beam and it is given in equation 5.2} \quad k_1 = \frac{48 EI}{l^3} \quad (5.2)$$

m_1 = mass of the simply supported beam with a mass m in the middle of the beam.

$$m_1 = M + 0.5m \quad (5.3)$$

k_2 = stiffness of the cantilever beam and it is given in equation 5.4

$$k_2 = \frac{3 EI}{l^3} \quad (5.4)$$

There are 8 cantilever beams which are connected in parallel. So $k_2 = \frac{3EI}{l^3} \times 8$

m_2 = mass of the cantilever beam is given by equation 5.5

$$m_2 = M_0 + 0.236 m \quad (5.5)$$

Since the system is undamped, the forced harmonic solution will be of the form

$$x_j(t) = X_j \sin \omega t, \quad j=1, 2 \quad (5.6)$$

Substitute in equation 5.1, we get

$$\begin{aligned} -k_2 a_1 + a_2(-m_2 \omega^2 + k_2) &= 0 \\ a_1(-m_1 \omega^2 + k_1 + k_2) - k_2 a_2 &= 0 \end{aligned} \quad (5.7)$$

By simplifying the above equation we get

$$\omega^4 - \left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2} \right) \omega^2 + \frac{k_1 k_2}{m_1 m_2} = 0 \quad (5.8)$$

We can obtain the steady state amplitude of the masses m_1 and m_2 as

$$X_1 = \frac{(k_2 - m_2 \omega^2) F_0}{(k_1 + k_2 - m_1 \omega^2)(k_2 - m_2 \omega^2) - k_2^2} \quad (5.9)$$

$$X_2 = \frac{k_2 F_0}{(k_1 + k_2 - m_1 \omega^2)(k_2 - m_2 \omega^2) - k_2^2} \quad (5.10)$$

We are primarily interested in reducing the amplitude of the beam which is simply supported (X_1). In order to make the amplitude of m_1 zero, the numerator of equation 5.9 should be set equal to zero. This gives equation 5.11 and 5.12

$$\omega^2 = \frac{k_2}{m_2} \quad (5.11)$$

If the mass m_1 , before the addition of the dynamic vibration absorber, operates near its resonance, $\omega^2 = \omega_1^2 = \frac{k_1}{m_1}$

. Thus, if the absorber is designed such that $\omega^2 = \frac{k_2}{m_2} = \frac{k_1}{m_1}$ (5.12)

The amplitude of vibration of the beam, while operating at its original resonant frequency, will be zero.

By defining $\delta_{st} = \frac{F_0}{k_1}$, $\omega_1 = \left(\frac{k_1}{m_1}\right)^{1/2}$ as the natural frequency of the main system, and $\omega_2 = \left(\frac{k_1}{m_1}\right)^{1/2}$ as the natural

frequency of the absorber or auxiliary system, the above equation can be rewritten as

$$\frac{X_1}{\delta_{st}} = \frac{1 - \left(\frac{\omega}{\omega_2}\right)^2}{\left[1 + \frac{k_2}{k_1} - \left(\frac{\omega}{\omega_1}\right)^2\right] \left[1 - \left(\frac{\omega}{\omega_2}\right)^2\right] - \frac{k_2}{k_1}} \quad (5.13)$$

$$\frac{X_2}{\delta_{st}} = \frac{1}{\left[1 + \frac{k_2}{k_1} - \left(\frac{\omega}{\omega_1}\right)^2\right] \left[1 - \left(\frac{\omega}{\omega_2}\right)^2\right] - \frac{k_2}{k_1}} \quad (5.14)$$

Above equation 5.13 and 5.14 can be used to plot the variations of vibration amplitudes of the main and auxiliary masses of a vibration absorber, as a function of the frequency ratio.

The equation is in the form of quadratic forms, by solving the equation, we get,

$$\omega_1 = 333.53 \text{ rad/s}, \quad \omega_2 = 726.41 \text{ rad/s}$$

Therefore the frequency of the system is given by,

$$f_i = \frac{\omega_i}{2\pi} \text{ Where } i = 1, 2$$

$$f_1 = 53.11 \text{ Hz.}, \quad f_2 = 115.67 \text{ Hz.}$$

The variations of vibration amplitude of the main and auxiliary masses of vibration absorber is being plotted using the equation 5.13 and 5.14 for the frequency ratio of $f=0.96$ and for the mass ratio of 0.35 using MATLAB.

Beam with Damped Two 1dof DVAS

In figure 18, the model that describes the transverse vibration of a beam, with damping and a DVA of MDOF, is presented.

Figure18: Beam with Damped two 1DOF DVA [6]

The displacement of the structure at position x and time t is assumed to be a superposition of modal amplitudes multiplied by normal mode shapes. The total kinetic energy, strain energy and dissipation function can be written as

$$T = \frac{1}{2} \sum_{i=1}^n M_i \dot{q}_i^2 + \frac{1}{2} \sum_{v=1}^r m_v \dot{z}_v^2 \quad (5.15)$$

$$V = \frac{1}{2} \sum_{i=1}^n \omega_i^2 M_i q_i^2 + \frac{1}{2} \sum_{v=1}^r k_v (z_v - z_{0v})^2 \quad (5.16)$$

$$D = \frac{1}{2} \sum_{i=1}^n c_{vi} \dot{q}_i^2 + \frac{1}{2} \sum_{v=1}^r c_v (\dot{z}_v - \dot{z}_{0v})^2 \quad (5.17)$$

The coordinate z_{0v} is the displacement of the beam at the point of attaching and z_v is the displacement of DVA mass. M_i , c_{vi} , and ω_i are the i th modal mass, damping constants and natural frequencies of the beam, k_v , c_v , and m_v are the stiffness, damping constants, and masses of the DVA respectively.

The forced vibration solution of a beam can be determined using the mode superposition principle. For this, the deflection of the beam is assumed as

$$w(x, t) = \sum_{i=1}^n \phi_i(x) q_i(t) \quad (5.18)$$

Where $\phi_i(x)$ the i^{th} mode shape of the beam and $q_i(t)$ is generalized coordinates, or modal amplitudes, which depend on t [7]. Lagrange's equations that include the dissipation function D and a primary harmonic excitation at the point $x = x_f$, $F e^{i\omega t} \delta(x - x_f)$ are expressed by

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{s}_k} \right) - \frac{\partial T}{\partial s_k} + \frac{\partial D}{\partial \dot{s}_k} + \frac{\partial V}{\partial s_k} = Q_k + \sum_{v=1}^r \lambda_v \frac{\partial f_v}{\partial s_k} \quad (5.19)$$

Where λ_v Lagrange's multipliers and subscript r are the number of restrictions of the problem and $k = 1, \dots, n + 2r$. Q_k is the generalized force $Q_k = F e^{i\omega t} \phi(x_f)$ being x_f the point where force is applied. It is also necessary to include the equation that represents the restrictions.

$$f_v = \sum_{i=1}^n \phi_i(x_v) q_i(t) - z_{0v} = 0 \quad (5.20)$$

Which allow eliminating the redundant coordinates of the problem. Talking these expressions into account and replacing the expression of T, V, D and Q_k in equation 5.19. Lagrange's equation can thus be written as

$$M_i \ddot{q}_i + 2\xi_i M_i \omega_i^2 \dot{q}_i + M_i \omega_i^2 q_i = F \phi_i(x_f) e^{i\omega t} + \sum_{v=1}^r \lambda_v \phi_i(x_v) \quad i = 1, \dots, n \quad (5.21)$$

$$m \ddot{z} + c_v (\dot{z}_v - \dot{z}_{0v}) + k_v (z_v - z_{0v}) = 0, \quad (5.22)$$

$$c_v (\dot{z}_v - \dot{z}_{0v}) + k_v (z_v - z_{0v}) = \lambda_v \quad (5.23)$$

Where $\xi_i = c_{vi} / 2M_i \omega_i$ and $v = 1, \dots, r$. This system describes the dynamics of a beam and the DVA

separately. To consider them as a whole, equation 5.21, 5.22 and 5.23 can be written as

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{f_p\} + \{f_s\} \quad (5.24)$$

To produce a system of n equations by n unknowns where $[M]_{n \times n}$ results in a diagonal matrix whose elements are the modal masses $[C]_{n \times n}$ is the modal damping matrix, and $[K]_{n \times n}$ is the modal stiffness matrix. $\{f_p\}$ are the external forces and $\{f_s\}$ are the forces applied by the DVA. Assuming all variables are harmonic:

$q_i = \bar{q}_i e^{i\omega t}$, $z = \bar{z}_i e^{i\omega t}$, $z_0 = \bar{z}_0 e^{i\omega t}$, it is possible to obtain the modal amplitude vector $\{\bar{q}\}$.

$$\{\bar{q}\} = ([K_d] + [\phi][K_{abs}][\phi]^T)^{-1} F \{\phi(x_f)\} \quad (5.25)$$

Where $[K_d] = [K] - \omega^2[M] + i\omega[C]$ is the dynamic stiffness of the beam and $[\phi]_{n \times v}$ is the matrix of modal amplitudes whose components are the modal amplitudes of the i^{th} mode evaluated at the v^{th} absorbers position. $\{\phi\}$ is the n vector of mode shapes.

Here the uncoupled 2DOF system (two 1DOF DVA) where $[K_{abs}] = \text{diag}(K_i)$ and (K_i) is given as [12]

$$K_i = \frac{-\omega^2 m_i (1 + i2\xi_{ai}(\omega/\omega_{ai}))}{1 - (\omega/\omega_{ai})^2 + i2\xi_{ai}(\omega/\omega_{ai})} \quad (5.26)$$

Where, $\xi_{ai} = c_i / 2M_i\omega_{ai}$ and $\omega_{ai} = \sqrt{\frac{k_i}{m_i}}$.

Thus the response of the beam after the DVA is connected to the system is given by substituting equation 5.24 in equation 5.18 we get,

$$w(x, t) = \sum_{i=1}^n \phi_i(x) ([K] - \omega^2[M] + i\omega[C] + [\phi][K_{abs}][\phi]^T)^{-1} F \{\phi(x_f)\} (\cos \omega t + i \sin \omega t) \quad (5.27)$$

RESULTS AND DISCUSSIONS

Response of the Beam without Absorber

Forced response of the beam as a continuous system without absorber is calculated using equation 1.33. The force is applied at a distance of 0.227 m from the left end of the beam. The response of the beam for two different values of forces is taken for the study, i.e., F_0 . The response graph is taken from the first natural frequency. figure 19 shows the amplitude of vibration of the beam without the absorber.

Figure 19: Response of Beam without Absorber

The amplitude of vibration of the beam is calculated as 0.5×10^{-2} m and 2×10^{-2} m to applied force of 100 and 400 N respectively.

Response of the Beam with Tuned Absorber

Case 1: Considering as undamped two DOF systems. Forced response of the beam with absorber tuned to the first natural frequency is considered as a two degree of freedom system was calculated using equation 5.13 and 5.14. The force is applied at the same distance as applied to the beam without the absorber. The response graph is taken from the first natural frequency. Here the amplitude of vibration of the beam is zero.

Case 2: Beam is considered as a continuous system with cantilevered mass absorber considered as undamped spring-mass system. The spring mass system is attached to the middle of the beam. In this case the absorber is tuned to a frequency of 87 Hz. It is found that the response of the beam is very close to zero, i.e., is 5×10^{-4} which is shown in figure 20.

Figure 20: Response of Beam with Translational Absorber

Case 3: Beam with Translational Absorber and Rotational Absorber

Modal Analysis of the Beam and Absorber

Modal analysis is the study of a dynamic system in terms of natural frequencies of vibration, mode shapes and damping factors. These properties are also called vibration properties of the model and can be formulated over a range of frequency for a certain structure to obtain the model. The behavior of a structure as a response to external forces and also the structural dynamic properties like mass, stiffness, damping can be analyzed with a comparison to the model.

Figure 21: (a) Modal Analysis of Beam, (b) Modal Analysis of the Absorber

The figure 21 (a) shows the modal analysis of beam in 2D with its first natural frequency is equal to 90.363 Hz, (b) shows the modal analysis of absorber in 2D with the absorber natural frequency is equal to 81.28 Hz.

Figure 22: (a) Modal Analysis of the Combined System in 2D, (b) Modal Analysis of the combined system in 3D

The figure 22 (a) & (b) shows the modal analysis of the combined system with the absorber placed at the middle of the simply supported beam in 2D and 3D i.e., the behavior of the system when it is exciting. The natural frequency of the system is 53.282Hz.

Harmonic Analysis of Beam and Absorber

Here the harmonic analysis of the beam along is carried out to find the maximum amplitude of vibration that occurs at the point of natural frequencies. The applied frequency is made equal to that of the frequency of the beam. The amplitude of vibration of beam is observed to be 0.75×10^{-2} m and 2×10^{-2} m for force equal to 100 and 400 N respectively. Figure 23 (a) & (b) shows the amplitude of beam at force equal to 100 and 400 N respectively.

Here harmonic analysis of beam with absorber is carried out to find the amplitude of vibration that occurs at the point of natural frequencies. The applied frequency is made equal to that of the frequency of the beam. The absorber frequency with is attached to the beam is made equal to that of the excited frequency. The amplitude of vibration of beam is observed to be 1.5×10^{-4} m and 1.2×10^{-3} m for force equal to 100 and 400 N respectively. Figure 24 (a) & (b) shows the amplitude of beam at applied force equal to 100 and 400 N respectively. But it is observed that, at different points on the beam the amplitude of vibration is different.

Figure 23: (a) Harmonic Analysis of the Beam with Load applied at Various Places, (b) Harmonic Analysis of Beam with Load at Particular Point

Figure 24: (a) Harmonic Analysis of the Combined with Load at Various Positions,
(b) Harmonic analysis of combined system with Applied Force

Absorber kept in Various Position of the Beam

In this study, the absorber is made to keep at various positions of the beam and analysis is being carried out. The figure 25 shows the position of the absorber placed on the beam and the node point where the force is applied. From this study it is clear that when the translational type of absorber is placed in the node where the load is applied, the vibration at that node is reduced to a maximum extent. Here load is applied at node 4 and when the absorber is placed at that point the amplitude of vibration at that node is zero. Figure 26 (a) and (b) shows the amplitude of the beam with and without absorber respectively.

Figure 25: Beam with TTA under Point Harmonic Load

Figure 26: (a) Response of the Beam with and Without Translational Absorber,
(b) Response of the Beam with TTA at Various Positions

Comparison of Beam Response

From the table 4 we can compare the theoretical and ANSYS results, which provide you the necessary detail how the DVA controls the vibration of the system.

Table 4 :Comparison of Theoretical and ANSYS Result

		Theory		Ansys
		As Continuous System	As Spring Mass System	
Modal analysis	Beam	90.667 Hz	90.619 Hz	90.592 Hz
	Absorber	87.396 Hz	84.286 Hz	83.722 Hz
	Combined system	53.96 Hz	53.11 Hz	53.28 Hz
Harmonic analysis	Beam	$2 \times 10^{-2} \text{m}$	$2.25 \times 10^{-2} \text{m}$	$2.12 \times 10^{-2} \text{m}$
(response of beam)	Combined system	$1.5 \times 10^{-3} \text{m}$	$1.9 \times 10^{-3} \text{m}$	$2 \times 10^{-3} \text{m}$

CONCLUSIONS

In this study, a self-adaptive dynamic vibration absorber has been designed using a dual cantilevered mass arrangement. This absorber is effective in controlling single as well as multiple frequency vibrations on a simply supported beam. It has been proved with finite element analysis and the Euler beam theory that the forced beam vibration under point harmonic excitation can be isolated by attaching the proposed absorber. Expressions for the beam response are derived from different types of absorber arrangement. Both numerical and finite element analysis has been done for prediction of vibration isolation using the proposed dynamic absorber. The numerical result shows that beam vibration under point harmonic excitations can be isolated in a region of the beam by the proposed absorber but not the traditional translational absorber alone.

Forced vibration of a beam is calculated and compared with the finite element analysis for the cases of the beam carrying the proposed absorber, the beam carrying only the traditional translational absorber, and the beam carrying no absorber. Two different types of arrangement are studied and it is observed that Beam along with the dual mass cantilever can suppress the vibration only in one axis. Whereas the other absorber can isolate the vibration in multiple axis. Translational motion absorber can isolate the vibration more effectively only at the point where it is attached. Furthermore, isolation of vibration takes place when the absorber is attached to the node where the load is exactly applied. But by attaching the combined translational motion and rotational motion absorber to the beam at the same node can isolate the vibration more effectively on the remaining nodes. Hence the beam with combined translational type and rotational type absorber has the best performance of vibration suppression in both single as well as in multiple axis transverse vibrations.

REFERENCES

1. Cha, C. P., (2005) 'Enforcing nodes at required locations in a harmonically excited structure using simple oscillators', *Journal of Sound and Vibration* 279 (3–5(21)) 799–816.
2. Den Hartog, J. P., (1985) 'Mechanical Vibrations', Dover Publications Inc.
3. Dayou J, (2006) 'Fixed-points theory for global vibration control using vibration neutralizer', *Journal of Sound and Vibration* 292, 765–776.

4. Febbo M, Vera S. A. Feb (2008) 'Optimization of a two degree of freedom system acting as a dynamic vibration absorber', *Journal of Vibration and Acoustics, ASME*, vol.130.
5. James, M. L., Smith, G. M., Wolford, J. C., Whaley, P. W., (1989), 'Vibration of Mechanical and Structural Systems', Harper & Row Publishers Inc.
6. Juang, J. N. (1984), 'Optimal Design of a Passive Vibration Absorber for a Truss Beam', *Journal of Guidance, Control and Dynamics*, Vol. 7, No. 6, Nov.
7. Lee-Glauser, G., Juang, J.-N., Sulla, J. L. (1995), 'Optimal Active Vibration Absorber: Design and Experimental Results', *Journal of Vibration and Acoustics*, Vol. 117.
8. Ravi Prasad & K Prahlada Rao et al., MATLAB Program for Traditional Design and Optimization of 30 KW SCIM Motor, *International Journal of Mechanical and Production Engineering Research and Development (IJMPERD)*, Volume 4, Issue 7, November - December 2015, pp. 81-116
9. Nishimura, H., Yoshida, K., Shimogo, T. (1990), 'Optimal Active Dynamic Vibration Absorber for Multi-Degree-of-Freedom Systems', *JSME International Journal*, Vol. 33, No. 4, Ser. III, 513.
10. Pun, D., Liu, Y. B., (2000), 'on the design of piecewise linear vibration absorber', *non linear dynamics*, 22:393-413.
11. Ryan, M. W., Franchek, M. A., Bernhard, R. (1994), 'Adaptive-Passive Vibration Control of Single Frequency Excitations Applied to Noise Control', *Noise-Con94*, 461-466.
12. Rita, A. D., Mc Garvey, J. H., Jones, R., (1978), 'Helicopter Rotor Isolation Evaluation Using the Dynamic Anti-resonant Vibration Isolator', *Journal of the American Helicopter Society*, Vol. 23, No. 1, Jan, 22-29.
13. Seto, W. (1964), 'Schaum's outline of Theory and Problems of Mechanical Vibrations', McGraw Hill, Singapore, Chap. 5, 128-150.
14. Snowdon, J. C. (1968), 'Vibration of Shock in Damped Mechanical Systems', Wiley, New York.
15. Thomson, W. T. (1993), 'Theory of Vibration with Applications', Prentice Hall, 281-286.
16. Wong W. O, Tang S. L, Cheung Y. L, Cheng L, (2007) 'Design of a dynamic vibration absorber for vibration isolation of beams under point or distributed loading', *Journal Of Sound And Vibration* 301 898-908.
17. Wang, K., Kim, Y., Shea, B. (1994), 'Structural vibration control via electro rheological-fluid-based actuators with adaptive viscous and frictional damping', *Journal of Sound and Vibration*, Vol 177,227-237.
18. Wang, B. P., Kitis, L., Pilkey, W. D., Palazzolo, A. (1985), 'Synthesis of Dynamic Vibration Absorbers', *Journal of Vibration, Stress and Reliability in Design*, Vol. 107, Apr, 161.