VIBRATIONAL CHARACTERISTICS MODELING OF A PZT5A MEMS RESONATOR DISC METALLIZED WITH ALTERNATING ELECTRICAL SOURCE

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ABSTRACT

With the purpose of integrating new functionalities in micro electro mechanical resonators, frequency response of such resonators has been gained more importance since long time ago. Previous studies have only concentrated on geometries with full metallization and centralized partial metallization. This paper proposes a study of wave propagation, in case of a homogeneous circular disk excited by an AC electrical source with partial annular metallization using Legendre polynomial approach. Our study consists of solving equation of motion and characteristic equations of piezoelectricity with appropriate boundary and continuity conditions. Numerical calculation provides both modal and harmonic analysis, whatever the mode order. In addition to dispersion curves, mechanical and electrical dispersion profiles were presented in a 3D configuration. The impact of varying metallization rates on resonance frequencies is highlighted. The validity of Legendre polynomial method is confirmed through a comparison between polynomial and analytical results provided in the literature.

KEYWORDS: Legendre Polynomial, MEMS Resonator, Partial Metallization & Frequency Response

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INTRODUCTION

Micro-Electro-Mechanical systems (MEMS) technology gathered more attention since long time. Used as detectors, actuators, sensors, filters and transducers, in the frequencies upper to MHz range, they play a key role in evolutioning electronic engineering technology going from mobile telecommunication, electric filtering, and medical monitoring to non destructive evaluation and so on [1-10]. Such applications involve large variety different geometries, hollow cylinders, rectangular plates, rods and multilayered structure. For fabrication and design of such systems, efficient tools of modeling are required.

Piezoelectric MEMS modeling concerns of presenting and analyzing completely their electro-acoustic response and vibration characteristics, which are totally determined, resolving simultaneously three governing differential equations of motion and Maxwell equations [11-12]. Several investigations have been carried out on the acoustic response of MEMS resonators under electrostatic actuation.

W. T. Thomson presented in 1950 computational method to solve the propagation equations in multilayered structures known as transfer matrix method [13]. In order to overcome the problem of instability

Knopoff developed global matrix method. Unlike the transfer matrix method, global matrix method [14] is proven efficient, especially in inhomogeneous structures, but is computationally expensive requiring more time and memory resources. In 1967, Eer Nisse [15] performed a vibration method to check the vibration characteristics of piezoelectric disks that the research paid more attention looking for efficient tools of modeling. Ivina [16] presented a numerical analysis of the normal modes of circular piezoelectric plates of finite dimensions. A Rayleigh-Ritz method was applied by Kharouf et al [17] to perform the vibrational axisymmetric characteristics of hollow cylinders and disks made from piezoelectric materials. Later on, an alternative computational method was proposed to calculate the stiffness matrix components, in case of anisotropic layered structures [18]. The method consists of introducing a layer stiffness matrix associated with specific recursive algorithm. A Recursive surface impedance matrix methods for ultrasonic wave propagation in piezoelectric multilayer was also proposed by B. Collet [19].

C. Huang Huang [20] investigated transverse vibration analysis and measurement for the piezoceramic annular plate with different boundary conditions. C. H. Huang and C. C Ma [21] studied the Vibration characteristics for piezoelectric cylinders using an amplitude-fluctuation electronic speckle pattern interferometer method. In addition, Kunkel et al [22] reported the vibration behavior of a ceramic disk composed of PZT5A with the diameter to thicknesses ratio range from 0.2 to 10 using the finite elements method. Guo et al [23] studied the vibration characteristics of disks composed of PZT5A with diameter to thicknesses ratio ranging from 10 to 20 using the same method. The finite element method is limited by its major drawbacks, in case of high radiating geometries; the method requires more computational time and storage, which makes the calculation more expensive.

In order to overcome the drawbacks cited above, Legendre polynomial method is proposed to solve wave propagation problem of piezoelectric resonators. The method was proved as an accurate tool of modeling giving excellent precision for various structures [24-30]. Legendre polynomial method is based on a decomposition of the mechanical and electrical displacement fields in a double series of Legendre polynomial basis. El Maimouni et al [27] extended the Legendre polynomial method to restore the frequency response of MEMS piezoelectric resonator disk partially metallized, case of centralized metallization.

The aim of this paper is to illustrate the potentialities of the polynomial approach in studying the vibrational behavior of a cylindrical ring MEMS resonator made from PZT5A, driven to vibrate by alternating current (AC) source, in a specific case of partial metallization instead of a full metallization. The purpose is reducing the number of unknown and thus reducing the computer storage and time. The mechanism consist of dividing the cylindrical geometry into two symmetrical parts, two electrodes are applied from the top and the bottom of the outer parts of the cylinder.

In section 2, the mathematic formulations basis of orthogonal polynomial method is described and shown how boundary, continuity and symmetry condition are directly incorporated in governing equations. In section 3, numerical results based on adopted formulation including resonance and resonance frequencies, input electrical impedance and dispersion curves. Our numerical simulation is developed using Matlab Software and has been validated using analytical data.

FORMULATION OF THE PROBLEM

Let us consider a cylindrical structure, schematically shown in figure 1. The cylinder has an infinite height with R and H representing the radius and thicknesses, respectively. A uniform polarization treatment is carried out through H direction. Two infinitely thin electrical electrodes, assumed to be perfect, are applied from the top and the bottom of the outer part of the disc faces driven to vibrate by electrical source. The problem of vibration in this paper is dealt in cylindrical coordinates $O r \phi z$. The ability of our model to solve wave propagation is, cylindrical structure depends on the assumption that the electrical source is axisymmetric, and thus the propagation does not depend on azimuthal direction ϕ . The body forces are also assumed to be zero.

Figure 1: Schematic View of a Piezoelectric Disc Partially Metallized

With electric excitation source $V_0 e^{j\omega t}$

The translational equation of motion, characterizing the dynamic behavior of studied structure is given by [31-32]:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{1}{r} \frac{\partial \sigma_{r\phi}}{\partial \phi} + \frac{\sigma_{rr} - \sigma_{\phi\phi}}{r} = \rho \frac{\partial^2 u}{\partial t^2} \quad (1a)$$

$$\frac{\partial \sigma_{r\phi}}{\partial r} + \frac{\partial \sigma_{\phi z}}{\partial z} + \frac{1}{r} \frac{\partial \sigma_{\phi\phi}}{\partial \phi} + \frac{2\sigma_{r\phi}}{r} = \rho \frac{\partial^2 v}{\partial t^2} \quad (1b)$$

$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{1}{r} \frac{\partial \sigma_{\phi z}}{\partial \phi} + \frac{\sigma_{rz}}{r} = \rho \frac{\partial^2 w}{\partial t^2} \quad (1c)$$

$$\frac{1}{r} \frac{\partial (r D_r)}{\partial r} + \frac{1}{r} \frac{\partial (D_\phi)}{\partial \phi} + \frac{\partial (D_z)}{\partial z} = 0 \quad (1d)$$

Where, $\partial^2 U(M, t) / \partial t^2$ denote the acceleration of the acoustic field with $U(M, t) = (u, v, w)$ is the mechanical displacement components evaluated at the point M at the time t. σ_{ij} and D_i are the stress and electrical displacement components.

From this part, a specific change of variables either for physical constant and mechanical displacement components u , v and w will be adopted as follow:

Normalized dielectric permittivity components $\bar{\epsilon}_{ij} = \epsilon_{ij} / \epsilon_{33}$; Normalized piezoelectric $\bar{e}_{ij} = e_{ij} / \sqrt{C_{33}^D \epsilon_{33}}$;

Normalized elastic stiffness constants $\bar{C}_{ij} = C_{ij} / C_{33}^D$; Normalized electric displacement components $\bar{D}_k = D_k / \epsilon_{33}$; Normalized stress components $\bar{\sigma}_{ij} = \sigma_{ij} / C_{33}^D$, Normalized mechanical displacement $\bar{u} = u(m) \times 10^{10}$, $\bar{v} = v(m) \times 10^{10}$ and $\bar{w} = w(m) \times 10^{10}$. Where, $C_{33}^D = C_{33} + \epsilon_{33}^2 / \epsilon_{33}$. We have also: $q_1 = R / r$, $q_3 = 2z / H$ and $a_0 = R_0 / R$

The e superscript (j=M, N) refers, whatever if we are describing the metalized or non metalized area.

Normalized stress in equation 1 are expressed using generalized Hook's laws as a function of mechanical displacement and independent elastic constants as follow:

$$\bar{\sigma}_{rr}^{(j)} = \left[\bar{C}_{11} \frac{1}{R} \frac{\partial \bar{u}^{(j)}}{\partial q_1} + \bar{C}_{12} \frac{1}{R} \frac{\bar{u}^{(j)}}{q_1} + \bar{C}_{13} \frac{2}{H} \frac{\partial \bar{w}^{(j)}}{\partial q_3} + \bar{e}_{31} \frac{1}{\beta H} \frac{2}{\partial q_3} \frac{\partial \phi^{(j)}}{\partial q_3} \right] 10^{-10} \quad (2a)$$

$$\bar{\sigma}_{\phi\phi}^{(j)} = \left[\bar{C}_{12} \frac{1}{R} \frac{\partial \bar{u}^{(j)}}{\partial q_1} + \bar{C}_{11} \frac{1}{R} \frac{\bar{u}^{(j)}}{q_1} + \bar{C}_{13} \frac{2}{H} \frac{\partial \bar{w}^{(j)}}{\partial q_3} + \bar{e}_{31} \frac{1}{\beta H} \frac{2}{\partial q_3} \frac{\partial \phi^{(j)}}{\partial q_3} \right] 10^{-10} \quad (2b)$$

$$\bar{\sigma}_{zz}^{(j)} = \left[\bar{C}_{13} \frac{1}{R} \frac{\partial \bar{u}^{(j)}}{\partial q_1} + \bar{C}_{13} \frac{1}{R} \frac{\bar{u}^{(j)}}{q_1} + \bar{C}_{33} \frac{2}{H} \frac{\partial \bar{w}^{(j)}}{\partial q_3} + \bar{e}_{33} \frac{1}{\beta H} \frac{2}{\partial q_3} \frac{\partial \phi^{(j)}}{\partial q_3} \right] 10^{-10} \quad (2c)$$

$$\bar{\sigma}_{rz}^{(j)} = \left[\bar{C}_{55} \left(\frac{2}{H} \frac{\partial \bar{u}^{(j)}}{\partial q_3} + \frac{1}{R} \frac{\partial \bar{w}^{(j)}}{\partial q_1} \right) + \bar{e}_{15} \frac{1}{\beta R} \frac{1}{\partial q_3} \frac{\partial \phi^{(j)}}{\partial q_3} \right] 10^{-10} \quad (2d)$$

The electric displacement components:

$$\bar{D}_{q_1}^{(j)} = \bar{e}_{15} \left(\frac{2\beta}{H} \frac{\partial \bar{u}^{(j)}}{\partial q_3} \right) + \bar{e}_{15} \left(\frac{\beta}{R} \frac{\partial \bar{w}^{(j)}}{\partial q_1} \right) - \bar{\epsilon}_{11} \left(\frac{1}{R} \frac{\partial \phi^{(j)}}{\partial q_1} \right) \quad (3a)$$

$$\bar{D}_{q_3}^{(j)} = \bar{e}_{31} \left(\frac{\beta}{R} \frac{\partial \bar{u}^{(j)}}{\partial q_1} \right) + \bar{e}_{31} \left(\frac{\beta}{R} \frac{\bar{u}^{(j)}}{q_1} \right) + \bar{e}_{33} \left(\frac{2\beta}{H} \frac{\partial \bar{w}^{(j)}}{\partial q_3} \right) - \bar{\epsilon}_{33} \left(\frac{2}{H} \frac{\partial \phi^{(j)}}{\partial q_3} \right) \quad (3b)$$

If we are interested by studying acoustic propagation in structure body, system of equation 1 can be written as bellow with the purpose of incorporation stress free boundary conditions:

$$\rho \pi(q_3) . \theta(q_1) . \left(\partial^2 U(M, t) / \partial t^2 \right) = \nabla . [\sigma(M) . \pi(q_3) . \theta(q_1)] \quad (4)$$

Where:

$$\pi(q_3) = \begin{cases} 1 & -1 \leq q_3 \leq 1 \\ 0 & \text{Elsewhere} \end{cases} \quad \theta(q_1) = \begin{cases} 1 & 0 \leq q_1 \leq 1 \\ 0 & \text{Elsewhere} \end{cases} \quad (5)$$

For bounding surfaces, following conditions must be fulfilled:

$$\begin{aligned}\bar{\sigma}_{rr}^{(M)}(a_0) &= \bar{\sigma}_{rr}^{(N)}(a_0) \quad ; \quad \bar{\sigma}_{rz}^{(M)}(a_0) = \bar{\sigma}_{rz}^{(N)}(a_0) \\ \bar{D}_{q_1}^{(M)}(a_0) &= \bar{D}_{q_1}^{(N)}(a_0) \quad ; \quad \bar{D}_{q_3}^{(N)}(q_3 = \pm l) = \bar{D}_{q_1}^{(M)}(q_1 = l) = 0 \\ \bar{\sigma}_{rz}^{(M)}(q_3 = \pm l) &= \bar{\sigma}_{rr}^{(M)}(q_1 = +l) = \bar{\sigma}_{zz}^{(M)}(q_3 = \pm l) = \bar{\sigma}_{rz}^{(M)}(q_1 = \pm l) = \bar{\sigma}_{zz}^{(N)}(q_3 = \pm l) = \bar{\sigma}_{rz}^{(N)}(q_3 = \pm l) = 0\end{aligned}$$

We aim now to express mechanical and electrical displacement. Their expressions are expanded on a double series of Legendre polynomial basis:

Non metalized area: $0 \leq q_1 \leq a_0$:

$$\begin{cases} \bar{u}^{(N)}(q_1, q_3, t) = \sum_{mn} p_{m,2n}^{(q_1, N)} Q_m^{(N)}(q_1) Q_{2n}(q_3) e^{j\alpha t} \\ \bar{w}^{(N)}(q_1, q_3, t) = \sum_{mn} p_{m,2n+1}^{(q_3, N)} Q_m^{(N)}(q_1) Q_{2n+1}(q_3) e^{j\alpha t} \\ \bar{\varphi}^{(N)}(q_1, q_3, t) = \bar{\varphi}^{(M)}(a_0, q_3, t) + (q_1 - a_0) \sum_{mn} r_{m,2n+1}^{(N)} Q_m^{(N)}(q_1) Q_{2n+1}(q_3) e^{j\alpha t} \end{cases} \quad (6)$$

Metalized area $0 \leq q_1 \leq a_0$:

$$\begin{cases} \bar{u}^{(M)}(q_1, q_3, t) = \bar{u}^{(N)}(a_0, q_3, t) + (q_1 - a_0) \sum_{mn} p_{m,2n}^{(q_1, M)} Q_m^{(M)}(q_1) Q_{2n}(q_3) e^{j\alpha t} \\ \bar{w}^{(M)}(q_1, q_3, t) = \bar{w}^{(N)}(a_0, q_3, t) + (q_1 - a_0) \sum_{mn} p_{m,2n+1}^{(q_3, M)} Q_m^{(M)}(q_1) Q_{2n+1}(q_3) e^{j\alpha t} \\ \bar{\varphi}^{(M)}(q_1, q_3, t) = \left[\frac{V_0}{2} q_3^3 + (q_3^2 - 1) \sum_{mn} r_{m,2n+1}^{(M)} Q_m^{(M)}(q_1) Q_{2n+1}(q_3) \right] e^{j\alpha t} \end{cases} \quad (7)$$

where:

$$Q_m^{(N)}(q_3) = \sqrt{(2m+1)/a_0} P_m((2q_1/a_0) - 1);$$

$$Q_m^{(M)}(q_3) = \sqrt{(2m+1)/(1-a_0)} P_m(2q_1/(1-a_0) - (1+a_0)/(1-a_0));$$

$Q_m(q_3) = [(2m+1)/2] P_m(q_3)$ and $Q_n(q_3) = [(2m+1)/2] P_n(q_3)$ are the Legendre polynomial of degree m and n respectively.

The expansions coefficients $p_{m,2n}^{(q_1, i)}$ and $r_{m,n}^{(i)}$ are chosen to satisfy the continuity of the mechanical and electrical displacement components, when $(q_1 = a_0)$, we have:

$$\bar{u}^{(2)}(q_1 = a_0) = \bar{u}^{(1)}(q_1 = a_0) \quad \bar{w}^{(2)}(q_1 = a_0) = \bar{w}^{(1)}(q_1 = a_0) \quad \bar{\varphi}^{(2)}(q_1 = a_0) = \bar{\varphi}^{(1)}(q_1 = a_0).$$

The term $q_3^2 - 1$ in $\bar{\varphi}^{(2)}$ impose that: $\bar{\varphi}^{(2)}(q_3 = +l) - \bar{\varphi}^{(2)}(q_3 = -l) = V_0$

The calculation process is done through following steps:

- Mechanical and electrical displacements components expressed in Equation.7 and 8 are integrated in the system of equation.1.
- The system of equations.1 is multiplied by $Q_j^{(*,N)}(q_1) \times Q_k^*(q_3)$ and $Q_j^{(*,M)}(q_1) \times Q_k^*(q_3)$ for area N and M respectively for the orthogonalization process. The orthogonality of polynomials allows obtaining a plenty of zero within resulting eigen-matrix which make the calculation fast and simple.
- (q_1) Is then integrated from 0 to a_0 for area (N) and from a_0 to 1 for area (M). The same (q_3) is integrated from -1 to 1.

The symmetry condition, of studied structure and electrical source, and the parity condition of filed quantities allows obtaining a system with an important reduction on the number of terms regarding $Q(q_3)$. Finally, a system of $6 \times (m+1) \times (n+1)$ equations with $6 \times (m+1) \times (n+1)$ unknowns is obtained. Such a system can be rewrite as following form:

$$A_{mnjk} P_{m,n}^{(b)} + B_{mnjk} r_{m,n} + f_{jk}^1 V_0 = -\pi^2 \Omega^2 M_{mnjk} P_{m,n}^{(b)} \quad (8)$$

$$C_{mnjk} p_{m,n}^{(b)} + D_{mnjk} r_{m,n} + f_{jk}^2 V_0 = 0 \quad (9)$$

Where normalized angular frequency is expressed as: $\Omega = \omega(H / \pi) \sqrt{\rho / C_{33}^D}$

From Equation.9, we have:

$$r_{m,n} = -[D_{mnjk}]^{-1} [C_{mnjk} p_{m,n}^{(b)} + f_{jk}^2 V_0] \quad (10)$$

Substituting equation.10 in equation.8, we obtain:

$$[AA_{mnjk} + \pi^2 \Omega^2 M_{mnjk}] p_{m,n}^{(b)} = BB_{mnjk} V_0 \quad (11)$$

Later on, normalized electric admittance is calculated throughout harmonic analysis and resonance and anti resonance frequencies throughout modal analyses.

Harmonic Analyses

We aim through harmonic analysis to express the normalized electric input admittance. We start form the expressions of the displacement current density $J = i\omega D_z$ and the average of electrical current that flows through the electrode $I_0 = j\omega \epsilon_{33} \iint D_z(r, z) dS$. Obtained expression is:

$$\bar{Y}(\Omega) = 1 - \frac{1}{V_0} \alpha_{m,n}^{(b)} p_{m,n}^b(\Omega) \quad (12)$$

Modal Analyses

The cancellation of electrical source ($V_0 = 0$) in equation.11 allows determining resonant frequencies.

$$\Omega_r^2 p_{m,n}^{(b)} = (1/\pi^2) AA_{mnjk} [M_{mnjk}]^{-1} \quad (13)$$

In case electric input admittance equation 12 is vanished ($I=0$), anti resonance frequencies are calculated by following expression:

$$[\Omega_a^2] = (1/\pi^2) [M_{mnjk}]^{-1} (BB_{mnjk} \alpha_{m,n}^{(b)} (\Omega_{Ant}) - AA_{mnjk}) \quad (14)$$

NUMERICAL RESULTS

Numerical calculations based on described formulation above are presented in this part. Piezoelectric materials studied in this work are PZT5A, ZnO, AlN and PIC. Physical properties of those materials are reported in table 1. The radius and the thicknesses of the disc are $H=1\text{mm}$, $R=15\text{mm}$. The summation over Legendre polynomials in equations 6 and 7 become negligible at higher order, we have limited the truncation number on $M=N=15$.

Table 1: Physical Properties of PZT5A [34], ZnO, AlN and PIC

Parameter	PZT5A	ZnO	AlN	PIC
ρ ($10^{+3} \text{ kg.m}^{-3}$)	7.750	5.676	3.260	7.800
Stiffness Constant C_{ij} ($\times 10^{11} \text{ N/m}^2$)				
C11	1.21	20.97	3.45	1.076
C12	0.754	12.11	1.25	0.631
C13	0.752	10.51	1.2	0.639
C33	1.11	21.09	3.95	1.004
C44	0.211	4.25	1.18	0.196
C66=(C11-C12)/2	0.226	4.43	1.1	0.222
Piezoelectric Constant e_{ij} (C.m^{-2})				
e15	12.3	-0.59	-0.48	11.97
e31	-5.4	-0.61	-0.58	-9.52
e33	15.8	1.14	1.55	15.14
Permittivity ϵ_{ij} ($\times 10^{-11} \text{ F.m}^{-1}$)				
ϵ_{11}	811.026	738	800	983.2
ϵ_{33}	734.88	783	950	818.5

Resonance and Anti Resonance Frequencies

The series of resonance frequencies are yielded from eigenvalues in case short circuited electrodes ($V_0 = 0$) while resonance frequencies are calculated in case of open circuited electrodes ($I = 0$).

For different values of metallization rate $\tau(\%) = (R_0/R) \times 100$ we calculate resonance and anti resonance frequencies. Obtained results using polynomial approach are compared with results calculated analytically [35].

For each value, an associated accuracy is calculated by following relations:

$$\epsilon_r(\%) = |(fr_A - fr_P) / fr_A| \times 100 ; \epsilon_a(\%) = |(fa_A - fa_P) / fa_A| \times 100$$

Where subscripts a, A and P denotes anti resonance, Analytic and Polynomial respectively

In the tables 2 and 3, we have presented the resonant and anti resonant frequencies in the case of the metallization rate $\tau = 20\%$ and $\tau = 70\%$ respectively.

Table 2: Normalized Resonance and Anti Resonance Frequencies of the 1st, 2nd and 5th Contour Modes, $\tau = 20\%$

Modes	fr_A (kHz)	fr_p (kHz)	ε_r (%)	fa_A (kHz)	fa_p (kHz)	ε_a (%)
1	66141	65414.8	1.11	80139	83472.68	3.99
3	281018	275591.9	1.96	282817	278831.5	1.42
5	481302	475766.2	1.16	482006	477518.7	0.93

Table 3: Normalized Resonance and Anti Resonance Frequencies of the 1st, 2nd and 5th Contour Modes, $\tau = 70\%$

Modes	fr_A (kHz)	fr_p (kHz)	ε_r (%)	fa_A (kHz)	fa_p (kHz)	ε_a (%)
1	80416	77417.5	3.72	84118	85874.34	2.04
3	307780	305012.1	0.89	326743	325558.5	0.36
5	531742	522882	1.66	534048.5	527687.3	1.20

As shown in these figures, the higher accuracy is less than 3.72%, which means that the polynomial approach is an efficient tool of modelling. Such discrepancy between analytical and polynomial results is occurred by the contrast at continuity condition between metallised and non metalized parts.

From those results, we can say that when metallization rate increase, the frequency resonance increase too which means that the frequency response of a such resonator can be controlled by the rate of metallization τ .

Normalized Electric Admittance

The vibration behaviour of MEMS resonator disc consists of analysing the normalized electric input admittance. Figure.2 and Figure.3 show the normalized electric admittance as function of normalized frequency for $\tau = 40\%$ and $\tau = 60\%$ respectively. A comparison with analytical results given by a method expressed in appendix which is valid only in a specific value of D/H.

Figure 2: Normalized Electric Input Admittance of PZT5A Resonator as a Function of Normalized Frequency of Contour Modes with Metallization Rate of 20% (a), 70% (b) M=N=15

Mechanical and Electrical Displacements Profiles

Our method allows also presenting the distribution of the mechanical displacement components through the resonator in a 3D configuration, according to equation 7 and 8. Figure 4 and figure 5 shows axial and radial components for two metallization rate, $\tau = 40\%$ and $\tau = 90\%$, respectively.

Figure 4: Radial (a) and Axial (b) Components of Displacement Profiles for $\tau = 70\%$, $f_r = 425356$ kHz, $M=N=15$

Figure 5: Electrical Displacement Profile for for $\tau = 70\%$, $f_r = 425356$ kHz, $M=N=15$

Dispersion Curves Profiles

To be complete and reliable, the polynomial approach must gives not only harmonic and modal analyses, but also dispersion curves and electromechanical coupling coefficient, whatever the materiel properties or mode order. Figure 6 gives obtained results of resonance and anti resonance frequencies profiles (a) and (b) respectively, as a function of diameter to thicknesses ratio. As the resonator performance is characterized by electromechanical coupling coefficient k^2 , figure 7 retrieves the later for the first mode as a function of diameter to thicknesses ratio D/H for different piezoelectric materials.

Figure 6: Resonance Frequency (a) and Anti Resonance Frequency (b) Profile Dispersion for $\tau = 50\%$, $M=N=15$

Figure 7: Electromechanical Coupling Coefficient for the First Mode as a Function of Diameter to Thicknesses Ratio D/H for Different Piezoelectric Materials

As shown in figure above, the polynomial approach is able to destitute the electrometrical coupling coefficient, whatever the material properties. However, PZT5A presents a very important coupling coefficient in studied case.

CONCLUSIONS

An investigation of wave propagation problem has been outlined using a mapped orthogonal polynomial approach. Shown are the results, for a cylindrical micro electromechanical resonator made of piezoelectric material, including the range of frequencies in which, the device vibrate, electrical input admittance and dispersion curves. The effect of varying the metallization rate was also explained. Partial metallization allows obtaining important economics in time computing and data storage. With the position-dependent physical constant associated to a specific treatment of mechanical and electrical boundary and continuity conditions directly incorporated in the equation of motion, the method is simple to implement, and proven as an efficient tool of modelling.

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APPENDIX

Expression of analytical normalized electric input admittance

The normalized electric admittance is expressed by:

$$\bar{Y} = 1 + \frac{\frac{2d_{31}^2}{\epsilon_{33}^T s_{11}^E (1-v^E)^2}}{\left[1 - \frac{2d_{31}^2}{\epsilon_{33}^T s_{11}^E (1-v^E)^2}\right] \left[1 - \left(\frac{R_0}{R}\right)^2\right]} \times M$$

Where:

$$Minv = inv \begin{pmatrix} I_1(\eta R_0) & -I_1(\alpha R_0) & -Y_1(\alpha R_0) \\ B_0 s_{11}^E [\eta R I_0(\eta R_0)] - (1-\sigma) \frac{I_1(\eta R_0)}{\frac{R_0}{R}} & -\delta \left[\alpha R I_0(\alpha R_0) - (1-v^E) \frac{I_1(\alpha R_0)}{\frac{R_0}{R}} \right] & -\delta \left[\alpha R Y_0(\alpha R_0) - (1-v^E) \frac{Y_1(\alpha R_0)}{\frac{R_0}{R}} \right] \\ 0 & -\delta [\alpha R I_0(\alpha R) - (1-v^E) I_1(\alpha R)] & \delta [\alpha R Y_0(\alpha R) - (1-v^E) Y_1(\alpha R)] \end{pmatrix}$$

where:

$$B_0 s_{11}^E (1-\sigma) = \frac{1}{1+(v^E)} \quad (1-\sigma) = \frac{2(1-v^E)(1-k_p)}{2-(1-v^E)k_p^2} \quad \delta = \frac{1}{1-(v^E)^2}$$

s_{11}^E elastic compliance constant. v^E Poisson ratio. ϵ_{33}^T : Dielectric constant at constant stress. d_{31} : Piezoelectric constant.