THEORETICAL APPROACH TO PREDICT FRICTION COEFFICIENTS OF VISCOUS NON-NEWTONIAN LIQUIDS IN TURBULENT PIPE FLOW

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ABSTRACT

OBJECTIVES

To establish analytical expressions for the thickness of laminar sublayer and turbulent boundary layer and friction coefficients of turbulent purely viscous non-Newtonian pipe flows as the function of apparent Reynolds number.

METHODS/STATISTICAL ANALYSIS

The turbulent boundary layer concepts are adopted to solve the problem of non-Newtonian flow through a pipe. The Blasius approach is considered in solving laminar sublayer and turbulent core regions which are dealt separately. Further, the influence of polymer concentration or power law index on turbulent flow behavior along with friction coefficients is studied. Further simulation is performed using Ansys Fluent v14.5 to validate the results obtained from the theoretical analysis.

FINDINGS

The normalized thicknesses of laminar sublayer and turbulent core are predicted theoretically in terms of apparent Reynolds number, Re_a , and power law index, n . From the present analysis, it is found that the normalized thickness for laminar sublayer is 12.25. The variation of fluid viscosity at laminar sublayer with respect to the concentration of polymer and Reynolds number are studied. The thicknesses and velocities at laminar sublayer are found from present analysis and the same compared with the Newtonian results. The range of apparent Reynolds number considered for present work is 10000-50000 and concentration range of polymer is 0.5 to 2.0 wt%. The expression of friction coefficients obtained from analysis yielded results which compared with the empirical equations available in literature and found to be in good agreement. The reduction in friction coefficients is observed with increase in the concentration of polymer. Further drag reduction also validated in comparison with Newtonian fluid flows at given Reynolds number. The present results are also in good agreement with simulation results obtained from AnsysFluent v14.5.

APPLICATION / IMPROVEMENTS

The present work finds applications in various fields like chemical, pharmaceutical, food industries and in defense.

KEYWORDS: Friction Coefficients, Pseudoplastic, Laminar Sublayer & Turbulent Pipe Flows

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INTRODUCTION

Non-Newtonian liquids have applications in chemical, pharmaceutical, and food processing industries. They are also used for reducing drag and noise in defense applications. They are classified as purely viscous, viscoelastic and time -dependent liquids¹ according to their characteristics. The purely viscous non-Newtonian liquids are further classified as pseudoplastic, dilatant and viscoplastic liquids.

Several experimental investigations are available in the literature on viscous non-Newtonian liquids to study the friction and heat transfer coefficients in laminar and turbulent flows for various geometries, viz., in pipes²⁻⁶, in rectangular channels⁷⁻¹⁰, square ducts¹¹⁻¹², and over immersed bodies¹³. A survey of literature on non-Newtonian liquids is presented by Hartnett and Cho¹⁴.

The shear stress for non-Newtonian pipe flow is defined as

$$\tau_w = K \left(\left. \frac{\partial u}{\partial y} \right|_{y=0} \right)^n = K \left(A \frac{8U}{D} \right)^n \text{ where } A = \frac{3n+1}{4n} \quad (1)$$

K is the consistency index and n is the flow behavior index. Apparent Reynolds number is defined for the case of non-Newtonian liquids¹⁵ as

$$Re_a = \frac{\rho U D}{\mu_a} \text{ where } \mu_a = K \left(A \frac{8U}{D} \right)^{n-1} \quad (2)$$

μ_a is known as the apparent viscosity.

Some important empirical equations available in the literature on friction coefficients are given below for the case of the turbulent flow of viscous non-Newtonian liquids in pipes.

Friction Coefficients

Dodge and Metzner¹⁶ developed a semi-empirical equation in implicit form for fully developed turbulent flows for the friction coefficient, f as a function of generalized Reynolds number, Re' where $Re' = Re_a/A$.

$$\frac{1}{\sqrt{f}} = \frac{4}{n^{0.75}} \log \left[Re'^{1-\frac{n}{2}} \right] - \frac{0.4}{n^{1.2}} \quad (3)$$

Another equation proposed by Dodge and Metzner¹⁶ is given as

$$f = \frac{\alpha}{Re_a^\beta} \quad (4)$$

where $\alpha = (0.0665 + 0.01175 * n)$ and $\beta = (0.365 - 0.177 * n + 0.062 * n^2)$

Yoo³ conducted experiments with carbopol solutions and calculated friction coefficients from pressure drop data. He compared his experimental data with the empirical equation of Dodge and Metzner¹⁶. Yoo³ proposed another empirical equation from his experimental data in a form similar to Blasius equation.

$$f = 0.079 n^{0.675} Re^{-0.25} \quad (5)$$

Similarly, other correlations for friction coefficients proposed by various authors for turbulent flow of pseudo-plastic power-law liquids through pipes are presented below.

Shaver and Merrill¹⁷,

$$f = \frac{0.079}{n^5 (Re')^\alpha}, \text{ where } \alpha = 2.63/10.5^n \quad (6)$$

Irvine et al¹⁸,

$$f = \left(\frac{\alpha}{Re_a} \right)^{\frac{1}{1+3n}} \text{ where } \alpha = \frac{2(n+4)}{7n_A 3n^2} \quad (7)$$

Desouky and El Emam¹⁹,

$$f = 0.125n \sqrt{n} \left(0.0112 + (Re')^{-0.3185} \right) \quad (8)$$

El-Emam et al.²⁰,

$$f = \left[\frac{n}{(3.072 - 0.143n)} \left((Re')^\beta - 0.00065 \right) \right] / 4 \quad (9)$$

Here $\beta = n / (0.282 - 4.211n)$

Rudman et al²¹ dealt the turbulent flow of pseudo -plastic fluid flows with a direct numerical simulation using the spectral element-Fourier method. Further the above problem studies by Gao and Zhang²² using statistical approach and found Dodge and Metzner¹⁶ equation best fit to any experimental data found in the literature.

It is evident that above works are done by experimental, numerical simulation and statistical approaches. However theoretical analyses based on boundary layer phenomenon are not available in the literature for the problem of turbulent flow of pseudoplastic viscous liquids. Hence an attempt is made present in this direction. Further above problem solved using Ansys Fluent and results were verified with theoretical results.

MATHEMATICAL FORMULATION

The physical model considered is as follows. A viscous non-Newtonian pseudoplastic ($n < 1$) liquid is in fully developed turbulent flow through a circular pipe. The following velocity profiles are assumed in the laminar sublayer and turbulent core with the respective boundary conditions.

Laminar Sublayer

$$(0 \leq y \leq \delta_{l,m})$$

$$\frac{u}{u_l} = \frac{y}{\delta_{l,m}} \quad (10)$$

$\delta_{l,m}$ is the thickness of the laminar sublayer, and u_l is the velocity at the edge of the laminar sublayer.

Thus $\delta_{l,m}$ refers to momentum transfer.

Boundary Conditions

$$u = 0 \text{ at } y=0 \text{ and } u = u_1 \text{ at } y = \delta_{l,m} \quad (11)$$

For the turbulent core ($\delta_{l,m} < y \leq R$),

$$u = \frac{60U}{49} \left(\frac{y}{R} \right)^{1/7} \quad (12)$$

where U is the average velocity defined by the equation

$$U = \frac{1}{\pi R^2} \int_0^R 2\pi r u dr \quad (13)$$

where $r = R - y$

The shear stress at the pipe wall is obtained using Equation (10)

$$\tau_w = K \left(\frac{u_1}{\delta_{l,m}} \right)^n \quad (14)$$

The shear velocity u^* is obtained as follows.

$$u^* = \sqrt{\frac{\tau_w}{\rho}} = \left(\frac{K}{\rho} \right)^{1/2} \left(\frac{u_1}{\delta_{l,m}} \right)^{n/2} \quad (15)$$

u_1 , the velocity at $y = \delta_{l,m}$ is obtained from Equation (12) and it is given as

$$u_1 = \frac{60}{49} U \left(\frac{\delta_{l,m}}{R} \right)^{1/7} \quad (16)$$

The viscosity in the laminar sublayer, μ_{sl} is defined by the following equation

$$\mu_{sl} = K \left(\frac{\tau_w}{K} \right)^{\frac{n-1}{n}} = K \left(\frac{u_1}{\delta_{l,m}} \right)^{n-1} \quad (17)$$

u_1 in Equations. (14) and (15) is eliminated by making use of Equation (16). An equation is obtained for $\delta_{l,m}^+$, the normalized thickness of the laminar sublayer as presented

$$\delta_{l,m}^+ = \frac{\delta_{l,m} u^*}{\mu_{sl}} = \left(\frac{K}{\rho} \right)^{-\frac{1}{2}} u_1^{1-\frac{n}{2}} \delta_{l,m}^{\frac{n}{2}} \quad (18)$$

However, the value of $\delta_{l,m}^+$ is determined with the help of Blasius equation for Newtonian liquids by the following procedure. The Blasius equation for the friction coefficient for Newtonian liquid in turbulent flow through the pipe is given by

$$\frac{f}{2} = 0.0395 \left(\frac{\rho U D}{\mu} \right)^{\frac{1}{4}} \quad (19)$$

Further

$$\frac{f}{2} u^{*2} = \frac{\mu}{\rho} \frac{u_l}{\delta_{l,m}} \quad (20)$$

Making use of Equations (19) and (20), the following equation is obtained for $\delta_{l,m}^+$.

$$\delta_{l,m}^+ = \frac{\delta_{l,m} u^*}{\nu} = 12.2572 \quad (21)$$

It is assumed in the present analysis that the same value holds good for the thickness of normalized laminar sublayer, $\delta_{l,m}^+$ for the case of non-Newtonian liquid. Hence from Equations, (18) and (21), the following equation is obtained for $\delta_{l,m}$ for the case of non-Newtonian liquid.

$$\delta_{l,m} = B^n \left(\frac{K}{\rho} \right)^{1/n} u_l^{\frac{n-2}{2}} \quad (22)$$

where $B = 12.2572$

u_l appearing in Equation (22) is eliminated using Equation (16) to give the following equation for $\delta_{l,m}$

$$\frac{\delta_{l,m}}{R} = \left[\frac{K B^2}{\rho R^n} \left(\frac{49}{60 U} \right)^{(2-n)} \right]^{\frac{7}{2(1+3n)}} \quad (23)$$

Equation (23) is written as follows to express $\delta_{l,m}$ as a function of Re_a

$$\frac{\delta_{l,m}}{R} = \left[\frac{B (\sqrt{2})^n \left(\sqrt{\left(\frac{49}{60} \right)} \right)^{(2-n)}}{(\sqrt{8A})^{n-1}} \right]^{\frac{7}{(1+3n)}} \quad (24)$$

The following equation is obtained for u_l using Equations (16) and (22)

$$\frac{u_l}{U} = (B')^{\frac{1}{1+3n}} (Re_a)^{-\frac{1}{2+6n}} \quad (25)$$

$$\text{where } B' = \left[\frac{B (\sqrt{2})^n \left(\frac{60}{49} \right)^{\frac{7n}{2}}}{(\sqrt{8A})^{n-1}} \right]$$

Friction Coefficient

The friction coefficient for a viscous non-Newtonian liquid is obtained from the equation shown below.

$$\frac{f}{2} = \frac{\tau_w}{\rho U^2} = \frac{K}{\rho U^2} \left(\frac{u_l}{\delta_{l,m}} \right)^n \quad (26)$$

$\delta_{l,m}$ and u_l appearing in Equation (26) are replaced using Equation (24) and (25) respectively to yield the following equation for the friction coefficient as a function of apparent Reynolds number.

$$\frac{f}{2} = \left[\frac{C^n}{Re_a} \right]^{\frac{1}{1+3n}} \quad (27)$$

$$\text{where } C = \frac{8.255}{B^6} (8A)^{\frac{1-n}{n}}$$

It can be observed that Equation (27) becomes Blasius equation, when $n=1$, viz., for a Newtonian liquid.

Viscosity in the Laminar Sublayer, μ_{sl}

The following equation is obtained for the viscosity μ_s as defined in Equation (17) making use of Equations (22) and (25) for $\delta_{l,m}$ and u_l respectively.

$$\frac{\mu_{sl}}{\mu_a} = \left[8 \left(\frac{60}{49} \right)^{(5-6n)} (B)^6 (4A)^4 \right]^{\frac{1-n}{1-3n}} Re_a^{\frac{-3(1-n)}{(1-3n)}} \quad (28)$$

where μ_a is the apparent viscosity defined in Equation (2).

RESULTS AND DISCUSSIONS

The values of flow consistency index, K and flow behavior index, n , which are required in the calculation of f , are taken from the experimental data available in the literature for Carboxy Methyl Cellulose (CMC)²³ solutions of different concentrations in water. These values of K and n for CMC are presented in Tables 1. The power law index or flow behavior index, n are decreasing with increase in the concentration of CMC in water.

Table 1: Rheological Properties of CMC Solutions²³

S. No	Concentration of CMC in Water (%wt)	Flow Consistency Index, K	Flow Behaviour Index, n
1	0.5	0.0067	0.976
2	0.75	0.0315	0.922
3	1.09	0.056	0.863
4	1.25	0.113	0.831
5	1.5	0.391	0.736
6	1.8	1.1004	0.638
7	2	2.103	0.568

The density of the viscous non-Newtonian liquids is taken as follows: $\rho = 996 \text{ kg/m}^3$. The diameter of the pipe, through which the liquid flows, is taken as 2.5 cm. The calculation procedure is as follows. A certain value is considered for the apparent Reynolds number, Re_a in the turbulent range, viz., Re_a lying between 10000 and 50000. The mean velocity U is calculated from the equation given below.

$$U = \left[Re_a \frac{K}{\rho D} \left(\frac{D}{8A} \right)^{(1-n)} \right]^{\frac{1}{(2-n)}} \quad (29)$$

The results for friction coefficients are obtained from empirical equations (4) to (9), presented by various authors in literature¹⁶⁻²⁰ along with the present theoretical equation (27).

However to avoid ambiguity in the graph present results compared with Irvine¹⁸ and Desouky and El Emam¹⁹.

Friction Coefficients

The friction coefficients are calculated by Equation (27) of the present analysis and are shown plotted in Figure 1 for CMC solutions. The results indicate a decrease in friction coefficient with an increase in the polymer concentration in water, thus signifying a reduction in drag in CMC solutions in turbulent flow. The results from the empirical equations of Irvine¹⁸ and Desouky and El Emam¹⁹ given by Equation (7) and equation (8) are also shown in Figure 1. The agreement between the present analysis and empirical equations are found to be good. It is to be noted that the results are in good agreement at low Reynolds number and power law index close to 1 i.e., Newtonian behavior. Hence it can be stated that present theory is able to predict friction coefficients satisfactorily for viscous liquids over a wide range of concentrations and apparent Reynolds numbers.

Friction coefficient data for CMC solutions in turbulent pipe flows are obtained using AnsysFluent v14.5 and same compared with theoretical results presented in Table 2. The results obtained from present analysis are in tune with Ansys Fluent results with 3-17% deviations.

Table 2: Comparison of Present Results with Ansys Fluent Simulation Results

Apparent Reynolds Number, Re_a	n, Flow Behavior Index	f, Present Analysis	f, Fluent Values	%error
10000	0.976	0.007823	0.00721	7.96
50000		0.005193	0.00461	11.41
10000	0.863	0.007443	0.00647	13.07
50000		0.004753	0.00432	9.11
10000	0.736	0.006976	0.00582	16.57
50000		0.004224	0.00407	3.64
10000	0.638	0.006586	0.00543	17.55
50000		0.003791	0.00397	-4.72
10000	0.568	0.006293	0.00532	15.46
50000		0.003471	0.00381	-9.76

The thickness of the laminar sublayer δ_l and the velocity at the edge of the laminar sublayer u_l are calculated as functions of the apparent Reynolds number, Re_a using Equations (24) and (25). (δ_l/R) and (u_l/U) are shown plotted in Figure 2, 3 respectively for CMC solutions. For the case of a Newtonian liquid, (δ_l/R) and (u_l/U) are obtained from Blasius equation [19] are as follows

$$\frac{\delta_l}{R} = \frac{123.28}{Re^{0.375}} \quad (30)$$

$$\frac{u_l}{U} = \left(\frac{\delta_l}{R} \right)^{1/7} \quad (31)$$

The laminar sublayer thickness and velocity for the case of Newtonian liquids are also plotted shown in Figure 2 and 3 respectively. (δ_l/R) for a Newtonian liquid is found to be higher in magnitude compared to that of a non-Newtonian liquid. (u_l/U) is found to be of the same order of magnitude for Newtonian as well as non-Newtonian liquids at lower concentrations of polymer solutions. The sublayer thickness and velocities are decreasing with increase in apparent Reynolds number and

concentration of polymer.

The ratio (μ_{sl}/μ_a) is plotted as a function of Re_a in Figure 4 for CMC solutions. It is found that the ratio (μ_{sl}/μ_a) tends to unity with a decrease in Re_a . It can be observed from Figure 4 that at higher power law index or lower concentrations of CMC fluid viscosity closes to unity. Non-linearity can be observed in Figure 4 at higher polymer concentrations or low power law index which is the indication of laminarization of flow.

The percentage of drag reduction is shown in Figure 5 for CMC solutions. The percentage drag reduction is defined as

$$\%DR = (f_{nf} - f_{nnf})/f_{nf} * 100 \quad (32)$$

where f_{nf} and f_{nnf} are the friction coefficients for Newtonian and non-Newtonian liquids respectively. The friction coefficient f_{nf} for a Newtonian liquid is calculated from Blasius equation, Equation (19), while that for a non-Newtonian liquid, f_{nnf} is calculated from Equation (27). It can be observed from Figure 5 that drag reductions of 20% and 35% occur at Reynolds numbers 10000 and 50000 respectively for the CMC solution shown in the figure for the higher concentration of CMC in water. As concentration increases reduction in drag can be observed.

CONCLUSIONS

- A theoretical analysis is accomplished to obtain an expression for friction coefficient for the case of the turbulent flow of a viscous non-Newtonian liquid in a pipe.
- The theoretical analysis yields an equation (27) for the friction coefficients which agree satisfactorily with the experimental data for a wide range of system parameters such as apparent Reynolds number (10000 to 50000) and wide range concentrations (0.5-2 %wt) of Carboxy Methyl Cellulose (CMC).
- The present analysis provided expressions for laminar sublayer thickness and velocity ratios at sublayer which are very significant parameters in the analysis.
- The results indicate that the friction coefficients for the viscous non-Newtonian liquids are less than those for the Newtonian liquids. Further, the friction coefficients decrease with an increase in concentration for CMC.
- Ansys Fluent simulation results are in good agreement with theoretical results obtained from present analysis with an error ranging from 3% to 17.55%.
- The equation for friction coefficient obtained from present analysis verified with correlations available in literature apart from the equations compared in this work.

Nomenclature

A	$(3n+1)/(4n)$
B	defined in Equation 18
C	Constant presented in equation (24)
d	diameter of the pipe, m
f	friction coefficient
K	flow consistency index
l	length of the pipe
n	flow behaviour index

Table Contd.,	
R	radius of the pipe
Re_a	apparent Reynolds number
Re'	generalized Reynolds number, (Re_a/A)
U	average velocity
u	velocity
u_l	velocity at laminar sublayer
u^*	$\sqrt{\frac{\tau_w}{\rho}}$, shear velocity
y	position coordinate normal to the wall
Greek Letters	
$\delta_{l,m}$	thickness of laminar sublayer
δ_l^+	Normalized laminar sublayer thickness defined in Equation (18)
μ_a	apparent viscosity
ρ	density
τ_w	wall shear stress
μ_{sl}	Viscosity in laminar sublayer

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Figure 1: Comparison of Friction Coefficients of Present Analysis with Correlations in Literature

Figure 2: Shows the Variation of Laminar Sublayer Thickness (δ_l) with Change in Concentration and Apparent Reynolds Number

Figure 3. Variation of the Velocity at the Edge of the Laminar Sublayer (u_l/U) with the Apparent Reynolds Number for CMC Solutions of Different Concentrations

Figure 4: Effect of the Apparent Reynolds Number on (μ_{sl}/μ_a) for CMC Solutions of Different Concentration

Figure 5: Effect of Polymer Concentration and Apparent Reynolds Number on Drag Reduction for CMC Solutions