EXTENSION OF NEUTROSOPHICSEMI OPEN SETS

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ABSTRACT

Neutrosophic concepts mostly utilized in vague collection of items. Already we extend feebly open sets, semi open sets and its properties to neutrosophic space. Many Researchers extend the concepts of generalized closure operator into neutrosophy theory. Focal point of this write-up is to initiate neutrosophic *semi open sets concerning the operator neutrosophic generalized closure.

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1. INTRODUCTION

In 1998, Smarandache [11] introduced the set on three components involving the membership, indeterminacy and non-membership values belongs to $[0^-, 1^+]$ a nonstandard unit interval. He represents the above three components respectively as T, I, F .

Let T, I, F be real standard or not standard subset of $[0, 1^+]$, with

$$\sup_- T = t_sup, \inf_- = t_inf$$

$$\sup_- I = i_sup, \inf_- = i_inf$$

$$\sup_- F = f_sup, \inf_- = f_inf$$

$$n - sup = t_sup + i_sup + f_sup$$

$$n - inf = t_inf + i_inf + f_inf, T, I, F \text{ are called neutrosophic components.}$$

The preliminary concepts required in our work please refer [5, 6, 12]. The rest of this article is assembled come after. In section 2, the abstraction of neutrosophicsemi open* sets is introduced along with its belongings are investigated. Section 3 deals with some properties on neutrosophicsemi closed* sets.

2. NEUTROSOPHICSEMI OPEN* SETS

This segment deals with a new class of sets, namely neutrosophicsemi open* make use of the neutrosophic generalized closure operator.

Proposition 2.1. Neutrosophic generalized closure operator (is denoted by $Ngcl$) operator is a Kuratowski closure operator on neutrosophic topological spaces.

Hereafter, “ $Ngcl$ ” operator is designated by Ncl^* .

Definition 2.2. Let $(\mathcal{X}, \mathfrak{T}_N^*)$ be a neutrosophic topological space on $\mathcal{X}$ generated by Ncl^* . That is $\mathfrak{T}_N^* = \{A : Ncl^*(A^c) = A^c, A \text{ is a neutrosophic subset of } \mathcal{X}\}$.

Theorem 2.3. $\mathfrak{T}_N \subseteq \mathfrak{T}_N^*$ with equality if and only if each neutrosophic generalized closed set in $(\mathcal{X}, \mathfrak{T}_N)$ is neutrosophic closed set in $(\mathcal{X}, \mathfrak{T}_N^*)$.

Proof. Let $\mathcal{L}$ be a neutrosophic closed set. Since $\mathcal{L} \subseteq Ncl^*(\mathcal{R}) \subseteq Ncl(\mathcal{L})$ implies that $\mathcal{L}$ is neutrosophic closed in $\mathfrak{T}_N^*$. Thus $\mathfrak{T}_N \subseteq \mathfrak{T}_N^*$. Also let $\mathfrak{T}_N = \mathfrak{T}_N^*$ and let $\mathcal{R}$ be neutrosophic generalized closed in $\mathfrak{T}_N$. Then $\mathcal{R} = Ncl^*(\mathcal{R})$. That implies $\mathcal{R}$ is neutrosophic closed in $\mathfrak{T}_N = \mathfrak{T}_N^*$. Thus each neutrosophic generalized closed set in $(\mathcal{X}, \mathfrak{T}_N)$ is neutrosophic closed set in $(\mathcal{X}, \mathfrak{T}_N^*)$.

Conversely, let each neutrosophic generalized closed set in $(\mathcal{X}, \mathfrak{T}_N^*)$ is neutrosophic closed set in $(\mathcal{X}, \mathfrak{T}_N)$. Then $Ncl^*(\mathcal{L}) = Ncl(\mathcal{L})$. Hence $\mathfrak{T}_N = \mathfrak{T}_N^*$.

Definition 2.4. A neutrosophic subset G of a neutrosophic topological space $(\mathcal{X}, \mathfrak{T}_N)$ is called a **neutrosophic semi open*** if there is a neutrosophic open set U in $\mathcal{X}$ such that $U \subseteq G \subseteq Ncl^*(U)$.

Theorem 2.5. A neutrosophic subset G is neutrosophic semi open* if and only if $G \subseteq Ncl^*(Nint(G))$.

Proof: If G is neutrosophic semi open*, then there is a neutrosophic open set U such that $U \subseteq G \subseteq Ncl^*(U)$. Now $U \subseteq G$ implies $U = Nint(U) \subseteq Nint(G) \Rightarrow G \subseteq Ncl^*(U) \subseteq Ncl^*(Nint(G))$.

Conversely, assume that $G \subseteq Ncl^*(Nint(G))$. Take $U = Nint(G)$. Then U is a neutrosophic open set in $\mathcal{X}$ such that $U \subseteq G \subseteq Ncl^*(U)$. Therefore G is neutrosophic semi open*.

Example 2.6. Let $\mathcal{X} = \{\alpha, \beta, \delta\}$ and $\mathfrak{T}_N = \{0_N, \wp, \mathbb{Q}, 1_N\}$. Then $(\mathcal{X}, \mathfrak{T}_N)$ is a neutrosophic topological space. The neutrosophic closed sets are $\mathfrak{T}_N^c = \{1_N, \mathcal{R}, S, 0_N\}$ where,

$$\wp = \langle x, \left(\frac{\alpha}{0.4}, \frac{\beta}{0.4}, \frac{\delta}{0.5}\right), \left(\frac{\alpha}{0.4}, \frac{\beta}{0.4}, \frac{\delta}{0.54}\right), \left(\frac{\alpha}{0.3}, \frac{\beta}{0.4}, \frac{\delta}{0.4}\right) \rangle,$$

$$\mathbb{Q} = \langle x, \left(\frac{\alpha}{0.4}, \frac{\beta}{0.5}, \frac{\delta}{0.6}\right), \left(\frac{\alpha}{0.4}, \frac{\beta}{0.5}, \frac{\delta}{0.6}\right), \left(\frac{\alpha}{0.2}, \frac{\beta}{0.3}, \frac{\delta}{0.3}\right) \rangle,$$

$$\mathcal{R} = \langle x, \left(\frac{\alpha}{0.6}, \frac{\beta}{0.6}, \frac{\delta}{0.5}\right), \left(\frac{\alpha}{0.6}, \frac{\beta}{0.6}, \frac{\delta}{0.5}\right), \left(\frac{\alpha}{0.7}, \frac{\beta}{0.6}, \frac{\delta}{0.7}\right) \rangle,$$

$$S = \langle x, \left(\frac{\alpha}{0.6}, \frac{\beta}{0.5}, \frac{\delta}{0.4}\right), \left(\frac{\alpha}{0.6}, \frac{\beta}{0.5}, \frac{n\delta}{0.4}\right), \left(\frac{\alpha}{0.8}, \frac{\beta}{0.8}, \frac{\delta}{0.7}\right) \rangle,$$

$$\text{Define } T = \langle x, \left(\frac{\alpha}{0.5}, \frac{\beta}{0.6}, \frac{\delta}{0.5}\right), \left(\frac{\alpha}{0.5}, \frac{\beta}{0.8}, \frac{\delta}{0.7}\right), \left(\frac{\alpha}{0}, \frac{\beta}{0.2}, \frac{\delta}{0.1}\right) \rangle.$$

Then $Nint(T) = \langle x, \left(\frac{\alpha}{0.4}, \frac{\beta}{0.4}, \frac{\delta}{0.5}\right), \left(\frac{\alpha}{0.4}, \frac{\beta}{0.4}, \frac{\delta}{0.54}\right), \left(\frac{\alpha}{0.3}, \frac{\beta}{0.4}, \frac{\delta}{0.4}\right) \rangle$ also $Ncl^*(Nint(T)) = \langle x, \left(\frac{\alpha}{0.5}, \frac{\beta}{0.6}, \frac{\delta}{0.7}\right), \left(\frac{\alpha}{0.5}, \frac{\beta}{0.8}, \frac{\delta}{0.6}\right), \left(\frac{\alpha}{0.7}, \frac{\beta}{0.2}, \frac{\delta}{0}\right) \rangle$. It can be verified that $T \subseteq Ncl^*(Nint(T))$. Hence T is a neutrosophic semi open set*.

Theorem 2.7. Each open set is semi open in general topological spaces. But this result is also true in neutrosophic spaces, that is each neutrosophic open set is neutrosophic semi open*.

Proof: Take U is neutrosophic open in κ . Then $Nint(U) = U$. Therefore $U \subseteq Ncl^*(U) = Ncl^*(Nint(U))$. Hence by Theorem 2.5, U is neutrosophicsemi open*.

Example 2.8. Let $\kappa = \{\eta, \vartheta\}$ and $\mathfrak{S}_N = \{0_N, A, B, C, D, 1_N\}$. Their corresponding closed sets are $\mathfrak{S}_N^c = \{1_N, F, G, H, I, 0_N\}$ where, $A = \langle x, \left(\frac{\eta}{0.2}, \frac{\vartheta}{0.9}\right), \left(\frac{\eta}{0.1}, \frac{\vartheta}{0.8}\right), \left(\frac{\eta}{0.4}, \frac{\vartheta}{0.3}\right) \rangle$, $B = \langle x, \left(\frac{\eta}{0.3}, \frac{\vartheta}{0.3}\right), \left(\frac{\eta}{0.7}, \frac{\vartheta}{0.2}\right), \left(\frac{\eta}{0.7}, \frac{\vartheta}{0.7}\right) \rangle$, $C = \langle x, \left(\frac{\eta}{0.3}, \frac{\vartheta}{0.9}\right), \left(\frac{\eta}{0.7}, \frac{\vartheta}{0.8}\right), \left(\frac{\eta}{0.4}, \frac{\vartheta}{0.3}\right) \rangle$,
 $D = \langle x, \left(\frac{\eta}{0.2}, \frac{\vartheta}{0.3}\right), \left(\frac{\eta}{0.1}, \frac{\vartheta}{0.2}\right), \left(\frac{\eta}{0.7}, \frac{\vartheta}{0.7}\right) \rangle$, $E = \langle x, \left(\frac{\eta}{0.8}, \frac{\vartheta}{0.1}\right), \left(\frac{\eta}{0.9}, \frac{\vartheta}{0.2}\right), \left(\frac{\eta}{0.6}, \frac{\vartheta}{0.7}\right) \rangle$, $F = \langle x, \left(\frac{\eta}{0.7}, \frac{\vartheta}{0.7}\right), \left(\frac{\eta}{0.3}, \frac{\vartheta}{0.8}\right), \left(\frac{\eta}{0.3}, \frac{\vartheta}{0.3}\right) \rangle$, $G = \langle x, \left(\frac{\eta}{0.7}, \frac{\vartheta}{0.1}\right), \left(\frac{\eta}{0.3}, \frac{\vartheta}{0.2}\right), \left(\frac{\eta}{0.6}, \frac{\vartheta}{0.7}\right) \rangle$, $H = \langle x, \left(\frac{\eta}{0.8}, \frac{\vartheta}{0.7}\right), \left(\frac{\eta}{0.9}, \frac{\vartheta}{0.8}\right), \left(\frac{\eta}{0.3}, \frac{\vartheta}{0.3}\right) \rangle$. Define $I = \langle x, \left(\frac{\eta}{0.4}, \frac{\vartheta}{0.3}\right), \left(\frac{\eta}{0.3}, \frac{\vartheta}{0.8}\right), \left(\frac{\eta}{0.3}, \frac{\vartheta}{0.4}\right) \rangle$. Then $Nint(I) = \langle x, \left(\frac{\eta}{0.2}, \frac{\vartheta}{0.3}\right), \left(\frac{\eta}{0.1}, \frac{\vartheta}{0.2}\right), \left(\frac{\eta}{0.7}, \frac{\vartheta}{0.7}\right) \rangle$ also $Ncl^*Nint(I) = \langle x, \left(\frac{\eta}{0.7}, \frac{\vartheta}{0.7}\right), \left(\frac{\eta}{0.3}, \frac{\vartheta}{0.8}\right), \left(\frac{\eta}{0.3}, \frac{\vartheta}{0.3}\right) \rangle$. It can be verified that $I \subseteq N^*cl(Nint(I))$. So I is neutrosophicsemi open* nevertheless not neutrosophic open.

Remark 2.9.(i) In any neutrosophic space $(\kappa, \mathfrak{S}_N)$, it can be verified that neutrosophicsemi open* sets are 0_N and 1_N .

Theorem 2.10. If $\{A_\alpha\}$ is a collection of neutrosophicsemi open* sets, then the arbitrary union $\cup A_\alpha$ is also neutrosophicsemi open*.

Proof: Since A_α is neutrosophicsemi open* for each α , a neutrosophic open set U_α in κ such that $U_\alpha \subseteq A_\alpha \subseteq Ncl^*(U_\alpha)$. Then $\cup U_\alpha \subseteq \cup A_\alpha \subseteq \cup Ncl^*(U_\alpha) \subseteq Ncl^*(\cup U_\alpha)$. Since $\cup U_\alpha$ is neutrosophic open, $\cup A_\alpha$ is Neutrosophicsemi open*.

Remark 2.11. It can be verified that the finite intersection of neutrosophicsemi open* sets need not be neutrosophicsemi open*.

The following example reveals the above fact.

Example 2.12. Let $\kappa = \{p, q\}$, $\mathfrak{S}_N = \{0_N, G_1, G_2, 1_N\}$ be a neutrosophic topology on κ , where $G_1 = \langle x, \left(\frac{p}{0.3}, \frac{q}{0.2}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}\right), \left(\frac{p}{0.7}, \frac{q}{0.6}\right) \rangle$ and $G_2 = \langle x, \left(\frac{p}{0.2}, \frac{q}{0.1}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}\right), \left(\frac{p}{0.4}, \frac{q}{0.3}\right) \rangle$. Then neutrosophic subset $A = \langle x, \left(\frac{p}{0.3}, \frac{q}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.3}\right), \left(\frac{p}{0.7}, \frac{q}{0.8}\right) \rangle$ and

$B = \langle x, \left(\frac{p}{0.2}, \frac{q}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.4}\right), \left(\frac{p}{0.8}, \frac{q}{0.6}\right) \rangle$ are neutrosophicsemi open* sets but $A \cap B = \langle x, \left(\frac{p}{0.2}, \frac{q}{0.2}\right), \left(\frac{p}{0.4}, \frac{q}{0.3}\right), \left(\frac{p}{0.8}, \frac{q}{0.8}\right) \rangle$ is not neutrosophicsemi open*.

But a neutrosophicsemi open* set intersect with a neutrosophic open set is neutrosophicsemi open* as shown in Theorem 2.14.

Theorem 2.13. If G is neutrosophic open and $Ncl^*(G \cap H) \subseteq Ncl^*(G \cap Ncl^*(H))$ and hence $Ncl^*(G) \cap H \subseteq Ncl^*(G \cap H)$, where H is a neutrosophic subset of a neutrosophic topological space.

Theorem 2.14. A neutrosophic semi open* set intersect with a neutrosophic open set is neutrosophic semi open*.

Proof: Let G be neutrosophicsemi open* and H be a neutrosophic open set. Then by definition, neutrosophic open set U such that $U \subseteq G \subseteq Ncl^*(U)$. Since H is neutrosophic open, then $U \cap H \subseteq G \cap H \subseteq Ncl^*(U) \cap H \subseteq Ncl^*(U \cap H)$. Hence $G \cap H$ is neutrosophicsemi open*.

Example 2.15. Let $\kappa = \{\varsigma, \xi\}$, $\mathfrak{I}_N = \{0, G_1, G_2, 1\}$ be a neutrosophic topology on κ , where $G_1 = \langle x, \left(\frac{\varsigma}{0.3}, \frac{\xi}{0.2}\right), \left(\frac{\varsigma}{0.5}, \frac{\xi}{0.4}\right), \left(\frac{\varsigma}{0.7}, \frac{\xi}{0.6}\right) \rangle$ and $G_2 = \langle x, \left(\frac{\varsigma}{0.2}, \frac{\xi}{0.1}\right), \left(\frac{\varsigma}{0.4}, \frac{\xi}{0.3}\right), \left(\frac{\varsigma}{0.8}, \frac{\xi}{0.7}\right) \rangle$. Then neutrosophic subset $G = \langle x, \left(\frac{\varsigma}{0.3}, \frac{\xi}{0.2}\right), \left(\frac{\varsigma}{0.4}, \frac{\xi}{0.3}\right), \left(\frac{\varsigma}{0.7}, \frac{\xi}{0.8}\right) \rangle$ is neutrosophic semi open* set and $H = \langle x, \left(\frac{\varsigma}{0.2}, \frac{\xi}{0.1}\right), \left(\frac{\varsigma}{0.4}, \frac{\xi}{0.3}\right), \left(\frac{\varsigma}{0.8}, \frac{\xi}{0.7}\right) \rangle$ is neutrosophic open but $G \wedge H = \langle x, \left(\frac{\varsigma}{0.2}, \frac{\xi}{0.2}\right), \left(\frac{\varsigma}{0.4}, \frac{\xi}{0.3}\right), \left(\frac{\varsigma}{0.8}, \frac{\xi}{0.8}\right) \rangle$ is not neutrosophic semi open*.

Remark 2.16. The collection of neutrosophic semi open* sets [denoted by $NS-O^*(X, \tau)$] forms a neutrosophic topology on κ if and only if it is closed under finite intersection.

Proposition 2.17. Union of neutrosophic semi open* G and a neutrosophic open, $G \cup U$ is neutrosophic semi open*.

Proof: Follows from Theorem 2.7 and Theorem 2.10.

Definition 2.18. For a neutrosophic topological space $(\kappa, \mathfrak{I}_N)$, let $\tau_{NS^*} = \{U \in NS-O^*(\kappa, \mathfrak{I}_N) : U \cap A \in NS-O^*(\kappa, \mathfrak{I}_N) \text{ for all } A \in NS-O^*(\kappa, \mathfrak{I}_N)\}$.

Theorem 2.19. τ_{NS^*} is a neutrosophic topology finer than $(\kappa, \mathfrak{I}_N)$.

Proof: Clearly $0_N, 1_N \in \mathfrak{I}_{NS^*}$. Let $U_\alpha \in \mathfrak{I}_{NS^*}$ and $U = \bigcup U_\alpha$. Since $U_\alpha \in NS-O^*(\kappa, \mathfrak{I}_N)$, by using Theorem 2.10, $U \in NS-O^*(\kappa, \mathfrak{I}_N)$. Let $A \in NS-O^*(\kappa, \mathfrak{I}_N)$. Then $U_\alpha \cap A \in NS-O^*(\kappa, \mathfrak{I}_N)$, for each α and hence by Theorem 2.10, $U \cap A = (\bigcup U_\alpha) \cap A = \bigcup (U_\alpha \cap A) \in NS-O^*(\kappa, \mathfrak{I}_N)$. Therefore $U \in \mathfrak{I}_{NS^*}$. Now let $U_1, U_2, \dots, U_n \in \mathfrak{I}_{NS^*}$. Then $U_1, U_2, \dots, U_n \in NS-O^*(X, \mathfrak{I}_N)$ and by definition of $\mathfrak{I}_{NS^*}$, we get $\bigcap_{k=1}^n U_k \in NS-O^*(\kappa, \mathfrak{I}_N)$. If $A \in NS-O^*(X, \mathfrak{I}_N)$, then by repeated application of the condition, we have $(\bigcap_{k=1}^n U_k) \cap A \in NS-O^*(X, \mathfrak{I}_N)$. Hence $A \in \mathfrak{I}_{NS^*}$. This shows that $\mathfrak{I}_{NS^*}$ is a neutrosophic topology on X .

Let $V \in \mathfrak{I}$. By using Theorem 2.7, $V \in NS-O^*(\kappa, \mathfrak{I}_N)$. Also by Theorem 2.14, $V \cap A \in NS-O^*(\kappa, \mathfrak{I}_N)$ for all $A \in NS-O^*(\kappa, \mathfrak{I}_N)$. Hence $A \in \tau_{NS^*}$. Thus τ_{NS^*} is finer than $(\kappa, \mathfrak{I}_N)$.

Theorem 2.20. Neutrosophic semi open* set is neutrosophic semi open.

Proof: Let G be a neutrosophic semi open* set. Then there is a neutrosophic open set U in X such that $U \subseteq G \subseteq Ncl^*(U)$. Note that $Ncl^*(U) \subseteq Ncl(U)$. Therefore $U \subseteq G \subseteq Ncl(U)$. Hence G is neutrosophic semi open.

The reverse of the above is not true.

Example 2.21. Let $\kappa = \{x\}$ and $\mathfrak{I} = \{0_N, 1_N, G, H, I, J\}$ where $G = \langle x, \frac{\mu}{0.5}, \frac{\gamma}{0.2}, \frac{\sigma}{0.6} \rangle$, $H = \langle x, \frac{\mu}{0.7}, \frac{\gamma}{0.4}, \frac{\sigma}{0.8} \rangle$, $I = \langle x, \frac{\mu}{0.7}, \frac{\gamma}{0.4}, \frac{\sigma}{0.6} \rangle$, $J = \langle x, \frac{\mu}{0.5}, \frac{\gamma}{0.2}, \frac{\sigma}{0.8} \rangle$. The corresponding neutrosophic closed sets are $\mathfrak{I}^c = \{0_N, 1_N, \langle x, \frac{\mu}{0.6}, \frac{\gamma}{0.8}, \frac{\sigma}{0.5} \rangle, \langle x, \frac{\mu}{0.8}, \frac{\gamma}{0.6}, \frac{\sigma}{0.7} \rangle, \langle x, \frac{\mu}{0.6}, \frac{\gamma}{0.6}, \frac{\sigma}{0.7} \rangle, \langle x, \frac{\mu}{0.8}, \frac{\gamma}{0.8}, \frac{\sigma}{0.5} \rangle\}$. Let $\mathfrak{K} = \langle x, \frac{\mu}{0.5}, \frac{\gamma}{0.2}, \frac{\sigma}{0.8} \rangle$. Then $Nint(\mathfrak{K}) = \langle x, \frac{\mu}{0.5}, \frac{\gamma}{0.2}, \frac{\sigma}{0.8} \rangle$ follows that $Ncl^*(Nint(\mathfrak{K})) = \langle x, \frac{\mu}{0.4}, \frac{\gamma}{0.2}, \frac{\sigma}{0.6} \rangle$. Thus $\mathfrak{K} \not\subseteq Ncl^*(Nint(\mathfrak{K}))$ implies $\mathfrak{K}$ is neutrosophic semi open*. And it can be verified that $\mathfrak{K} \subseteq \langle x, \frac{\mu}{0.5}, \frac{\gamma}{0.2}, \frac{\sigma}{0.8} \rangle \subseteq Ncl(Nint(\mathfrak{K})) = \langle x, \frac{\mu}{0.8}, \frac{\gamma}{0.8}, \frac{\sigma}{0.5} \rangle$. It can be seen that $\mathfrak{K}$ is neutrosophic semi open* set.

Theorem 2.22. If L is neutrosophic semi open* then $Ncl^*(L) = Ncl^*(Nint(L))$.

Proof: By definition, $L \subseteq Ncl^*(Nint(L))$. Hence $Ncl^*(L) \subseteq Ncl^*(Nint(L))$. Also we have $Ncl^*(Nint(L)) \subseteq Ncl^*(L)$. Hence $Ncl^*(Nint(L)) = Ncl^*(L)$.

Conversely, Take $M = Nint(L)$. Then M is neutrosophic open set in X such that $M \subseteq L \subseteq$

$Ncl^*(L) = Ncl^*(Nint(L)) = Ncl^*(L)$. Then by Definition 2.4, L is neutrosophicsemi open*.

Theorem 2.23. Let M be neutrosophicsemi open* and $N \subseteq X$ such that $M \subseteq N \subseteq Ncl^*(M)$. Then N is neutrosophicsemi open*.

Proof: Since M is neutrosophicsemi open*, $M \subseteq Ncl^*(Nint(M))$. Since $Nint(M) \subseteq Nint(N)$, $Ncl^*(Nint(M)) \subseteq Ncl^*(Nint(N))$. Therefore by the above theorem, $N \subseteq Ncl^*(M) = Ncl^*(Nint(M)) \subseteq Ncl^*(Nint(N))$. Hence by Definition 2.4, N is neutrosophicsemi open*.

3. NEUTROSOPHICSEMI CLOSED* SET

Definition 3.1. Neutrosophic semi closed* set is known as the complement of a neutrosophic semi open*.

The neutrosophic generalized interior operator is symbolized by $Nint^*$.

Definition 3.2. A neutrosophic semiregular* is known as both neutrosophicsemi open* and neutrosophicsemi closed*.

Theorem 3.3. A neutrosophic set $\mathcal{M}$ is neutrosophicsemi closed* iff $Nint^*(\mathcal{F}) \subseteq \mathcal{M} \subseteq \mathcal{F}$, where $\mathcal{F}$ is a neutrosophic closed set.

Proof: Suppose $\mathcal{M}$ is neutrosophicsemi closed*. Then by Definition 3.1, $\mathcal{M}^c$ is neutrosophicsemi open*. Then by Definition 2.4, there exists a neutrosophic open set $\mathcal{F}$ in κ such that $\mathcal{F} \subseteq \mathcal{M}^c \subseteq Ncl^*(\mathcal{F})$ which implies $\mathcal{F}^c \supseteq \mathcal{M} \supseteq (Ncl^*(\mathcal{F}))^c$. Since $(Ncl^*(\mathcal{F}))^c = Nint^*(\mathcal{M}^c)$, $\mathcal{F}^c \supseteq \mathcal{M} \supseteq Nint^*(\mathcal{F}^c)$ where $\mathcal{F}^c$ is neutrosophic closed in κ .

Suppose a neutrosophic closed set $\mathcal{F}$ satisfying $Nint^*(\mathcal{F}) \subseteq \mathcal{M} \subseteq \mathcal{F}$ which implies $(Nint^*(\mathcal{F}))^c \supseteq \mathcal{M}^c \supseteq \mathcal{F}^c$. Since $(Nint^*(\mathcal{F}))^c = Ncl^*(\mathcal{F}^c)$, we have $Ncl^*(\mathcal{F}^c) \supseteq \mathcal{M}^c \supseteq \mathcal{F}^c$ where $\mathcal{F}^c$ is a neutrosophic open set. Hence by Definition 2.1, $\mathcal{M}^c$ is neutrosophicsemi open*. Hence the result.

Theorem 3.4. The Characterization of neutrosophicsemi closed*. That is, A is neutrosophicsemi closed* iff $Nint^*(Ncl(A)) \subseteq A$.

Proof: By Theorem 2.5, $A^c \subseteq Ncl^*(Nint(A^c))$. Taking complement on both sides and using the properties, $Nint^*(Ncl(A)) \subseteq A$.

Conversely, assume that $Nint^*(Ncl(A)) \subseteq A$. Take $\mathcal{M} = Ncl(A)$. Then a neutrosophic closed set $\mathcal{M}$ in X such that $Nint^*(\mathcal{M}) \subseteq A \subseteq \mathcal{M}$ and hence A is neutrosophicsemi closed*.

Theorem 3.5. Neutrosophic closed set is neutrosophicsemi closed*.

Proof: If $\mathbb{N}$ is Neutrosophic closed set. Then $\mathbb{N}^c$ is neutrosophic open. By Theorem 3.7, $\mathbb{N}^c$ is neutrosophicsemi open*. Hence $\mathbb{N}$ is neutrosophicsemi closed*.

Example 3.6. Let $\kappa = \{\mathcal{g}, \mathcal{h}, \mathcal{E}\}$ and $\mathfrak{N} = \{0_N, A, B, C, D, 1_N\}$. Then (κ, τ) is a neutrosophic topological space. The neutrosophic closed sets are $\mathfrak{N}^c = \{1_N, E, F, G, H, 0_N\}$ where

$$A = \langle \mathcal{X}, \left(\frac{\mathcal{g}}{0}, \frac{\mathcal{h}}{0.2}, \frac{\mathcal{E}}{0.9}\right), \left(\frac{\mathcal{g}}{0.1}, \frac{\mathcal{h}}{0.8}, \frac{\mathcal{E}}{0.4}\right), \left(\frac{\mathcal{g}}{0.5}, \frac{\mathcal{h}}{0.4}, \frac{\mathcal{E}}{0.3}\right) \rangle$$

$$B = \langle \mathcal{X}, \left(\frac{\mathcal{g}}{0.4}, \frac{\mathcal{h}}{0.3}, \frac{\mathcal{E}}{0.3}\right), \left(\frac{\mathcal{g}}{0.3}, \frac{\mathcal{h}}{0.7}, \frac{\mathcal{E}}{0.2}\right), \left(\frac{\mathcal{g}}{0.3}, \frac{\mathcal{h}}{0.1}, \frac{\mathcal{E}}{0.7}\right) \rangle,$$

$$C = \langle x, \left(\frac{g}{0.4}, \frac{h}{0.3}, \frac{\varepsilon}{0.9}\right), \left(\frac{g}{0.3}, \frac{h}{0.8}, \frac{\varepsilon}{0.4}\right), \left(\frac{g}{0.3}, \frac{h}{0.1}, \frac{\varepsilon}{0.3}\right) \rangle,$$

$$D = \langle x, \left(\frac{g}{0}, \frac{h}{0.2}, \frac{\varepsilon}{0.9}\right), \left(\frac{g}{0.1}, \frac{h}{0.7}, \frac{\varepsilon}{0.2}\right), \left(\frac{g}{0.5}, \frac{h}{0.4}, \frac{\varepsilon}{0.7}\right) \rangle$$

$$E = \langle x, \left(\frac{g}{1}, \frac{h}{0.8}, \frac{\varepsilon}{0.1}\right), \left(\frac{g}{0.9}, \frac{h}{0.2}, \frac{\varepsilon}{0.6}\right), \left(\frac{g}{0.5}, \frac{h}{0.6}, \frac{\varepsilon}{0.7}\right) \rangle,$$

$$F = \langle x, \left(\frac{g}{0.6}, \frac{h}{0.7}, \frac{\varepsilon}{0.7}\right), \left(\frac{g}{0.7}, \frac{h}{0.3}, \frac{\varepsilon}{0.8}\right), \left(\frac{g}{0.7}, \frac{h}{0.9}, \frac{\varepsilon}{0.3}\right) \rangle,$$

$$G = \langle x, \left(\frac{g}{0.6}, \frac{h}{0.7}, \frac{\varepsilon}{0.1}\right), \left(\frac{g}{0.7}, \frac{h}{0.2}, \frac{\varepsilon}{0.6}\right), \left(\frac{g}{0.7}, \frac{h}{0.9}, \frac{\varepsilon}{0.7}\right) \rangle,$$

$$H = \langle x, \left(\frac{g}{1}, \frac{h}{0.8}, \frac{\varepsilon}{0.1}\right), \left(\frac{g}{0.9}, \frac{h}{0.3}, \frac{\varepsilon}{0.8}\right), \left(\frac{g}{0.5}, \frac{h}{0.6}, \frac{\varepsilon}{0.3}\right) \rangle$$

$$\text{Define } I = \langle x, \left(\frac{g}{0.5}, \frac{h}{0.6}, \frac{\varepsilon}{0.5}\right), \left(\frac{g}{0.5}, \frac{h}{0.2}, \frac{\varepsilon}{0.4}\right), \left(\frac{g}{0.8}, \frac{h}{1}, \frac{\varepsilon}{0.3}\right) \rangle.$$

$$\text{Then } Ncl(I) = \langle x, \left(\frac{g}{0.6}, \frac{h}{0.7}, \frac{\varepsilon}{0.7}\right), \left(\frac{g}{0.7}, \frac{h}{0.3}, \frac{\varepsilon}{0.8}\right), \left(\frac{g}{0.7}, \frac{h}{0.9}, \frac{\varepsilon}{0.3}\right) \rangle \text{ also it can be verified that}$$

$$Nint^*(Ncl(I)) = 0_N \subseteq I. \text{ Thus } I \text{ is neutrosophic semi closed}^* \text{ but not neutrosophic closed.}$$

Remark 3.7. We have seen that 0_N and 1_N are always neutrosophic semi closed* sets.

Theorem 3.8. If collection of neutrosophic semi closed* sets $\{F_\alpha\}$, then arbitrary intersection, $\cap F_\alpha$ is also neutrosophic semi closed*.

Proof: Since each F_α is neutrosophic semi closed*, F_α^c is neutrosophic semi open*. By Theorem 2.10, $\cup F_\alpha^c$ is neutrosophic semi open*. That implies $(\cap F_\alpha)^c$ is neutrosophic semi open*. Hence by Definition 3.1, $\cap F_\alpha$ is neutrosophic semi closed*.

Remark 3.9. We have found the result that finite union of two neutrosophic semi closed* sets need not be neutrosophic semi closed* as seen from the following examples.

Example 3.10. Let $\kappa = \{\kappa, \varrho\}$. Then the neutrosophic topology $\mathfrak{I} = \{0_N, G_1, G_2, 1_N\}$, where $G_1 = \langle x, \left(\frac{\kappa}{0.3}, \frac{\varrho}{0.2}\right), \left(\frac{\kappa}{0.5}, \frac{\varrho}{0.4}\right), \left(\frac{\kappa}{0.7}, \frac{\varrho}{0.6}\right) \rangle$ and

$$G_2 = \langle x, \left(\frac{\kappa}{0.2}, \frac{\varrho}{0.1}\right), \left(\frac{\kappa}{0.4}, \frac{\varrho}{0.3}\right), \left(\frac{\kappa}{0.5}, \frac{\varrho}{0.4}\right) \rangle. \text{ The corresponding neutrosophic closed sets are } = \{0_N, G_2, G_3, 1_N\}$$

$$\text{where } G_3 = \langle x, \left(\frac{\kappa}{0.7}, \frac{\varrho}{0.8}\right), \left(\frac{\kappa}{0.5}, \frac{\varrho}{0.6}\right), \left(\frac{\kappa}{0.3}, \frac{\varrho}{0.4}\right) \rangle \text{ and}$$

$$G_4 = \langle x, \left(\frac{\kappa}{0.8}, \frac{\varrho}{0.9}\right), \left(\frac{\kappa}{0.6}, \frac{\varrho}{0.7}\right), \left(\frac{\kappa}{0.5}, \frac{\varrho}{0.6}\right) \rangle$$

$$\text{Define neutrosophic subsets } G_5 = \langle x, \left(\frac{\kappa}{0.3}, \frac{\varrho}{0.2}\right), \left(\frac{\kappa}{0.4}, \frac{\varrho}{0.3}\right), \left(\frac{\kappa}{0.4}, \frac{\varrho}{0.5}\right) \rangle \text{ and}$$

$$G_6 = \langle x, \left(\frac{\kappa}{0.2}, \frac{\varrho}{0.2}\right), \left(\frac{\kappa}{0.4}, \frac{\varrho}{0.4}\right), \left(\frac{\kappa}{0.4}, \frac{\varrho}{0.5}\right) \rangle. \text{ Then it can be found that}$$

$$Ncl(G_5) = \langle x, \left(\frac{\kappa}{0.7}, \frac{\varrho}{0.8}\right), \left(\frac{\kappa}{0.5}, \frac{\varrho}{0.6}\right), \left(\frac{\kappa}{0.3}, \frac{\varrho}{0.4}\right) \rangle \text{ and}$$

$$Ncl(G_6) = \langle x, \left(\frac{\kappa}{0.7}, \frac{\varrho}{0.8}\right), \left(\frac{\kappa}{0.5}, \frac{\varrho}{0.6}\right), \left(\frac{\kappa}{0.3}, \frac{\varrho}{0.4}\right) \rangle,$$

$$Nint^*(Ncl(G_5)) = \langle x, \left(\frac{\kappa}{0.2}, \frac{\varrho}{0.1}\right), \left(\frac{\kappa}{0.4}, \frac{\varrho}{0.3}\right), \left(\frac{\kappa}{0.5}, \frac{\varrho}{0.4}\right) \rangle \subseteq G_5 \text{ and}$$

$$Nint^*(Ncl(G_6)) = \langle x, \left(\frac{\kappa}{0.2}, \frac{\varrho}{0.1}\right), \left(\frac{\kappa}{0.4}, \frac{\varrho}{0.3}\right), \left(\frac{\kappa}{0.5}, \frac{\varrho}{0.4}\right) \rangle \subseteq G_6, \text{ hence } G_5 \text{ and } G_6 \text{ are neutrosophic semi closed* sets but}$$

$$G_5 \vee G_6 = \langle x, \left(\frac{\kappa}{0.3}, \frac{\varrho}{0.2}\right), \left(\frac{\kappa}{0.4}, \frac{\varrho}{0.4}\right), \left(\frac{\kappa}{0.4}, \frac{\varrho}{0.4}\right) \rangle \text{ is not neutrosophicsemi closed*}. \text{ For, } Ncl(G_5 \vee G_6) =$$

$$\langle x, \left(\frac{\kappa}{0.7}, \frac{\varrho}{0.8}\right), \left(\frac{\kappa}{0.5}, \frac{\varrho}{0.6}\right), \left(\frac{\kappa}{0.3}, \frac{\varrho}{0.4}\right) \rangle, Nint^*(Ncl(G_5 \vee G_6)) = \langle x, \left(\frac{\kappa}{0.3}, \frac{\varrho}{0.2}\right), \left(\frac{\kappa}{0.5}, \frac{\varrho}{0.4}\right), \left(\frac{\kappa}{0.7}, \frac{\varrho}{0.6}\right) \rangle \not\subseteq G_5 \vee G_6.$$

But the union of a neutrosophicsemi closed* set and a neutrosophic closed set is neutrosophicsemi closed* as shown in Theorem 3.11.

Theorem 3.11. Union of neutrosophicsemi closed* and neutrosophic closed is neutrosophicsemi closed*.

Proof: Let us take F be neutrosophicsemi closed*. Then F^C is neutrosophicsemi open*. Also we take G is neutrosophic closed, G^C is neutrosophic open. By Theorem 2.14, $F^C \cap G^C = (F \cup G)^C$ is neutrosophicsemi open*. Finally $F \cup G$ is neutrosophicsemi closed*.

Corollary 3.12. Finite intersection of neutrosophicsemi closed* and neutrosophic closed is neutrosophicsemi closed*.

Proof: We take F is neutrosophic closed and E is neutrosophic closed, by Theorem 3.5, F is neutrosophicsemi closed*. Then by Theorem 3.11, $E \cap F$ is neutrosophicsemi closed*.

Theorem 3.13. Each neutrosophicsemi closed* set is neutrosophicsemi closed.

Proof: Let F be a neutrosophicsemi closed* set. Then F^C is neutrosophicsemi open* set. By Theorem 2.20, F^C is neutrosophicsemi open. Hence F is neutrosophicsemi closed.

But each neutrosophic closed set may not be neutrosophic semi closed* as shown in the following examples.

Example 3.14. Let $\kappa = \{a\}$ and $\mathfrak{S} = \{0_N, 1_N, U, V, W, X\}$ where $U = \langle a, \frac{\mu}{0.5}, \frac{\gamma}{0.2}, \frac{\sigma}{0.6} \rangle$,

$$V = \langle a, \frac{\mu}{0.7}, \frac{\gamma}{0.4}, \frac{\sigma}{0.8} \rangle, W = \langle a, \frac{\mu}{0.7}, \frac{\gamma}{0.4}, \frac{\sigma}{0.6} \rangle, X = \langle a, \frac{\mu}{0.5}, \frac{\gamma}{0.2}, \frac{\sigma}{0.8} \rangle.$$

The corresponding closed sets are $\mathfrak{S}^C = \{0_N, 1_N, Y, Z, S, T\}$ where $Y = \langle a, \frac{\mu}{0.5}, \frac{\gamma}{0.8}, \frac{\sigma}{0.4} \rangle$,

$$Z = \langle a, \frac{\mu}{0.3}, \frac{\gamma}{0.6}, \frac{\sigma}{0.2} \rangle, S = \langle a, \frac{\mu}{0.3}, \frac{\gamma}{0.6}, \frac{\sigma}{0.4} \rangle, T = \langle a, \frac{\mu}{0.5}, \frac{\gamma}{0.8}, \frac{\sigma}{0.2} \rangle.$$

Let $O = \langle a, \frac{\mu}{0.5}, \frac{\gamma}{0.8}, \frac{\sigma}{0.2} \rangle$. Then it can be seen that $Ncl(O) = \langle a, \frac{\mu}{0.5}, \frac{\gamma}{0.8}, \frac{\sigma}{0.2} \rangle$. and $Nint^*(Ncl(O)) = \langle a, \frac{\mu}{0.5}, \frac{\gamma}{0.2}, \frac{\sigma}{0.4} \rangle \not\subseteq \langle a, \frac{\mu}{0.5}, \frac{\gamma}{0.8}, \frac{\sigma}{0.2} \rangle$. But

$$Nint(Ncl(O)) = \langle a, \frac{\mu}{0.5}, \frac{\gamma}{0.8}, \frac{\sigma}{0.2} \rangle \subseteq \langle a, \frac{\mu}{0.5}, \frac{\gamma}{0.8}, \frac{\sigma}{0.2} \rangle. \text{ Therefore } O \text{ is neutrosophicsemi closed but not}$$

neutrosophicsemi closed*.

Theorem 3.15. Let A be neutrosophicsemi closed* and B be a neutrosophic subset such that $Nint^*(A) \subseteq B \subseteq A$. Then B is neutrosophicsemi closed*.

Proof: A is neutrosophicsemi closed*, A^C is neutrosophicsemi open*. Now $Nint^*(A) \subseteq B \subseteq A$ implies $(Nint^*(A))^C \supseteq B^C \supseteq A^C$. That is $Ncl^*(A) \supseteq B^C \supseteq A^C$. Therefore by Definition 2.4, B^C is neutrosophicsemi open*. Hence B is neutrosophicsemi closed*.

Theorem 3.16. A is neutrosophic semi closed* if and only if $Nint^*(A) = Nint^*(Ncl(A))$.

Proof: Since A is neutrosophic semi closed*, $Nint^*(Ncl(A)) \subseteq A$. Hence $Nint^*(Ncl(A)) \subseteq (Nint^*(A))$. Also we have $Nint^*(A) \subseteq Nint^*(Ncl(A))$. Finally $Nint^*(Ncl(A)) = Nint^*(A)$.

Conversely, Take $U = Ncl(A)$. Then neutrosophic closed set U in κ such that $Nint^*(A) = Nint^*(Ncl(A)) = Nint^*(A) \subseteq A \subseteq U$. Then by Theorem 3.3, A is neutrosophic semi open*.

CONCLUSIONS

In this article, we had shown that the ideas of neutrosophic semi open* and neutrosophic semi closed* sets and its properties. Further, we have exhibited that the collection of neutrosophic semi open*(closed*) sets lies in the middle of the collection of neutrosophic semi open(closed) sets and the collection of neutrosophic open(closed) sets. In addition, some of their characterizations and fundamental properties of neutrosophic semi open*(closed*) sets are also discussed.

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