ON RADIO HERONIAN MEAN D-DISTANCE NUMBER OF CYCLE RELATED GRAPHS

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ABSTRACT

A Radio Heronian Mean D-distance Labeling of an associated graph is a one-to-one $h: V(D) \rightarrow G \ni$ two distinct vertices u and v of G , $d^D(u, v) + \left\lceil \frac{f(u) + \sqrt{f(u)f(v)} + f(v)}{3} \right\rceil \geq \text{diam}^D(G) + 1$ where $d^D(u, v)$ means D-distance among u and v and $\text{diam}^D(G)$ means D-diameter of G . The $\text{rhmn}^D(f)$ is the max $\{G\}$ and $\text{rhmn}^D(G)$ is the min $\{\text{rhmn}^D(f)\}$ assumed control over all radio heronian D-distance marking $f(G)$.

KEYWORDS: Radio Heronian D-Distance Number of a Cycle, Wheel, Gear, Helm and Friendship Graph

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1. INTRODUCTION

A graph $G = (V, E)$ finite not directed graph without loops or multiple edges. The $O(G)$ and size of G are indicated by p and q respectively. Graph labeling was introduced by Alexander Rosa in 1967. Radio mean labeling was presented by S. Somasundaram and R. Ponraj in 2004. Harmonic mean labeling was presented by S. Somasundaram and S. S. Sandhya in 2012.

D. Reddy Babu et al. was introduced D-distance concept. T. Nicholas and K. John Bosco was introduced radio D-distance concept in 2017. Radio mean D-distance was introduced by T. Nicholas and K. John Bosco in 2017. Heronian mean labeling was introduced by S. S. Sandhya in 2017. K. John Bosco and Dinesh was presented radio Heronian mean D-distance labeling of fundamental graphs in 2021. We are introducing radio Heronian mean D-distance number of cycle related graphs.

Theorem: 1.1

The radio Heronian D-distance mean number of a cycle,

$$\text{rhmn}^D(C_n) = \begin{cases} 2n - 2, & \text{if } n \text{ is even}, n \geq 4 \\ 4n - 9, & \text{if } n \text{ is odd}, n \geq 3 \end{cases}$$

Proof:

Let $V(C_n) = \{v_1, v_2, \dots, v_n\}$ be the vertex set. It is $\text{diam}^D(C_n) = \left(\frac{3n}{2}\right) + 2$ (if n even) and $\text{diam}^D(C_n) = \left(\frac{3(n-1)}{2}\right) + 2$ (if n odd). Define the function f as

if n is even

$$f(v_{2k-1}) = x_k, \quad 1 \leq k \leq n - 5$$

$$f(v_{n+2-2k}) = x_{n-5+k}, \quad 1 \leq k \leq n-5$$

$$f(x_k) = \{n-2+k, 1 \leq k \leq n\}$$

if n odd

$$f(v_{2k-1}) = x_k, \quad 1 \leq k \leq n-5$$

$$f(v_{n+1-2k}) = x_{n-5+k}, \quad 1 \leq k \leq n-5$$

$$f(v_n) = x_n$$

$$f(x_k) = \{3n-9+k, 1 \leq k \leq n\}$$

First Case: n even, Check on radio Heronian mean D-distance condition.

If v_k and v_l are not adjacent $|k-l| \leq \frac{n}{2}$

$$\begin{aligned} d^D(v_k, v_l) + \left\lceil \frac{f(v_l) + \sqrt{f(v_k)f(v_l)} + f(v_k)}{3} \right\rceil &\geq \left(\frac{3n}{2}\right) + 2 + \left\lceil \frac{n-2+k + \sqrt{(n-2+k)(n-2+l)} + n-2+l}{3} \right\rceil \\ &\geq \left(\frac{3n}{2}\right) + 3 \end{aligned}$$

Second Case: n odd, Check on radio Heronian mean D-distance condition,

$$d^D(u, v) + \left\lceil \frac{f(u) + \sqrt{f(u)f(v)} + f(v)}{3} \right\rceil \geq 1 + \text{diam}^D(C_n) = 3\left(\frac{n-1}{2}\right) + 3 \forall \text{ pair of } u, v \text{ where } u \neq v.$$

If v_k and v_l are adjacent,

$$\begin{aligned} d^D(v_k, v_j) + \left\lceil \frac{f(v_l) + \sqrt{f(v_k)f(v_l)} + f(v_k)}{3} \right\rceil &\geq 5 + \left\lceil \frac{k-9+3n + \sqrt{(k-9+3n)(l-9+3n)} + l-9+3n}{3} \right\rceil \\ &\geq \left(\frac{3(n-1)}{2}\right) + 3 \end{aligned}$$

$$\text{Hence } rhmn^D(C_n) = \begin{cases} 2n-2, & \text{if } n \text{ is even, } n \geq 4 \\ 4n-9, & \text{if } n \text{ is odd, } n \geq 3 \end{cases}$$

Theorem: 1.2

The radio Heronian D-distance mean number of a wheel graph,

$$rhmn^D(W_n) = \begin{cases} 7, & n = 3 \\ 8, & n = 4 \\ 9, & n = 5 \\ 3\left(\frac{n}{2}\right) + 2 & \text{if } n \text{ is even, } n \geq 6 \\ 3\left(\frac{n-1}{2}\right) + 4 & \text{if } n \text{ is odd, } n \geq 7 \end{cases}$$

Proof:

Let $V(W_n) = \{v_0, v_1, \dots, v_n\}$ be vertices and v_0 be central vertex.

It is $\text{diam}^D(G_n) = n+8$. Define the function f as

if n is even

$$f(v_0) = x_0$$

$$f(v_{2k-1}) = x_k, \quad 1 \leq k \leq \frac{n}{2}$$

$$f(v_{n-2k}) = x_{(\frac{n}{2})+k}, \quad 1 \leq k \leq \left(\frac{n}{2}\right) - 1$$

$$f(v_n) = x_n$$

$$f(x_k) = \begin{cases} 5, & k = 0 \\ \frac{n}{2} + k + 2, & 1 \leq k \leq n \end{cases}$$

if n odd

$$f(v_0) = x_0$$

$$f(v_{2k-1}) = x_k, \quad 1 \leq k \leq \frac{n-1}{2}$$

$$f(v_{n+1-2k}) = x_{(\frac{n-1}{2})+k}, \quad 1 \leq k \leq \left(\frac{n-1}{2}\right)$$

$$f(v_n) = x_n$$

$$f(x_k) = \begin{cases} 6, & k = 0 \\ \frac{n-1}{2} + k + 2, & 1 \leq k \leq n \end{cases}$$

First Case: n even, Check on radio Heronian mean D-distance condition.

If v_0 and v_k are adjacent,

$$d^D(v_0, v_k) + \left\lceil \frac{f(v_0) + \sqrt{f(v_0)f(v_k)} + f(v_k)}{3} \right\rceil \geq n + 4 + \left\lceil \frac{5 + \sqrt{(5)(n+2k+4/2)} + n + 2k + 4/2}{3} \right\rceil \geq n + 9$$

Second Case: n odd, Check on radio Heronian mean D-distance condition

If v_0 and v_k are adjacent,

$$d^D(v_0, v_k) + \left\lceil \frac{f(v_0) + \sqrt{f(v_0)f(v_k)} + f(v_k)}{3} \right\rceil \geq n + 4 + \left\lceil \frac{6 + \sqrt{(n-1+2k+4/2)(6)} + n - 1 + 2k + 4/2}{3} \right\rceil \geq n + 9$$

$$\text{Hence } rhmn^D(W_n) = \begin{cases} 7, & n = 3 \\ 8, & n = 4 \\ 9, & n = 5 \\ 3\left(\frac{n}{2}\right) + 2 & \text{if } n \text{ is even, } n \geq 6 \\ 3\left(\frac{n-1}{2}\right) + 4 & \text{if } n \text{ is odd, } n \geq 7 \end{cases}$$

Theorem: 1.3

The radio Heronian D-distance mean number of a gear graph,

$$rhmn^D(G_n) = \begin{cases} 13, n = 3 \\ 16, n = 4 \\ 2n + 9, n \geq 5 \end{cases}$$

Proof:

Let $\{v_0, v_1, \dots, v_n\}$ and $\{u_1, u_2, \dots, u_n\}$ be vertices and v_0 be central vertex.

It is $diam^D(G_n) = n + 14$. Define the function f as

$$f(v_0) = 9, f(u_k) = 9 + k, 1 \leq k \leq n, f(v_k) = n + 9 + k, 1 \leq k \leq n.$$

Check the radio Heronian mean D-distance condition,

$$d^D(u, v) + \left\lceil \frac{f(v) + \sqrt{f(u)f(v)} + f(u)}{3} \right\rceil \geq 1 + diam^D(G_n) = n + 15 \forall \text{ pair of } u, v \text{ where } u \neq v.$$

If v_0 and v_k are adjacent,

$$d^D(v_0, v_k) + \left\lceil \frac{f(v_0) + \sqrt{f(v_0)f(v_k)} + f(v_k)}{3} \right\rceil \geq n + 5 + \left\lceil \frac{9 + \sqrt{(9n + 9k + 81)} + n + 9 + l}{3} \right\rceil \geq n + 15$$

If v_0 and u_k are non adjacent,

$$d^D(v_0, u_k) + \left\lceil \frac{f(v_0) + \sqrt{f(v_0)f(u_k)} + f(u_k)}{3} \right\rceil \geq n + 7 + \left\lceil \frac{9 + \sqrt{(9k + 81)} + 9 + k}{3} \right\rceil \geq n + 15$$

$$\text{Hence } rhmn^D(G_n) = \begin{cases} 13, n = 3 \\ 16, n = 4 \\ 2n + 9, n \geq 5 \end{cases}$$

Theorem: 1.4

The radio Heronian mean D-distance number of a Helm graph,

$$rhmn^D(H_n) = 2n + 9, n \geq 3.$$

Proof:

Let $\{v_0, v_1, \dots, v_n\}$ and $\{u_1, u_2, \dots, u_n\}$ be vertices and v_0 be central vertex and also $\{u_1, u_2, \dots, u_n\}$ are the pendent vertices. It is $diam^D(H_n) = n + 14$

Define the function h as $h(v_0) = 9, h(u_k) = k + 9, 1 \leq k \leq n, h(v_k) = n + k + 9, 1 \leq k \leq n$.

Check on radio Heronian mean D-distance condition.

If v_0 and v_k are adjacent,

$$d^D(v_0, v_k) + \left\lceil \frac{f(v_0) + \sqrt{f(v_0)f(v_k)} + f(v_k)}{3} \right\rceil \geq n + 5 + \left\lceil \frac{9 + \sqrt{(9n + 9k + 81)} + n + 9 + l}{3} \right\rceil \geq n + 15$$

If v_0 and u_k are non adjacent,

$$d^D(v_0, u_k) + \left\lceil \frac{f(v_0) + \sqrt{f(v_0)f(u_k)} + f(u_k)}{3} \right\rceil \geq n + 7 + \left\lceil \frac{9 + \sqrt{(9k + 81)} + k + 9}{3} \right\rceil \geq n + 15$$

Hence $rhmn^D(H_n) = 2n + 9, n \geq 3$.

Theorem: 1.5

The radio Heronian mean D-distance number of a friendship graph,

$$rhmn^D(C_3^{(t)}) = \{3t + 2, n \geq 2\}.$$

Proof:

Let $V(C_3^{(t)}) = \{v_0, v_1, \dots, v_{2t}\}$ where v_0 is central vertex. It is $diam^D(T(P_n)) = 2t + 6$. Define the function f as

$$f(v_0) = x_0$$

$$f(v_{2k-1}) = x_k, \quad 1 \leq k \leq 2t$$

$$f(v_{2(t+1)-2k}) = x_{n+k}, \quad 1 \leq k \leq 2t$$

$$f(x_k) = \begin{cases} 2, & k = 0 \\ t + 2 + k, & 1 \leq k \leq t \\ 2t + 2 + k, & 1 \leq k \leq t \end{cases}$$

Check the radio Heronian mean D-distance condition.

If v_0 and v_k are adjacent,

$$d^D(v_0, v_k) + \left\lceil \frac{f(v_0) + \sqrt{f(v_0)f(v_k)} + f(v_k)}{3} \right\rceil \geq 2t + 3 + \left\lceil \frac{3 + \sqrt{(3t + 3k + 6)} + t + 2 + k}{3} \right\rceil \geq 2t + 7$$

$$d^D(v_0, v_k) + \left\lceil \frac{f(v_0) + \sqrt{f(v_0)f(v_k)} + f(v_k)}{3} \right\rceil \geq 2t + 3 + \left\lceil \frac{3 + \sqrt{(6t + 3k + 6)} + 2t + 2 + k}{3} \right\rceil \geq 2t + 7$$

Hence $rhmn^D(C_3^{(t)}) = \{3t + 2, n \geq 2\}$.

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