ON RADIO SQUARE SUM D-DISTANCE NUMBER OF GRAPHS

K. JOHN BOSCO¹ & G.VISHMA GEORGE²

¹Assistant professor, Department of Mathematics, St. Jude's College, Thoothoor, Tamil Nadu, India

²Research scholar, Manonmaiusundaranar University, St. Jude's College, Thoothoor, Tamil Nadu, India

ABSTRACT

Let $d^D(u, v)$ denotes the D-distance between two distinct vertices of a connected graph G , and $diam^D(G)$ be the D-diameter of G . A Radio square sum D-distance coloring of a connected graph G is an one-to-one mapping $c: V(G) \rightarrow \mathbb{N}$ such that condition holds every two distinct vertices u and v $d^D(u, v) + |c(u)|^2 + |c(v)|^2 \geq 1 + diam^D(G)$. The $rssn^D(c)$ is the maximal span of G and $rssn^D(G)$ is the minimal span among all $rssn^D(c)$ of G . A graph admitting radio square sum D-distance coloring is called radio square sum D-distance graphs. We discussed a new concept radio square sum D-distance number of certain graphs.

KEYWORDS: Sum Labeling, Square Sum Labeling, Radio Square Sum D-Distance Number

Received: Feb 03, 2021; **Accepted:** Feb 23, 2021; **Published:** Mar 17, 2021; **Paper Id.:** IJMCARJUN20213

1. INTRODUCTION

Radio labelling is an extension of distance two labelling which is motivated by channel assignment problem introduced by Hale[5]. By a graph $G = (V(G), E(G))$. We mean a simple, connected graph without loops or multiple edges. The concept of square sum labeling was introduced by V. Ajitha [6].

We assign positive integers to the vertices of graph subject to a restriction concerning the distance between vertices u and v . Let graph G with diameter d and number of vertices n to an extent that $1 \leq k \leq d$. The radio k -coloring of G is a task to assign c of colors to vertices of G for following condition that $d(u, v) + |c(u) - c(v)| \geq 1 + k$ among all u, v of G , Chatrand et al, [2,3] was presented radio labeling of graphs. D. Reddy babu presented the idea of D-distance et al. [11]

Radio D-distance and Radio mean D-distance labeling was presented by Dr.T,Nicholas and Dr .K.John Bosco in 2017 [7,8]. The connected u - v path s is defined as $l^D(s) = l(s) + \deg(v) + \deg(u) + \sum \deg(w)$ and $l(s)$ is the length of the path. D-distance of graph G is defined as $d^D(u, v) = \min\{l^D(s)\}$ where the minimum is taken over all u - v paths s in G .

We introduced the concept of radio square difference D-distance labeling of a graph G [9,10]. In this we introduce a new concept radio square sum D-distance labelling of graphs.

2.MAIN RESULT

Theorem .1.1.

For n , Radio square sum D-distance number of complete graphs, K_n , $rssn^D(K_n) = n$.

Proof:

Let $V(K_n) = \{v_1, v_2, v_3, \dots, v_n\}$ be the vertex set .

Then $d^D(v_i, v_j) = 2n - 1$

$$\text{diam}^D(K_n) = 2n - 1.$$

The radio square sum D -distance condition,

$$d^D(u, v) + |[c(u)]^2 + [c(v)]^2| \geq \text{diam}^D(G) + 1, \text{ for all } (u, v) \text{ where } u \neq v.$$

$$d^D(v_i, v_j) + |[c(v_i)]^2 + [c(v_j)]^2| \geq \text{diam}^D(K_n) + 1,$$

which implies $|[c(v_i)]^2 + [c(v_j)]^2| \geq 1$.

Therefore $\text{rssn}^D(K_n) = n$.

Theorem .1.2.

For $n \geq 2$, Radio square sum D -distance number of star graphs $K_{1,n}$, $\text{rssn}^D(K_{1,n}) = n + 1$.

Proof:

Let $V(K_{1,n}) = \{v_0, v_1, v_2, v_3, \dots, v_n\}$.

Then $d^D(v_i, v_{i+1}) = n + 4, 1 \leq i \leq n$.

So $\text{diam}^D(K_{1,n}) = n + 4$.

The radio square sum D -distance condition becomes, $d^D(u, v) + |[c(u)]^2 + [c(v)]^2| \geq \text{diam}^D(K_{1,n}) + 1$

For (v_0, v_i) , $d^D(u, v) + |[c(v_0)]^2 + [c(v_i)]^2| \geq n + 5$

$$n + 2 + |[1]^2 + (i + 1)^2| \geq n + 5,$$

For (v_i, v_{i+1}) , $d^D(v_i, v_{i+1}) + |[c(v_i)]^2 + [c(v_{i+1})]^2| \geq n + 5$

$$n + 4 + |(i + 1)^2 + (i + 2)^2| \geq n + 5$$

Define the function f as $f(v_0) = 1$ and

$$f(v_i) = i + 1, 1 \leq i \leq n.$$

Therefore $\text{rssn}^D(K_{1,n}) = n + 1$, if $n \geq 2$.

Theorem.1.3.

For $n \geq 2$. The radio square sum D -distance number of subdivision star graph $S(K_{1,n})$,

$$\text{rsdn}^D(S(K_{1,2})) = 2n + 1$$

Proof:

Let $V(S(K_{1,n})) = \{v_0, v_1, v_3, \dots, v_n, u_1, u_2, u_3, \dots, u_n\}$, and

$E(S(K_{1,n})) = \{v_0v_i, v_iu_i, 1 \leq i \leq n\}$ where v_0 is the apex vertex.

Then $d^D(v_i, u_i) = 4, 1 \leq i \leq n$,

$$d^D(v_0, u_i) = n + 5, 1 \leq i \leq n,$$

$$d^D(v_i, v_j) = n + 6, 1 \leq i, j \leq n,$$

$$d^D(u_i, u_j) = n + 10$$

$$\text{diam}^D(S(K_{1,n})) = n + 10.$$

Case 1. When n is odd

$$f(v_0) = 1, f(u_i) = i + 2, f(v_i) = n + i + 1$$

We check radio square sum D-distance condition ,

$$\text{For}(u_i, u_j), \quad d^D(u_i, u_j) + |[c(u_i)]^2 + [c(u_j)]^2| \geq n + 11$$

$$n + 10 + |(i + 2)^2 + (j + 2)^2| \geq n + 11,$$

$$\text{For}(v_i, v_j), \quad d^D(v_i, v_j) + |[c(v_i)]^2 + [c(v_j)]^2| \geq n + 11$$

$$n + 6 + |(n + i + 1)^2 + (n + j + 1)^2| \geq n + 11,$$

$$\text{For}(v_i, u_i), \quad d^D(v_i, u_i) + |[c(v_i)]^2 + [c(u_i)]^2| \geq n + 11$$

$$4 + |(n + i + 1)^2 + (i + 2)^2| \geq n + 11,$$

$$\text{For}(v_i, u_i), \quad d^D(v_i, u_i) + |[c(v_0)]^2 + [c(u_i)]^2| \geq n + 11$$

$$n + 5 + |(1)^2 + (i + 2)^2| \geq n + 11,$$

Case 2.

When n is even

$$c(v_0) = 1,$$

$$c(u_i) = i + 2,$$

$$c(v_i) = n + i + 2$$

The radio square sum D-distance condition ,

$$d^D(u_i, u_j) + |[c(u_i)]^2 + [c(u_j)]^2| \geq n + 11$$

$$n + 10 + |[c(u_i)]^2 + [c(u_j)]^2| \geq n + 11,$$

$$d^D(v_i, v_j) + |[c(v_i)]^2 + [c(v_j)]^2| \geq n + 11$$

$$n + 6 + |(n + i + 2)^2 + (n + j + 2)^2| \geq n + 11,$$

$$d^D(v_i, u_i) + |[c(v_i)]^2 + [c(u_i)]^2| \geq n + 11$$

$$4 + |(n + i + 2)^2 + (i + 2)^2| \geq n + 11,$$

$$d^D(v_0, u_i) + |[c(v_0)]^2 + [c(u_i)]^2| \geq n + 11,$$

$$n + 5 + |(1)^2 + (i + 2)^2| \geq n + 11,$$

Then $c(v_i) = n + i + 1, 1 \leq i \leq n$ and

$$c(u_i) = i + 2, 1 \leq i \leq n.$$

$$\text{Hence, } rsn^D(S(K_{1,n})) = \begin{cases} 2n + 1, & \text{if } n \text{ is odd} \\ 2n + 2, & \text{if } n \text{ is even} \end{cases}$$

Theorem 1.4.

$$\text{Radio sum D-distance number of Fan graphs, } rsn^D(F_n) = \begin{cases} n + 1, & \text{if } 2 \leq n \leq 10 \\ n + 2, & \text{if } n \geq 11 \end{cases}$$

Proof:

$$\text{Let } V(F_n) = \{v_0, v_1, v_2, \dots, v_n\}.$$

$$\text{It is clear that } \text{diam}^D(F_n) = n + 8,$$

We verify the radio square sum D-distance condition,

$$d^D(u, v) + |[c(u)]^2 + [c(v)]^2| \geq \text{diam}^D(F_n) + 1$$

$$v_0 = 1,$$

$$v_{2i+1} = i + 3, 0 \leq i \leq \frac{n}{2} - 1,$$

$$v_{2i} = n - i + 3, 1 \leq i \leq \frac{n}{2},$$

If v_0 and v_{2i+1} are adjacent,

$$d^D(v_0, v_{2i+1}) + |[c(v_0)]^2 + [c(v_{2i+1})]^2| \geq n + 9$$

$$n + 4 + |1^2 + (i + 3)^2| \geq n + 9,$$

If v_0 and v_{2i} are adjacent,

$$d^D(v_0, v_{2i}) + |[c(v_0)]^2 + [c(v_{2i})]^2| \geq n + 9$$

$$n + 3 + |(1)^2 + (n - i + 3)^2| \geq n + 9,$$

If v_{2i} and v_{2i+1} are adjacent

$$d^D(v_{2i+1}, v_{2i}) + |[c(v_{2i+1})]^2 + [c(v_{2i})]^2| \geq n + 9$$

$$6 + |(i + 3)^2 + (n - i + 3)^2| \geq n + 9,$$

$$d^D(v_{2i+1}, v_{2i}) + |[c(v_{2i+1})]^2 + [c(v_{2i})]^2| \geq n + 9$$

$$5 + |(i + 3)^2 + (n - i + 3)^2| \geq n + 9$$

If v_i and v_j are non adjacent,

$$d^D(v_{2i+1}, v_{2i}) + |[c(v_{2i+1})]^2 + [c(v_{2i})]^2| \geq n + 9$$

$$n + 8 + |(i + 3)^2 + (n - i + 3)^2| \geq n + 9$$

$$\text{Hence, } rsn^D(F_n) = \begin{cases} n + 1, & \text{if } 2 \leq n \leq 10 \\ n + 2, & \text{if } n \geq 11 \end{cases}$$

Theorem:1.5.

For $m \geq 5$, Radio square sum D-distance number of Book with triangle page graphs,

$$K_2 + mK_1, rsn^D(K_2 + mK_1) = m + n - 3$$

Proof:

Let $V(K_2 + mK_1) = \{v_1, v_2, u_1, u_2, u_3, \dots, u_m\}$ and

$$E(K_2 + mK_1) = \{v_1v_2, v_1u_i, v_2u_i \mid 1 \leq i \leq m\},$$

Then $d^D(v_i, u_j) = m + 4, \quad 1 \leq i, j \leq m,$

$$d^D(v_1, v_2) = 2m + 3,$$

$$d^D(u_i, u_j) = m + 7, \quad 1 \leq i, j \leq m,$$

and $\text{diam}^D(K_2 + mK_1) = 2m + 3.$

We check radio square sum D-distance condition is

$$d^D(u_i, u_j) + |[c(u_i)]^2 + [c(u_j)]^2| \geq 2m + 4$$

$$n + 10 + |(m + i - 3)^2 + (m + j - 3)^2| \geq 2m + 4,$$

$$d^D(v_1, v_2) + |c(v_1)^2 + c(v_2)^2| \geq 2m + 4$$

$$2m + 3 + |(1)^2 + (2)^2| \geq 2m + 4,$$

$$d^D(v_i, u_j) + |c(v_i)^2 + c(u_j)^2| \geq 2m + 4$$

$$m + 7 + |(i)^2 + (m + j - 3)^2| \geq 2m + 4,$$

Hence, $rsn^D(K_2 + mK_1) = m + n - 3$ if $m \geq 5$.

Theorem.1.6.

For $n \geq 3$, Radio square sum D-distance number of comb graphs, $rsn^D(P_n \odot K_1) = 2n$

Proof:

Let $V(P_n \odot K_1) = \{v_i, u_i, 1 \leq i \leq n\}$ and

$$E = \{v_i, v_{i+1}, v_i, u_i, 1 \leq i \leq n\}$$

Let $c(u_1) = 1,$

$c(u_n) = 2$ and

$$c(u_i) = i + 1,$$

$$c(v_i) = 2n - i + 1$$

We verify the radio square sum D-distance condition,

$$d^D(u, v) + |[c(u)]^2 + [c(v)]^2| \geq \text{diam}^D(F_n) + 1$$

$$d^D(u_i, u_j) + |[c(u_i)]^2 + [c(u_j)]^2| \geq 4n + 2,$$

$$4n + 1 + |(i + 1)^2 + (j + 1)^2| \geq 4n + 2,$$

$$d^D(u_i, u_j) + |[c(u_i)]^2 + [c(u_j)]^2| \geq 4n + 2$$

$$4n - 6 + |(i + 1)^2 + (j + 1)^2| \geq 4n + 2,$$

$$d^D(u_1, u_2) + |[c(u_1)]^2 + [c(u_2)]^2| \geq 4n + 2$$

$$4n + 1 + |(1)^2 + (2)^2| \geq 4n + 2,$$

$$d^D(v_i, v_j) + |[c(v_i)]^2 + [c(v_j)]^2| \geq 4n + 2$$

$$n + 6 + |(2n - i + 1)^2 + (2n - j + 1)^2| \geq 4n + 2,$$

$$d^D(v_i, v_j) + |[c(v_i)]^2 + [c(v_j)]^2| \geq 4n + 2$$

$$4n - 3 + |(2n - i + 1)^2 + (2n - j + 1)^2| \geq 4n + 2,$$

$$d^D(v_i, v_j) + |[c(v_i)]^2 + [c(v_j)]^2| \geq 4n + 2$$

$$4n - 7 + |(2n - i + 1)^2 + (2n - j + 1)^2| \geq 4n + 2,$$

$$d^D(u_i, u_j) + |c(u_i)^2 + c(u_j)^2| \geq 4n + 2$$

$$4n - 7 + |(i + 1)^2 + (j + 1)^2| \geq 4n + 2,$$

Hence, $\text{rssn}^D(P_n \odot K_1) = 2n$ if $n \geq 3$

REFERENCES

1. F. Buckley and F. Harary, *Distance in Graphs*, Addition- Wesley, Redwood City, CA, 1990.
2. G. Chartrand, D. Erwin, F. Harary, and P. Zhang, "Radio labeling of graphs," *Bulletin of the Institute of Combinatorics and Its Applications*, vol. 33, pp. 77-85, 2001.
3. G. Chartrand, D. Erwin, and P. Zhang, Graph labeling problem suggested by FM channel restrictions, *Bull. Inst. Combin. Appl.*, 43, 43-57(2005).
4. J.A. Gallian, A dynamic survey of graph labeling, *Electron. J. Combin.* 19 (2012) #Ds6.
5. W.K. Hale, Frequency assignment: Theory and applications, *Proc. IEEE* 68 (1980), pp.1497-1514.
6. V. Ajitha, s. Arumugam, K. A. Germina, On square sum graphs AKCE, *International Journal of Graphs and Combinatorics AKCE* 6 No.1(2009) pp.(1-10)

7. T.Nicholas and K.JohnBosco , Radio D-distance number of some graphs, *International Journal of Engineering & Scientific Research* Vol.5 Issue 2, February 2017, ISSN: 2347-6532.
8. T.Nicholas, K.JohnBosco and M. Antony, Radio mean D-distance labeling of some graphs, *International Journal of Engineering & Scientific Research* Vol.5 Issue 2, February 2017, ISSN: 2347-6532. publication in *ArsCombinatoria*.
9. K.Johnbosco,G.Vishma George, On Radio square Difference D-distance number of some standard graphs, *IJRAR* January 2021, Volume 8, Issue 1 www.ijrar.org (E-ISSN 2348-1269, P- ISSN 2349-5138)
10. K.Johnbosco, G.Vishma George, On Radio square difference Dd-distance number of cycle related Graphs, *JETIR* January 2021, Volume 8, Issue 1 www.jetir.org (ISSN-2349-5162)
11. Reddy Babu, D., Varma, P.L.N., D-distance in graphs, *Golden Research Thoughts*, 2(2013), 53-58.