Analysis of Generalized Non-Active Power Theory for Compensation of Non-Periodic Disturbances

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Abstract. The principle and analysis of usage of the generalized non-active power theory for the parallel compensation of periodic and non-periodic current disturbances is presented in the paper. A non-linear unbalanced load generating periodic as well as non-periodic stochastic currents connected to an unsymmetrical non-sinusoidal voltage source is considered.

Key words

Generalized non-active power theory, electric power system, parallel compensation, non-periodic disturbances, averaging time interval, simulation

1. Introduction

The importance of the problems related to the presence of distorted waveforms in electrical systems has been pointed out since the early thirties, when two important electric power theories were formulated by Fryze and Budeanu in 1927, 1932, respectively.

In 1984, the IRP (Instantaneous Reactive Power) theory, called also p-q theory, was first published in English [1], giving a tool to compensate for the whole reactive power and the fluctuating instantaneous active power produced by a nonlinear load.

It has been mathematically as well as in some experiments proved by many authors that under asymmetrical voltage system the IRP theory is not able to produce the sinusoidal currents of the system [2]. Thus, several modifications, like the vectorial theory [3], d-q theory [4], or the p-q-r theory [5] have been suggested to improve the IRP theory. The original IRP theory and its modifications were formulated in the time domain, so they have been the mostly used theories for proposing compensation control algorithms of active and hybrid power filters working in the real time.

Although some applications of the CPC (Current Physical Components) or FBD (Fryze - Buchholz - Depenbrock) theories as the theoretical tools for generating reference signals for compensating devices were reported [6]-[8] as well, the IRP theory and its modifications still remain prevailing ones for these purposes. The main reason is that these theories are based on the time domain approach, what makes their implementation easier and more straightforward than in case of the CPC and FBD theories.

The power theories mentioned above have been mostly discussed, classified and compared in terms of their capabilities for the compensation of unwanted instantaneous power components in three-phase asymmetrical systems with non-sinusoidal, but periodic, voltages and currents.

Although several different approaches have been focused on non-periodic (subharmonic, interharmonic) voltage and current components identification [9]-[15] as well, the purpose was mainly to model or detect these disturbance waveforms for estimation of power quality indices.

But now, the compensation of subharmonic, non-periodic, and stochastic disturbances in the electric power systems has emerged as a very serious problem to be solved. The diversity of the features of these disturbances makes their compensation quite difficult. The compensation devices based on power electronics converters must work in the real time, so the time domain approach to control algorithms for the compensation of non-periodic disturbances seems to be promising.

In the Fryze's theory the active power P was calculated as the average value of $p = v i$ over one fundamental cycle $T = 1/f$. The active current i_a over this period was then the replica of the voltage v , related to the voltage by the conductance $G = P/V^2$, where V^2 was again calculated as the average value of v^2 over the same cycle. A serious

limitation, especially in accordance with problems related to the compensation of non-periodic and stochastic disturbances, is that the calculation of i_a is characterized by the time delay T .

The original Fryze's theory of the non-active current is extended in [16] by using a general averaging time interval T_C that can be chosen either one-half fundamental cycle, one full fundamental cycle T , that is the same as that in Fryze's theory, or multiple fundamental cycles, depending on the character of the load current, compensation objectives and the energy storage capacity of a power electronics-based compensator.

A principal and analysis of usage of this GNP (Generalized Non-active Power) theory for the parallel compensation of periodic and non-periodic disturbances is presented in the paper. A non-linear unbalanced load generating periodic as well as non-periodic stochastic currents connected to an unsymmetrical non-sinusoidal voltage source is considered.

2. Generalized non-active power theory

For the three-phase system, the voltage and current vectors may be expressed as follows

$$\begin{aligned} \mathbf{v} &= [v_a, v_b, v_c]^T \\ \mathbf{i} &= [i_a, i_b, i_c]^T \end{aligned} \quad (1)$$

The active current is defined in this generalized non-active power theory by

$$\mathbf{i}_a(t) = \frac{P(t)}{V_P^2(t)} \mathbf{v}_P(t) = Y(t) \mathbf{v}_P(t) \quad (2)$$

$$P(t) = \frac{1}{T_C} \int_{t-T_C}^t \mathbf{v}^T(\tau) \mathbf{i}(\tau) d\tau \quad (3)$$

$$V_P^2(t) = \frac{1}{T_C} \int_{t-T_C}^t \mathbf{v}_P^T(\tau) \mathbf{v}_P(\tau) d\tau \quad (4)$$

where $\mathbf{v}_P(t)$ is the reference voltage vector.

The non-active current is the remaining part of the current vector

$$\mathbf{i}_n(t) = \mathbf{i}(t) - \mathbf{i}_a(t) \quad (5)$$

If the current $\mathbf{i}(t)$ is the current of an unbalanced load, which may contain, in general, harmonic and also non-periodic or stochastic components, and the current $\mathbf{i}_a(t)$ is the demanded source current, then $\mathbf{i}_n(t)$ is the current that should be injected by a parallel compensator connected to the PCC (Point of Common Coupling).

By choosing $T_C \rightarrow 0$, the GNP theory gives the same results as those obtained by using the well known IRP theory. If $T_C \rightarrow \infty$, the non-active component in the non-periodic load current is completely eliminated, so the source current is only active.

3. Analysis of GNP theory

It was shown [17] that the precision of the calculation of the currents $\mathbf{i}_a, \mathbf{i}_n$ by using (2), (5) in unbalanced non-sinusoidal systems with non-periodic load currents is sufficient if T_C is chosen to be 5–10 times that of the fundamental period. Let us demonstrate some basic attributes of the GNP theory on an example of the one phase voltage and current

$$v = \sqrt{2} V \sin \omega t \quad (6)$$

$$\begin{aligned} i &= \sqrt{2} I (1 + \Delta \sin \Delta \omega t) \sin \omega t + \sqrt{2} I_h \sin \omega_h t = \\ &= (\sqrt{2} I + \sqrt{2} \Delta I \sin \Delta \omega t) \sin \omega t + \sqrt{2} I_h \sin \omega_h t = \\ &= \sqrt{2} I \sin \omega t + i_{np} + i_h \end{aligned} \quad (7)$$

The current contains three components: the fundamental, non-periodic (specifically subharmonic here), and harmonic one.

By using the concept of the GNP theory (2) - (4), the respective active current can be expressed as follows

$$i_a = Yv = \sqrt{2} I \sin \omega t + i_2 + i_{anp} + i_{ah} \quad (8)$$

where the expressions for the components are presented in the Appendix. If $T_C = kT/2$, $k=1, 2, 3, \dots$, the current component i_2 is equal to zero. By changing the value of T_C we can evaluate how the components i_{anp}, i_{ah} are more or less excluded from the active current component i_a . As it has been already said, for $T_C \gg T$ these two components are almost completely suppressed and we can get the active current practically in the correct form $i_a = \sqrt{2} I \sin \omega t$.

After some manipulations, the equation (D3) for i_{ah} in Appendix can be expressed as

$$i_{ah} = \frac{\sqrt{2} I_h}{2} \left[\begin{aligned} &G_h^+ \sin(\omega_h t - \phi_h^+) + G_h^- \sin(\omega_h t - \phi_h^-) - \\ &-G_h^+ \sin((\omega_h + 2\omega)t - \phi_h^+) - \\ &-G_h^- \sin((\omega_h - 2\omega)t - \phi_h^-) \end{aligned} \right] \quad (9)$$

where

$$\begin{aligned} G_h^+ &= \sin \phi_h^+ / \phi_h^+, \quad \phi_h^+ = (\omega_h + \omega)T_C / 2 \\ G_h^- &= \sin \phi_h^- / \phi_h^-, \quad \phi_h^- = (\omega_h - \omega)T_C / 2 \end{aligned} \quad (10)$$

We see that, in addition to the presence of the component with the frequency ω_h , two more components with frequencies $(\omega_h + 2\omega)$, $(\omega_h - 2\omega)$ have appeared in the spectrum of i_{ah} after processing the current i (7) in (2).

By using formulas (9)-(10), we can analyze how the current component with the frequency ω_h , which is present in the load current i_L , will appear in the active current i_{La} when the GNP theory is applied. Let us suppose that the frequency ω_h is not only an integer multiple of the fundamental frequency ω (harmonic component), but generally any other one, that is subharmonic $\langle 2\pi 1, 2\pi 49 \text{ Hz} \rangle$, harmonic $\langle 2\pi h 50 \text{ Hz}, h=1,$

2, 3,...) or interharmonic higher than $2\pi 50$ Hz. The thing is that these non-periodic components with frequencies different from integer multiples of the fundamental frequency ω and their possible compensation are of great interest today.

Fig. 1 shows the impact of the value of T_C on the rate I_{La}/I_L for different frequencies, where I_{La} , I_L are the respective magnitudes of the current components presented in $i_{La}(t)$, $i_L(t)$ (for i_{La} given by the first two terms in (9)). We see that all harmonics presented in $i_L(t)$ are completely suppressed for T_C being one full fundamental cycle T or any multiple fundamental cycles. We see that components of interharmonic frequencies are substantially suppressed as well. As for the subharmonic frequencies we can estimate the degree of the suppression of these components too. For example, if the magnitude of the load current fluctuates with the frequency 15 Hz, the current components of frequencies 35, 65 Hz, in addition to the component of the fundamental frequency 50 Hz, are presented in this current. These components remain in the spectrum of the current i_{La} , but their magnitudes are mitigated, for example, only to 18.7 and 24.4 % (for $T_C = 5T = 0.1s$, two greatest side lobes at the green line in Fig. 1) of those presented in the load current i_L .

Fig. 1. Impact of value of T_C on rate I_{La}/I_L ; a) for frequencies in range $<0, 700$ Hz>; b) zoom of part a)

Figs. 2, 3 show similar dependences of the terms G_h^+ , G_h^- (10) characterizing the magnitudes of two further components with frequencies $(\omega_h+2\omega)$, $(\omega_h-2\omega)$ which are pronounced, according to (9), in the spectrum of i_{La} as well. It is evident that $G_h^+=G_h^-=0$ for the frequencies being integer multiples of the fundamental frequency. Therefore these two additional components are zero for harmonic frequencies too.

Fig. 2. Impact of value of T_C on rate G_h^+ for different frequencies

Fig. 3. Impact of value of T_C on rate G_h^- for different frequencies

Tab. I Parameters of the load voltage and current used in simulation

Variable	Magnitude (p.u.)	Phase
$v_L = v_{L1} = v_{L1}^+ = v_{LP}$	1	0
i_{L1}^+	1	$-\pi/2$
i_{L1}^-	0.5	0
i_{L5}	0.2	0

We see that, especially for frequencies $<0,100$ Hz>, the term G_h^- is dominating over G_h^+ due to its denominator which is greater than that of G_h^+ in this frequency range. It needs to be emphasized that the values of G_h^+ , G_h^- for some specific frequency ω_h represent the magnitudes of two additional components in (9) with the frequencies $(\omega_h+2\omega)$, $(\omega_h-2\omega)$, respectively.

Let us demonstrate the use of the GNP strategy presented above on an example of the three phase unbalanced non-sinusoidal system with the load current i_L that contains, in addition to the active current i_{La} , also the reactive current i_{L1r} associated with the fundamental harmonic only, the fundamental harmonic unbalanced current i_{L1u} , and the harmonic current i_{Lh} .

Figs. 4-6 show the results of the simulation of a model based on (1)-(5) with $v_{LP} = v_{L1}$ (v_{LP} is selected as the fundamental positive sequence component), $T_C = 5T = 0.1s$, and with the parameters of the fundamental positive sequence components v_{L1}^+ , i_{L1}^+ , fundamental negative sequence component i_{L1}^- , and the fifth harmonic current i_{L5} declared in Tab. I.

In Fig. 4 we can see how the active current i_{La} in one phase is separated from i_L when the magnitude of the load current i_L is modulated by the subharmonic signal with the

magnitude $\Delta I_L=0.2$ p.u. and frequency 10 Hz. Due to the phase shift $-\pi/2$ between v_{L1} and i_{L1}^+ the current component i_{La} has the magnitude $\sqrt{2}/2$ p.u. Because $T_C=5T=0.1$ s is just the cycle period of the subharmonic frequency 10 Hz, the extracted active current does not contain any subharmonic component and, of course, any harmonic one as well.

Fig. 4. Extraction of active current from load current i_L specified by Tab. I and additionally modulated by the subharmonic signal with the magnitude $\Delta I_L=0.2$ p.u. and frequency 10 Hz

In Fig. 5 we can see the extraction of the active current i_{La} ($T_C=5T=0.1$ s) from the load current i_L in one phase when the magnitude of the load current i_L specified in Tab. I is modulated additionally by the white noise signal with the power spectral density 0.001875 and the correlation time of the noise 0.002. We see that the load current imitates fairly a waveform with a non-periodic stochastic character like that produced for example by an Electrical Arc Furnace (EAF).

The active current i_{La} is harmonic free, balanced and in phase with the source voltage v_L .

Fig. 5. Extraction of active current from load current i_L specified by Tab. I and additionally modulated by white noise signal with power spectral density 0.001875 and correlation time of noise 0.002

Finally, Fig. 6 presents the results of the extraction of the active current i_{La} when the additional current interharmonics (104, 117, 134, and 147 Hz) specified by [18] are superimposed to the components of the load current i_L imitates a current waveform produced, for example, by the EAF.

Fig. 6. Extraction of active current from load current i_L with superimposed interharmonics (104, 117, 134, and 147 Hz) specified by [18]

It is evident that if the non-active load current $i_{Ln}=i_L-i_{La}$ is completely compensated for by a parallel

compensator, only the active load current i_{La} is exchanges with the source, i.e. $i_S=i_{La}$.

The choice of the value of T_C has a great influence on the quality of the compensation of non-periodic current components. The values of $T_C=(5-10)T$ are high enough to compensate sufficiently for especially the interharmonics with frequencies above 100 Hz. The thing is that interharmonics can be more troublesome than harmonics and voltage fluctuations caused by a combination of interharmonics can cause voltage fluctuations in addition to waveform distortions, resulting in light flicker [12]. As for subharmonics, the lower their frequency is, the higher the value of T_C should be for a sufficient compensation, Fig. 1. The full compensation of the subharmonic with the frequency $\Delta\omega$ needs the averaging time interval $T_C=2\pi/\Delta\omega$ which, on the other hand, restricts the compensation dynamics in case of transient processes [17]. Another thing is that the energy storage requirement on a power electronics-based compensator is higher for lower T_C . That is why, a proper balance between a source current quality in steady states as well as in transients and energy storage requirements should be ensured for concrete conditions.

4. Parallel active power filter compensation

The three phase power system was simulated in the Matlab/SimulinkTM environment. The parallel APF (Active Power Filter) was based on the PWM (Pulse Width Modulated) converter. The converter was modeled as a 4-level Capacitor Clamped Inverter (CCI) working with the sampling frequency 1600 Hz. The converter reference current was $i_{CCI}^*=i_L-i_{La}$.

The nominal line-to-line voltage of the grid was 10 kV_{RMS}, without any disturbances. The parameters of the 400 kVA load, modeled as the three phase system of current sources, are in Tab. II.

Tab. II Parameters of non-linear load

Current component	Value in phase	Current vector phase shift
i_{L1}^+	$I_N=23.1$ A _{RMS}	$-\pi/2$
i_{L1}^-	$0.5 I_N$	0
i_{L5}	$0.2 I_N$	0
i_{L7}	$0.15 I_N$	0

In Fig. 7 we see the waveforms of the grid voltage v_L , load current i_L , and grid current i_S after compensation in all three phases. The magnitude of the load current i_L with components declared in Tab. II was additionally modulated by the subharmonic signal with the magnitude $\Delta I_L=0.2$ p.u. and frequency 5 Hz. We see that there is a phase shift between the grid voltage and the load current which is not the case for the grid current. Also the magnitudes of the grid currents in all three phases are approximately the same, although the load current is, due to the presence of the negative component i_{L1}^- , highly unbalanced. But, due to the choice $T_C=5T$, a significant part of the subharmonic component with the frequency 5 Hz is still present in the grid current in accordance with the results presented in Fig. 1. The full compensation of this current could be obtained by setting $T_C=10T$.

Fig. 7. Waveforms of grid voltage v_L (dotted blue curve), load current i_L (dashed green curve), and grid current i_S (full red curve) after compensation in all three phases. The magnitude of load current i_L with components declared in Tab. II was modulated by subharmonic signal with magnitude $\Delta I_L = 0.2$ p.u. and frequency 5 Hz

Recently, a three-phase four-leg unified series-parallel active filter system compensating for both periodic and non-periodic disturbances using the GNP theory has been presented [19].

5. Conclusion

A principal of usage of the generalized non-active power theory for the parallel compensation of periodic and non-periodic (subharmonic, interharmonic) current disturbances is presented in the paper. A non-linear unbalanced load generating periodic as well as non-periodic stochastic currents connected to an unsymmetrical non-sinusoidal voltage source is considered.

The impact of the value of T_C on the rate I_{La}/I_L for different frequencies has been studied in the frequency domain (I_{La} is a residuum of the load current magnitude I_L remaining in the active current after the applying GNP strategy). It was found that especially the interharmonic current components above 100 Hz are sufficiently suppressed for $T_C = (5 - 10)T$. As for subharmonics, the lower their frequency is, the higher the value of T_C should be for a sufficient compensation. On the other hand, the compensation dynamics in case of transient processes is becoming worse for T_C increasing.

The effectiveness of the generalized non-active power theory to mitigate non-periodic current disturbances is demonstrated on the example of a reactive, unbalanced load current with the 5th harmonic and several interharmonic components, additionally modulated either by the subharmonic component or the white noise signal by the simulations in the time domain.

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Appendix

The load current i (7) consists of three components: the fundamental, subharmonic, marked here as a non-periodic component (because the non-periodic current means here a current whose frequency is not an integer multiple of the fundamental frequency), and harmonic one. The calculated active current i_a (8) contains then the following three components i_2 , i_{anp} , i_{ah}

$$i_2 = \frac{\sqrt{2}}{T_C} \left[-\frac{I}{2\omega} \sin 2\omega\tau \right]_{t-T_C}^t \sin \omega t \quad (D1)$$

$$i_{anp} = \frac{\sqrt{2}}{T_C} \left[\begin{aligned} &-\frac{\Delta I}{\Delta\omega} \cos \Delta\omega\tau + \frac{\Delta I}{2(2\omega + \Delta\omega)} \cos(2\omega + \Delta\omega)\tau - \\ &-\frac{\Delta I}{2(2\omega - \Delta\omega)} \cos(2\omega - \Delta\omega)\tau \end{aligned} \right]_{t-T_C}^t \sin \omega t \quad (D2)$$

$$i_{ah} = \frac{\sqrt{2}}{T_C} \left[\frac{-I_h}{\omega + \omega_h} \sin(\omega + \omega_h)\tau + \frac{I_h}{\omega - \omega_h} \sin(\omega - \omega_h)\tau \right]_{t-T_C}^t \sin \omega t \quad (D3)$$

The terms marked by the bracket parenthesis, which result from the integration (3), mean the differences of the expressions placed inside between the times t and $t-T_C$.

The components i_2 , i_{anp} , i_{ah} are more or less excluded from the active current component i_a depending on the value of the general averaging time interval T_C and the frequencies $\Delta\omega$, ω_h .