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MODELING THE «POWER-SOCIETY» SYSTEM WITH TWO BUREAUCRATIC CLANS AND BIPOLAR REACTION OF THE SOCIETY*

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The paper studies the model of «Power-Society» system with two clans and bipolar reaction of the society. The «Power-Society» model describes the dynamics of distribution of power in hierarchy. This dynamics is influenced by society. Continuous-time «Power-Society» model has the form of parabolic equation in the case of continuous hierarchy, and the form of system of ordinary differential equations in the case of discrete hierarchy. The discrete-time model considered in this paper has the form of five dynamical equations. Bipolar reaction of the society refers to the situation with two stable distributions of power. In other words, for each government official two values are possible for the volume of power. Each of these values is considered by society as desirable. If each official holds the greater volume, we say that there is the «strong hand» distribution, if they all hold the smaller volume, this is the participatory distribution. Bureaucratic clans are an association of bureaucrats united by common interests and pursuing common goals, generally speaking, different from those of society as a whole. The paper considers a simple hierarchy of five officials, of which one is the head and four others form two competing clans. The system is studied numerically. It is shown, in particular, that in this system, the clan's lust for power significantly affects how quickly it manages to increase its power, however, the achieved amount of power itself almost does not depend on the lust for power, but is determined by the reaction of society.

Keywords: mathematical modeling, dynamic model, «Power – Society» system, political clans, numerical experiment.

Introduction. Historically, the concept of clan has been applied to a specific form of association of blood relatives. Over time, it acquired a political context. For example, Collins [1] and Shatz [2] consider clans as informal political communities formed on the basis of real or symbolically created family ties. In general, in political studies, clan is viewed as a hierarchical system of a closed nature, formed on the basis of the principles of personal loyalty and community of material interests.

The main method for studying political clans in this work is mathematical modeling - a scientific methodology [3], which was formed in the middle of the 20th century in connection with the work on the creation of a «nuclear shield», and subsequently became widespread, primarily in the tasks of natural and technical sciences. In recent years, mathematical modeling has been increasingly used in social sciences [4-8]. In a more concrete sense, this work develops a model of the "Power -

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Society" system (see, for example, [9-11]), which implements a systemic-social approach to the study of the power hierarchy.

1. Model

Let's consider a model of the «Power – Society» system with two opposing clans (Fig. 1). Here the clan is formalized as a part («subtree») of the hierarchy, which, as a rule, has a vertical structure and consists of instances pursuing common goals, which, generally speaking, do not coincide with the goals of society. To describe the phenomenon of "clannishness", we use a discrete version of the tree hierarchy model, since it provides the ability to analyze the topology of the power tree and set characteristics for each specific instance.

Figure 1. Five-point hierarchy with two clans

Consider the simplest five-point tree, which is two three-point branches with a common root vertex (Fig. 1). The value of power at the highest nod is denoted here by p_0 , at the nodes of the first (middle) layer – p_{11} , p_{12} , at the nodes of the second (lower) layer – p_{21} , p_{22} . We assume that the left subtree (p_{11} and p_{21}) form the left clan, and the right subtree (p_{12} and p_{22}) form the right clan. The monograph [10] examines a system with one clan and formulates possible clan strategies.

An effective way to increase the power of a clan is to increase its members' degree of their lust for power. The total reaction of the system is taken in the form

$$F_{ij}(t) = -k_1 (p_{ij}(t) - p_{ij}^1) (p_{ij}(t) - p_{ij}^2) (p_{ij}(t) - p_{ij}^3) + F_{H,ij}.$$

Here $p_{ij}(t)$ are current volumes of power, p_{ij}^1 and p_{ij}^3 are «ideal», i.e. attractive for the society volumes of power. Here k_1 stands for the intensity of the civil society reaction is proportional to the deviation of the level of power from the «ideal»; k is the parameter that characterizes the «difference» of powers between instances – the linear function). Then $F_{H,ij}$ refers to the degree of lust for power in the instance in question. This value is positive for the instances that are in the clans (F_{H1} for the first clan and F_{H2} for the second), and equals to zero ($F_H = 0$) for the highest hierarchy, which does not belong with any clan.

The system for dynamics of distribution of power in the hierarchy is as follows:

$$\begin{aligned} p_0(t+1) = & p_0(t) - k_0(p_0(t) - p_{11}(t)) - k_0(p_0(t) - p_{12}(t)) - \\ & - k_1(p_0(t) - p_0^1)(p_0(t) - p_0^2)(p_0(t) - p_0^3), \end{aligned} \quad (1)$$

$$p_{11}(t+1) = p_{11}(t) + k_0(p_0(t) - p_{11}(t)) - k_0(p_{11}(t) - p_{21}(t)) - k_1(p_{11}(t) - p_{11}^1)(p_{11}(t) - p_{11}^2)(p_{11}(t) - p_{11}^3) + F_{H1}, \quad (2)$$

$$p_{12}(t+1) = p_{12}(t) + k_0(p_0(t) - p_{12}(t)) - k_0(p_{12}(t) - p_{22}(t)) - k_1(p_{12}(t) - p_{12}^1)(p_{12}(t) - p_{12}^2)(p_{12}(t) - p_{12}^3) + F_{H2}, \quad (3)$$

$$p_{21}(t+1) = p_{21}(t) + k_0(p_{11}(t) - p_{21}(t)) - k_1(p_{21}(t) - p_{21}^1)(p_{21}(t) - p_{21}^2)(p_{21}(t) - p_{21}^3) + F_{H1}, \quad (4)$$

$$p_{22}(t+1) = p_{22}(t) + k_0(p_{12}(t) - p_{22}(t)) - k_1(p_{22}(t) - p_{22}^1)(p_{22}(t) - p_{22}^2)(p_{22}(t) - p_{22}^3) + F_{H2}. \quad (5)$$

where k_0 refers to the responsibility of the officials, $p_{ij}^1 < p_{ij}^2 < p_{ij}^3$ for all considered i, j . This system is derived from the calculation of the balance of power in the instances. System (1)-(5) is supplemented with initial conditions, i.e. values

$$p_0(0) = p_{0,start}, \quad p_{11}(0) = p_{11,start}, \quad p_{12}(0) = p_{12,start}, \quad p_{21}(0) = p_{21,start}, \quad p_{22}(0) = p_{22,start}. \quad (6)$$

2. Numerical experiments.

Experiment 1. Let the parameters be:

$$p_0^1 = 10, p_0^2 = 15, p_0^3 = 20, \quad p_{11}^1 = 5, p_{11}^2 = 7, p_{11}^3 = 9, \quad p_{12}^1 = 3, p_{12}^2 = 5, p_{12}^3 = 6, \\ p_{21}^1 = 2, p_{21}^2 = 4, p_{21}^3 = 5, \quad p_{22}^1 = 1, p_{22}^2 = 2, p_{22}^3 = 4, \\ F_{H1} = 0,04, F_{H2} = 0,03, k_0 = 0,01, k_1 = 0,02.$$

Initial conditions:

$$p_0(0) = 14, \quad p_{11}(0) = 2, \quad p_{12}(0) = 2, \quad p_{21}(0) = 1, \quad p_{22}(0) = 1.$$

The results are shown in Fig. 2. In this experiment, the left clan is somewhat more power-hungry than the right ($F_{H1} = 0,04 > F_{H2} = 0,03$). In addition, society wants him to have more power than the right ($p_{11}^k > p_{12}^k, p_{21}^k > p_{22}^k, k = 1, 2, 3$). The initial conditions for the two clans are the same: $p_{11}(0) = p_{12}(0), p_{21}(0) = p_{22}(0)$. However, the dynamics of the system is such that at some time interval (approximately from $t = 60$ to $t = 100$), the second clan has more power than the first.

The uneven growth of some variables is also noteworthy. For example, the function $p_{11}(t)$ has a boundary layer in the vicinity of the point $t = 0$, then almost does not increase for a long time, remaining in the vicinity of the values 5.5-5.6. Only after a long period does it resume active growth and approaches a stationary solution. The meaningful conclusion is that internal transition layers appear in the dynamics, which could hardly have been predicted without mathematical modeling.

Experiment 2. Let the parameters be:

$$p_0^1 = 10, p_0^2 = 15, p_0^3 = 20, \quad p_{11}^1 = 5, p_{11}^2 = 7, p_{11}^3 = 9, \quad p_{12}^1 = 3, p_{12}^2 = 5, p_{12}^3 = 6, \\ p_{21}^1 = 2, p_{21}^2 = 4, p_{21}^3 = 5, \quad p_{22}^1 = 1, p_{22}^2 = 2, p_{22}^3 = 4, \\ F_{H1} = 0,04, F_{H2} = 0,03, k_0 = 0,01, k_1 = 0,02.$$

Initial conditions:

$$p_0(0) = 14, \quad p_{11}(0) = 2, \quad p_{12}(0) = 7, \quad p_{21}(0) = 1, \quad p_{22}(0) = 3.$$

Figure 2. Experiment 1. The dynamics of the variables of model (1)-(6)

Figure 3. Experiment 2. The dynamics of the variables of model (1)-(6)

This experiment differs from the previous one in initial conditions. Namely, here is considered a situation in which the second clan initially has more power than the first. The results are shown in Fig. 3. Obviously, changing the initial values did not affect the outcome of the dynamic process.

In the following experiments, the influence of the first clan's lust for power on the final amount of power is examined.

Experiment 3. Let the parameters be:

$$\begin{aligned} p_0^1 = 10, p_0^2 = 15, p_0^3 = 20, p_{11}^1 = 5, p_{11}^2 = 7, p_{11}^3 = 9, p_{12}^1 = 3, p_{12}^2 = 5, p_{12}^3 = 6, \\ p_{21}^1 = 2, p_{21}^2 = 4, p_{21}^3 = 5, p_{22}^1 = 1, p_{22}^2 = 2, p_{22}^3 = 4, \\ F_{H1} = 0,035, F_{H2} = 0,03, k_0 = 0,01, k_1 = 0,02. \end{aligned}$$

Initial conditions:

$$p_0(0) = 14, p_{11}(0) = 2, p_{12}(0) = 2, p_{21}(0) = 1, p_{22}(0) = 1.$$

This experiment differs from Experiment 1 in the meaning of the lust for power of the first clan: here we have $F_{H1} = 0,035$ (instead of $F_{H1} = 0,04$). The results are shown in Fig. 4. The graph shows that the final values of the variables practically do not differ from those that we saw in Experiment 1, however, the inner transition layer of the variables $p_{11}(t), p_{21}(t)$ occurs later.

Experiment 4. Let the parameters be:

$$\begin{aligned} p_0^1 = 10, p_0^2 = 15, p_0^3 = 20, p_{11}^1 = 5, p_{11}^2 = 7, p_{11}^3 = 9, p_{12}^1 = 3, p_{12}^2 = 5, p_{12}^3 = 6, \\ p_{21}^1 = 2, p_{21}^2 = 4, p_{21}^3 = 5, p_{22}^1 = 1, p_{22}^2 = 2, p_{22}^3 = 4, \\ F_{H1} = 0,03, F_{H2} = 0,03, k_0 = 0,01, k_1 = 0,02. \end{aligned}$$

Initial conditions:

$$p_0(0) = 14, p_{11}(0) = 2, p_{12}(0) = 2, p_{21}(0) = 1, p_{22}(0) = 1.$$

This experiment differs from Experiment 3 in that now $F_{H1} = 0,03$. The main observation is that inner transition layer of the variables $p_{11}(t), p_{21}(t)$ has moved even further to the right.

Figure 4. Experiment 3. The dynamics of the variables of model (1)-(6)

Figure 5. Experiment 4. The dynamics of the variables of model (1)-(6)

Conclusion. This work examines a model of the Power-Society system with two clans and a bipolar reaction of society. In most of the experiments, the initial values of the amount of power were below the stationary ones. The main conclusions are as follows.

1. A clan's lust for power has a significant impact on how quickly it manages to achieve a stationary volume of power (which higher than the initial volume). However, the very meaning of this level almost does not depend on lust for power, but is determined by the reaction of society.

2. The lust for power of one of the clans has almost no effect on the power of the First Hierarchy, as well as on the power of the other clan. Thus, if, with an increase in the lust for power of one of the clans, the first hierarchy has the ability (due to the support of society) to maintain a constant volume of his power, then the volume of power of the other clan is also kept constant. In other words, under these conditions, an increase in the lust for power of one of the clans does not lead to the fact that it takes power from another clan.

References

1. Collins K. The Logic of Clan Politics: Evidence from the Central Asian Trajectories // World Politics. 2004. Vol. 56. No. 2. pp. 224-261.
2. Schatz E. Modern Clan Politics: The Power of «Blood» in Kazakhstan and Beyond. Seattle, 2004.
3. Samarskii A.A., Mikhailov A.P. Principles of Mathematical Modelling: Ideas, Methods, Examples. 2001. Taylor and Francis Group.
4. Petrov A.P., Proncheva O.G. Modeling Position Selection by Individuals during Informational Warfare with a Two-Component Agenda // Mathematical Models and Computer Simulations, 2020, Vol. 12, No. 2, pp. 154–163. DOI: 10.1134/S207004822002009X.
5. Petrov A.P., Lebedev S.A. Online Political Flashmob: the Case of 632305222316434 // Computational mathematics and information technologies. 2019. No 1. pp. 17-28. DOI: 10.23947/2587-8999-2019-1-1-17-28.
6. Petrov A., Proncheva O. Propaganda Battle with Two-Component Agenda // Proceedings of the MACSPro Workshop 2019. Vienna, Austria, March 21-23, 2019. pp. 28-38. CEUR Workshop Proceedings. Vol. 478. ISSN 1613-0073. <http://ceur-ws.org/Vol-2478/>
7. Petrov A.P., Proncheva O.G. Stationary States in a Model of Position Selection by Individuals // Computational Mathematics and Mathematical Physics, 2020, Vol. 60, No. 10, pp. 1737–1746. DOI: 10.1134/S0965542520100115.

8. Mikhailov A.P., Yukhno L.F. The formulation and preliminary study of the model of the hype dissemination of information in society // Computational mathematics and information technologies. 2019. No 2. P. 76-82.
9. Mikhailov A.P. Mathematical Modeling of Power Distribution in State Hierarchical Structures Interacting With Civil Society. Proceedings of 14-th IMACS World Congress, Atlanta, USA, 1994, Vol. 2, pp. 828.
10. Mikhailov A.P. Modeling the «Power – Society» system. M.: Fizmatlit, 2006. 144 p. (in Russian).
11. A. P. Mikhailov, E. A. Gorbatikov, Anticorruptional strategies analysis in the modified «power-society» model, Math. Models Comput. Simul., 8:6 (2016), 709–724.

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МОДЕЛИРОВАНИЕ СИСТЕМЫ «ВЛАСТЬ-ОБЩЕСТВО» С ДВУМЯ БЮРОКРАТИЧЕСКИМИ КЛАНАМИ И БИПОЛЯРНОЙ РЕАКЦИЕЙ ОБЩЕСТВА*

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В статье исследуется модель системы «Власть-Общество» с двумя кланами и биполярной реакцией общества. Модель «Власть-Общество» описывает динамику распределения власти в иерархии с учетом влияния общества. Модель «Власть-Общество» с непрерывным временем имеет форму параболического уравнения в случае непрерывной иерархии и форму системы обыкновенных дифференциальных уравнений в случае дискретной иерархии. Рассматриваемая в данной статье модель с дискретным временем представляет собой систему пять динамических уравнений. Биполярная реакция общества описывает ситуацию с двумя устойчивыми распределениями власти; другими словами, для каждого государственного чиновника возможны два значения объема власти, каждое из которых рассматривается обществом как желательное. Если каждый чиновник реализует больший объем власти из этих двух, то имеет место распределение «сильной руки», если все они реализуют меньший объем, то имеет место партиципаторное распределение. Под бюрократическим кланом понимается объединение бюрократов, имеющих общий интерес и преследующих общие цели, вообще говоря, отличные от целей общества в целом. В статье рассматривается иерархия из пяти должностных лиц, из которых один является первоиерархом, а четыре других образуют два конкурирующих клана. Система изучается численно. Показано, в частности, что в этой системе властолюбие клана существенно влияет на то, насколько быстро ему удастся увеличить свою власть, однако само достигнутое количество власти почти не зависит от властолюбия, а определяется реакция общества.

Ключевые слова: математическое моделирование, динамическая модель, система «Власть - Общество», политические кланы, численный эксперимент.

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