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Edge Trimagic Total Labeling of Mobius Ladder, Book and Dragon Graphs

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Abstract. A graph G = (V, E) with p vertices and q edges is said to be edge trimagic total labeling if there exists a bijection f: $V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p + q\}$ such that for each edge uv $\in$ E(G), the value of f(u) + f (uv) + f(v) is either k_1 or k_2 or k_3. In this paper, we prove that the graphs (n, *l*) - dragon, Mobius ladder M_n and Book B_n are edge trimagic total labeling.

Keywords: Graph, function, bijection, dragon, mobius ladder, book, trimagic labeling

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

A labeling of a graph is an assignment of integers to vertices or sometimes edges in a graph based upon certain criteria. Rosa in 1967, introduced the concept of graph labeling. Any graph labeling will have the following three common characteristics: (i) A set of numbers from which vertex labels are chosen (ii) A rule that assigns a value to each edge (iii) A condition that this value has to satisfy [13]. Graph labeling are of many types such as graceful, harmonious, elegant, cordial, magic, antimagic, bimagic etc. Harary [4] is referred to know about the notations in graph theory.

Magic labeling was introduced by Sedlacek [12]. Kotzing and Rosa [10], defined edge magic of a graph G with a bijection f: $V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ such that, for each edge uv $\in$ E(G), f(u) + f (uv) + f(v) is a magic constant. In [3] shows the cycle C_n with P_3 chords are edge magic total labeling. Edge bimagic labeling of graphs was introduced by Babujee [2] in 2004, defined by a graph G with a bijection f: $V \cup E \rightarrow \{1, 2, \ldots, p + q\}$ such that for each edge uv$\in$ E(G), the value of f(u) + f(uv) + f(v) is either k_1 or k_2. Magic and bimagic labeling for disconnected graphs are showed in [1].

In 2013, Jayasekaran et al. [5] introduced the edge Trimagic total labeling of graphs. An edge trimagic total labeling of a (p, q) graph G is a bijection f: $V \cup E \rightarrow \{1, 2, \ldots, p+ q\}$ such that for each edge uv $\in$ E(G), the value of f(u) + f(uv) + f(v) is equal to any of the distinct constants k_1 or k_2 or k_3. An edge trimagic total labeling is called a super edge trimagic total labeling of a graph G, if the vertices are labeled with the smallest possible integers i.e. 1, 2, …, p. Graphs such as Umbrella $U_{n,m}$, Dumb bell Db_n and Circular ladder CL(n) are proved to be edge trimagic and super edge trimagic total graphs in [7]. In [6], edge trimagic labeling of digraphs were discussed.

A (n, *l*) - dragon is formed by joining an end point of a path P_n to a point of cycle C_n (Koh, et al [11] call these tadpoles; Kim and Park [9] call them kites). Mobius ladder M_n is the graph obtained from the ladder $P_n \times P_2$ by joining the opposite end points of the two copies of P_n. The Book B_n is the graph $S_n \times P_2$ where S_n is the star with n+1 vertices.

For more references, we use dynamic survey of graph labeling by Gallian [8]. In this paper, we prove that the dragon, Mobius ladder and Book are edge trimagic total labeling graphs.

2. Main results

Theorem 2.1. The Mobius ladder M_n is an edge trimagic for odd n.

Proof: Let $V = \{u_i, v_i \,/ 1 \le i \le n\}$ be the vertex set and $E = \{u_iu_{i+1}, v_iv_{i+1} \,/ 1 \le i \le n-1\} \cup \{u_iv_i \,/ 1 \le i \le n\} \cup \{u_1v_n, v_1u_n\}$ be the edge set of the Mobius ladder M_n. Then M_n has 2n vertices and 3n edges. Define a bijection $f: V \cup E \to \{1, 2, \ldots, 5n\}$ such that

$$f(u_i) = \begin{cases} \dfrac{i+1}{2}, 1 \le i \le n \text{ and i is odd} \\ \dfrac{n+i+1}{2}, 1 \le i \le n \text{ and i is even} \end{cases}$$

$$f(v_i) = \begin{cases} n + \dfrac{n+i}{2}, 1 \le i \le n \text{ and i is odd} \\ n + \dfrac{i}{2}, 1 \le i \le n \text{ and i is even} \end{cases}$$

and $f(u_iu_{i+1}) = 5n-i-1$, $1 \le i \le n-1$; $f(v_iv_{i+1}) = 3n-i, 1 \le i \le n-1$; $f(u_iv_i) = 4n-i, 1 \le i \le n$; $f(u_1v_n) = 5n-1$ and $f(v_1u_n) = 5n$.

To prove this labeling is an edge trimagic total labeling.

For the edge u_1v_n, $f(u_1) + f(u_1v_n) + f(v_n) = 1+5n-1+2n = 7n = \lambda_1$.

For the edge v_1u_n, $f(v_1) + f(v_1u_n) + f(u_n) = n + \dfrac{n+1}{2} + 5n + \dfrac{n+1}{2} = 7n+1 = \lambda_2$.

Consider the edges u_iu_{i+1}, $1 \le i \le n-1$.

For odd i, $f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = \dfrac{i+1}{2} + 5n - i - 1 + \dfrac{n+i}{2} + 1 = \dfrac{11n+1}{2} = \lambda_3$.

For even i, $f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = \dfrac{n+i+1}{2} + 5n - i - 1 + \dfrac{i}{2} + 1 = \dfrac{11n+1}{2} = \lambda_3$.

Consider the edges v_iv_{i+1}, $1 \le i \le n-1$.

For odd i, $f(v_i) + f(v_iv_{i+1}) + f(v_{i+1}) = n + \dfrac{n+i}{2} + 3n - i + n + \dfrac{i+1}{2} = \dfrac{11n+1}{2} = \lambda_3$.

For even i, $f(v_i) + f(v_iv_{i+1}) + f(v_{i+1}) = n + \dfrac{i}{2} + 3n - i + n + \dfrac{n+i+1}{2} = \dfrac{11n+1}{2} = \lambda_3$.

Consider the edges u_iv_i, $1 \le i \le n$.

For odd i, $f(u_i) + f(u_iv_i) + f(v_i) = \frac{i+1}{2} + 4n - i + n + \frac{n+i}{2} = \frac{11n+1}{2} = \lambda_3$.

For even i, $f(u_i) + f(u_iv_i) + f(v_i) = \frac{n+i+1}{2} + 4n - i + n + \frac{i}{2} = \frac{11n+1}{2} = \lambda_3$.

Hence for each edge $uv \in E$, $f(u) + f(uv) + f(v)$ yields any one of the magic constants $\lambda_1 = 7n$, $\lambda_2 = 7n+1$ and $\lambda_3 = \frac{11n+1}{2}$. Therefore, the Mobius ladder M_n is an edge trimagic total labeling for odd n.

Corollary 2.2. The Mobius ladder M_n is super edge trimagic total labeling for odd n.

Proof: We proved that the Mobius ladder M_n is an edge trimagic total labeling for odd n with 2n vertices. The labeling given in Theorem 2.1 is as follows:

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & 1 \le i \le n \text{ and } i \text{ is odd} \\ \frac{n+i+1}{2}, & 1 \le i \le n \text{ and } i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} n + \frac{n+i}{2}, & 1 \le i \le n \text{ and } i \text{ is odd} \\ n + \frac{i}{2}, & 1 \le i \le n \text{ and } i \text{ is even} \end{cases}$$

Hence the 2n vertices get labels 1, 2, …, 2n. Therefore, the Mobius ladder M_n is super edge trimagic total labeling for odd n.

Example 2.3. An edge trimagic total labeling of the Mobius ladder M_7 is given in figure 1.

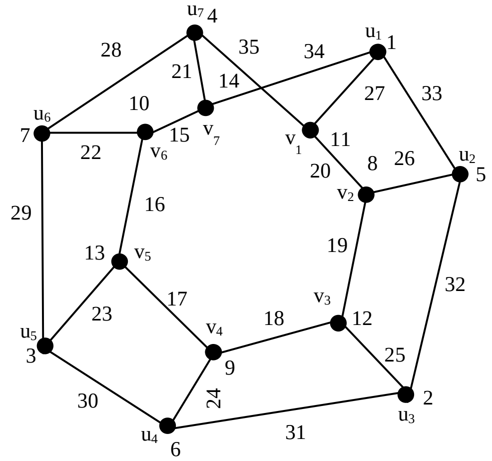

Figure1: M_7 with $\lambda_1 = 49$, $\lambda_2 = 50$ and $\lambda_3 = 39$

Theorem 2.4. The Book B_n is an edge trimagic graph.

Proof: Let $V=\{v_o, u_o, v_i, u_i \,/\, 1\leq i \leq n\}$be the vertex set and $E = \{v_ov_i, u_ou_i \,/\, 1\leq i \leq n\}\cup\{u_ov_o, u_iv_i \,/\, 1\leq i \leq n\}$ be the edge set of the graph B_n. Then B_n has 2n+2 vertices and 3n+1 edges.

Case 1. n is even.

Define a bijection $f: V\cup E\rightarrow \{1, 2, 3, \ldots, 5n+3\}$ such that

$f(v_o) = 1$; $f(u_o) = n+2$; $f(v_i) = 2n+3-i$, $1 \leq i \leq n$;

$$f(u_i) = \begin{cases} 2i, & 1\leq i \leq \frac{n}{2} \\ 2i - n + 1, & \frac{n}{2}+1 \leq i \leq n \end{cases}$$

$f(v_ov_i) = 4n+i+2$, $1 \leq i \leq n$; $f(u_1v_1) = 3n+2$ and $f(u_ov_o) = 5n+3$.

$$f(u_ou_i) = \begin{cases} 4n - 2i + 4, & 1\leq i \leq \frac{n}{2} \\ 5n - 2i + 3, & \frac{n}{2}+1 \leq i \leq n \end{cases}$$

$$f(u_i v_i) = \begin{cases} \frac{5}{2}n + 3 - i, & 2\leq i \leq \frac{n}{2} \\ \frac{7}{2}n + 2 - i, & \frac{n}{2}+1 \leq i \leq n \end{cases}$$

To prove this labeling is an edge trimagic total labeling.

For the edges $v_ov_i, 1 \leq i \leq n$, $f(v_o)+f(v_ov_i)+f(v_i) = 1+4n+i+2+2n+3-i = 6(n+1) = \lambda_1$.

For the edge v_ou_o, $f(v_o) + f(v_ou_o) + f(u_o) = 1+5n+3+n+2 = 6(n+1) = \lambda_1$.

Now consider the edges u_ou_i, $1 \leq i \leq n$.

For $1 \leq i \leq \frac{n}{2}$, $f(u_o) + f(u_ou_i) + f(u_i) = n+2+4n-2i+4+2i = 5n+6 = \lambda_2$.

For $\frac{n}{2}+1 \leq i \leq n$, $f(u_o) + f(u_ou_i) + f(u_i) = n+2+5n-2i+3+2i-n+1 = 5n+6= \lambda_2$.

For the edge v_1u_1, $f(v_1) + f(v_1u_1) + f(u_1) = 2n+2+3n+2+2 = 5n+6 = \lambda_2$.

Consider the edges u_iv_i, $2 \leq i \leq n$.

For $2 \leq i \leq \frac{n}{2}$, $f(u_i) + f(u_iv_i) + f(v_i) = 2i + \frac{5}{2}n + 3 - i + 2n + 3 - i = \frac{9}{2}n + 6 = \lambda_3$.

For $\frac{n}{2}+1 \leq i \leq n$, $f(u_i) + f(u_iv_i) + f(v_i) = 2i - n + 1 + \frac{7}{2}n + 2 - i + 2n + 3 - i = \frac{9}{2}n + 6 = \lambda_3$.

Hence for each edge $uv\in E$, $f(u) + f(uv) + f(v)$ yields any one of the magic constants $\lambda_1 = 6(n+1)$, $\lambda_2 = 5n + 6$ and $\lambda_3 = \frac{9}{2}n + 6$. Hence the Book B_n is an edge trimagic total labeling when n is even.

Case 2. n is odd.

Define a bijection $f : V \cup E \rightarrow \{1, 2, \ldots, 5n+3\}$ such that

$f(v_o) = 1$; $f(u_o) = n+2$; $f(v_i) = 2n+3-i$, $1 \le i \le n$;

$$f(u_i) = \begin{cases} \frac{i+1}{2}+1, 1 \le i \le n \text{ and i is odd} \\ \frac{n+i+1}{2}+1, 1 \le i \le n \text{ and i is even} \end{cases}$$

$f(v_o v_i) = 4n+i+2$, $1 \le i \le n$;

$$f(u_o u_i) = \begin{cases} 4n+3-\frac{i+1}{2}, 1 \le i \le n \text{ and i is odd} \\ 3n+\frac{n}{2}+2+\frac{1-i}{2}, 1 \le i \le n \text{ and i is even} \end{cases}$$

$$f(u_i v_i) = \begin{cases} 2n+\frac{n}{2}+2+\frac{i}{2}, 1 \le i \le n \text{ and i is odd} \\ 2n+2+\frac{i}{2}, 1 \le i \le n \text{ and i is even} \end{cases}$$

and $f(u_o v_o) = 5n + 3$.

To prove this labeling is an edge trimagic total labeling.
For the edges $v_o v_i$, $1 \le i \le n$, $f(v_o)+f(v_o v_i)\, f(v_i) = 1+4n+i+2+2n+3-i = 6(n+1) = \lambda_1$.
For the edge $v_o u_o$, $f(v_o) + f(v_o u_o) + f(u_o) = 1+ 5n + 3 + n + 2 = 6(n+1) = \lambda_1$.
Consider the edges $u_o u_i$, $1 \le i \le n$.

For odd i, $f(u_o) + f(u_o u_i) + f(u_i) = n+2+4n+3-\left(\frac{i+1}{2}\right)+\left(\frac{i+1}{2}\right)+1 = 5n+6 = \lambda_2$.

For even i, $f(u_o) + f(u_o u_i) + f(u_i) = n+2+3n+\frac{n}{2}+2+\frac{1-i}{2}+\frac{n+i+1}{2}+1 = 5n+6 = \lambda_2$.

Consider the edges $u_i v_i$, $1 \le i \le n$,

For odd i, $f(u_i) + f(u_i v_i)+f(v_i) = \frac{i+1}{2}+1+2n+\frac{n}{2}+2+\frac{i}{2}+2n+3-i = \frac{9n+13}{2} = \lambda_3$.

For even i, $f(u_i)+f(u_i v_i) + f(v_i) = \frac{n+3+i}{2}+2(n+1)+\frac{i}{2}+2n+3-i = \frac{9n+13}{2} = \lambda_3$.

Hence for each edge $uv \in E$, $f(u) + f(uv) + f(v)$ yields any one of the magic constants $\lambda_1 = 6(n+1)$, $\lambda_2 = 5n+6$ and $\lambda_3 = \frac{9n+13}{2}$. Therefore, the Book B_n is an edge trimagic total labeling when n is odd. From cases (1) and (2), the Book B_n is an edge trimagic total labeling.

Corollary 2.5. The Book B_n is a supper edge trimagic total labeling graph.
Proof: We proved that the Book B_n is an edge trimagic total labeling with 2n+2 vertices. The labeling given in the proof of Theorem 2.4 is as follows:
For even n, $f(v_o) = 1$; $f(u_o) = n+2$; $f(v_i) = 2n+3-i$, $1 \le i \le n$ and

$$f(u_i) = \begin{cases} 2i, & 1 \le i \le \dfrac{n}{2} \\ 2i - n + 1, & \dfrac{n}{2} + 1 \le i \le n \end{cases}$$

For odd n, $f(v_0) = 1$; $f(u_0) = n+2$; $f(v_i) = 2n+3-i$, $1 \le i \le n$ and

$$f(u_i) = \begin{cases} \dfrac{i+1}{2} + 1, & 1 \le i \le n \text{ and i is odd} \\ \dfrac{n+i+1}{2} + 1, & 1 \le i \le n \text{ and i is even} \end{cases}$$

Hence, the 2n+2 vertices get labels 1, 2, …, 2n+2. Therefore, the Book B_n is super edge trimagic total labeling graph.

Example 2.6. An edge trimagic total labeling of the Book B_{10} and B_7 are given in figure 2 and figure 3, respectively.

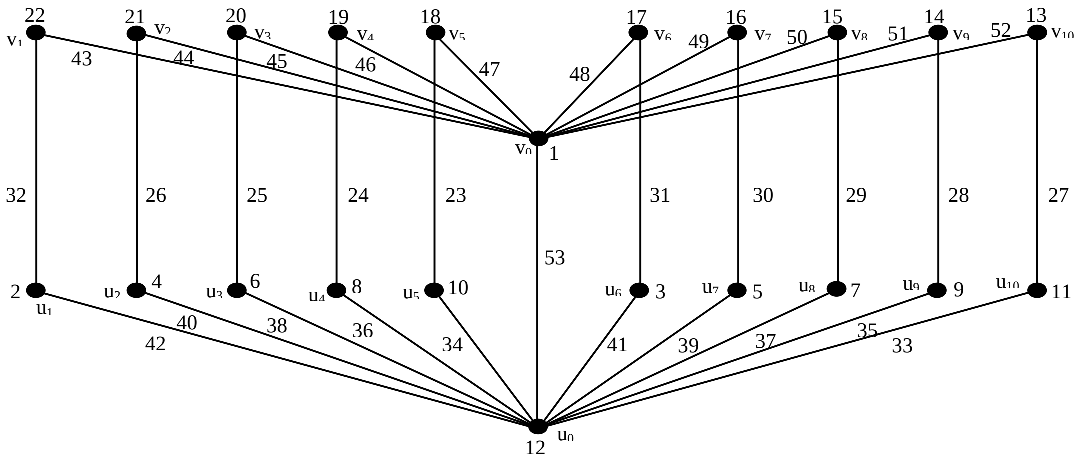

Figure 2: $B_{10}(S_{10} \times P_2)$ with $\lambda_1 = 66$, $\lambda_2 = 56$ and $\lambda_3 = 51$

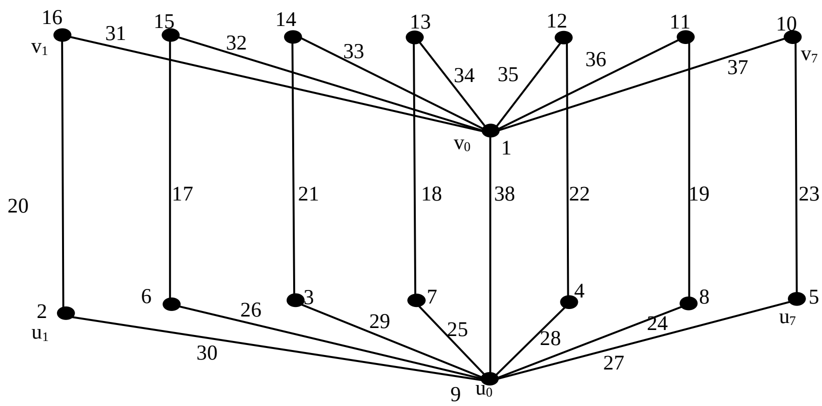

Figure 3: $B_7(S_7 \times P_2)$ with $\lambda_1 = 48$, $\lambda_2 = 41$ and $\lambda_3 = 38$

Theorem 2.7. The (n, l) – dragon graph is an edge trimagic total labeling.

Proof: Let V = $\{v_i / 1 \le i \le l\} \cup \{u_j / 1 \le j \le n\}$ be the vertex set and E = $\{v_iv_{i+1}/ 1\le i \le l\text{-}1\} \cup \{u_1v_1, u_1u_n\} \cup \{u_ju_{j+1} / 1 \le j \le n\text{-}1\}$ be the edge set.

Then (n, l) – dragon has n+l vertices and n+l edges.

Case 1. Both n and l are even.

Define a bijection f: $V \cup E \to \{1, 2, \ldots, 2(n+l)\}$ such that

$$f(v_i) = \begin{cases} \dfrac{l+i+3}{2}, & i \text{ is odd and } 1 \le i \le l \\ \dfrac{i+2}{2}, & i \text{ is even and } 1 \le i \le l \end{cases}$$

$f(u_j) = l+j$, $2 \le j \le n$; $f(u_1) = 1$; $f(v_iv_{i+1}) = 2(n+l-1)- i$, $1 \le i \le l-1$; $f(u_1u_2) = 2(n+l)-1$; $f(u_1u_n)=2(n+l)$; $f(u_1v_1) = 2(n+l-1)$ and

$$f(u_ju_{j-1}) = \begin{cases} 2n + l - 2j + 3, & 2 < j \le \dfrac{n}{2}+1 \\ 3n + l - 2j + 2, & \dfrac{n}{2}+1 < j \le n \end{cases}$$

To prove this labeling is an edge trimagic total labeling.

Consider the edges v_iv_{i+1}, $1 \le i \le l-1$.

For odd i, $f(v_i)+f(v_iv_{i+1})+f(v_{i+1})=\dfrac{l+i+3}{2}+2(n+l-1)-i+\dfrac{i+3}{2} = 2n+\dfrac{5}{2}l+1 = \lambda_1$.

For even i, $f(v_i)+f(v_iv_{i+1})+f(v_{i+1})=\dfrac{i+2}{2}+2(n+l-1)-i+\dfrac{l+i+4}{2} =2n+\dfrac{5}{2}l+1 = \lambda_1$.

For the edge u_1v_1, $f(u_1)+ f(u_1v_1) + f(v_1) = 1+2(n+l-1)+\dfrac{l+4}{2} = 2n+\dfrac{5}{2}l+1 = \lambda_1$.

For the edge u_1u_2, $f(u_1) + f(u_1u_2) +f(u_2) = 1+2(n+l)-1+l+2 = 2n+3l+2 = \lambda_2$.

For the edges u_ju_{j-1}, $2< j \le \dfrac{n}{2}+1$, $f(u_j) + f(u_ju_{j-1}) + f(u_{j-1}) = l+j+2n+l-2j+3+l+j-1$

$= 2n+3l+2 = \lambda_2$.

For the edge u_1u_n, $f(u_1) + f(u_1u_n) + f(u_n) = 1+2(n+l)+l+n = 3(n+l)+1 = \lambda_3$.

For the edges u_ju_{j-1}, $\dfrac{n}{2}+1 < j \le n$, $f(u_j) + f(u_ju_{j-1}) + f(u_{j-1}) = l+j+3n+l-2j+2+l+j-1 = 3(n+l)+1 = \lambda_3$.

Hence for each edge uv$\in$ E, f(u) + f(uv) + f(v) yields any one of the magic constants $\lambda_1 = 2n+\dfrac{5}{2}l+1$, $\lambda_2 = 2n+3l+2$ and $\lambda_3 = 3(n+l)+1$. Therefore, the (n, l) – dragon is an edge trimagic total labeling when both n and l are even.

Case 2. n is even and l is odd.

Define a bijection f: $V \cup E \to \{1, 2, \ldots, 2(n+l)\}$ such that

$$f(v_i) = \begin{cases} \dfrac{l+i+2}{2}, & \text{i is odd and } 1 \le i \le l \\ \dfrac{i+2}{2}, & \text{i is even and } 1 \le i \le l \end{cases}$$

$f(u_j) = l+j$, $2 \le j \le n$; $f(u_1) = 1$; $f(v_iv_{i+1}) = 2(n+l-1)-i$, $1 \le i \le l-1$; $f(u_1u_2) = 2(n+l)-1$; $f(u_1u_n) = 2(n+l)$; $f(u_1v_1) = 2(n+l-1)$ and

$$f(u_ju_{j-1}) = \begin{cases} 2n + l - 2j + 3, & 2 < j \le \dfrac{n}{2}+1 \\ 3n + l - 2j + 2, & \dfrac{n}{2}+1 < j \le n \end{cases}$$

To prove this labeling is an edge trimagic total labeling.

Consider the edges v_iv_{i+1}, $1 \le i \le l-1$.

For odd i, $f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = \dfrac{l+i+2}{2} + 2(n+l-1) - i + \dfrac{i+3}{2} = 2(n+l) + \dfrac{l+1}{2} = \lambda_1$.

For even i, $f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = \dfrac{i+2}{2} + 2(n+l-1) - i + \dfrac{l+i+3}{2} = 2(n+l) + \dfrac{l+1}{2} = \lambda_1$.

For the edge u_1v_1, $f(u_1)+f(u_1v_1)+f(v_1) = 1 + 2(n+l-1) + \dfrac{l+3}{2} = 2(n+l) + \dfrac{l+1}{2} = \lambda_1$.

For the edge u_1u_2, $f(u_1) + f(u_1u_2) + f(u_2) = 1+2(n+l)-1+l+2 = 2n+3l+2 = \lambda_2$.

For the edge u_1u_n, $f(u_1) + f(u_1u_n) + f(u_n) = 1+2(n+l)+l+n = 3(n+l)+1 = \lambda_3$.

For the edges u_ju_{j-1}, $2 < j \le \dfrac{n}{2}+1$, $f(u_j) + f(u_ju_{j-1}) + f(u_{j-1}) = l+j+2n+l-2j+3+l+j-1 = 2n+3l+2 = \lambda_2$.

For the edges u_ju_{j-1}, $\dfrac{n}{2}+1 < j \le n$, $f(u_j) + f(u_ju_{j-1}) + f(u_{j-1}) = l+j+3n+l-2j+2+l+j-1 = 3(n+l)+1 = \lambda_3$.

Hence for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ yields any one of the magic constants $\lambda_1 = 2(n+l) + \dfrac{l+1}{2}$, $\lambda_2 = 2n+3l+2$ and $\lambda_3 = 3(n+l)+1$. Therefore, the (n, l) – dragon is an edge trimagic total labeling when n is even and l is odd.

Case 3. Both n and l are odd.

Define a bijection $f: V \cup E \to \{1, 2, \ldots, 2(n+l)\}$ such that

$$f(v_i) = \begin{cases} \dfrac{l+i+2}{2}, & \text{i is odd and } 1 \le i \le l \\ \dfrac{i+2}{2}, & \text{i is even and } 1 \le i \le l \end{cases}$$

$$f(u_j) = \begin{cases} l+\dfrac{j+1}{2}, \ 2 \le j \le n \text{ and } j \text{ is odd} \\ l+\dfrac{n+j+1}{2}, \ 2 \le j \le n \text{ and } j \text{ is even} \end{cases}$$

$f(u_1) = 1$; $f(v_iv_{i+1}) = 2(n+l)-i$, $1 \le i \le l-1$; $f(u_1v_1) = 2(n+l)$; $f(u_1u_2) = n+l+1$; $f(u_1u_n) = n+l+2$ and $f(u_ju_{j+1}) = 2n+l+2-j$, $2 \le j \le n-1$.

To prove this labeling is an edge trimagic total labeling.

Consider the edges v_iv_{i+1}, $1 \le i \le l-1$.

For odd i, $f(v_i)+f(v_iv_{i+1})+f(v_{i+1})=\dfrac{l+i+2}{2}+2(n+l)-i+\dfrac{i+3}{2} = 2(n+l+1)+\dfrac{l+1}{2} = \lambda_1$.

For even i, $f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = \dfrac{i+2}{2}+2(n+l)-i+\dfrac{l+i+3}{2} = 2(n+l+1)+\dfrac{l+1}{2} = \lambda_1$.

For the edge u_1v_1, $f(u_1)+f(u_1v_1)+f(v_1)=1+2(n+l)+\dfrac{l+3}{2} = 2(n+l+1)+\dfrac{l+1}{2} = \lambda_1$.

For the edge u_1u_2, $f(u_1)+f(u_1u_2)+f(u_2)=1+n+l+1+l+\dfrac{n+3}{2} = n+2l+3+\dfrac{n+1}{2} = \lambda_2$.

For the edge u_1u_n, $f(u_1)+f(u_1u_n)+f(u_n)=1+n+l+2+l+\dfrac{n+1}{2} = n+2l+3+\dfrac{n+1}{2} = \lambda_2$.

Consider the edges u_ju_{j+1}, $2 \le j \le n-1$.

For odd j,

$f(u_j)+f(u_ju_{j+1})+f(u_{j+1})=l+\dfrac{j+1}{2}+2n+l+2-j+l+\dfrac{n+j+2}{2} = 2n+3l+3+\dfrac{n+1}{2} = \lambda_3$.

For even j,

$f(u_j)+f(u_ju_{j+1})+f(u_{j+1}) = l+\dfrac{n+j+1}{2}+2n+l+2-j+l+\dfrac{j+2}{2} = 2n+3l+3+\dfrac{n+1}{2} = \lambda_3$.

Hence, for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ yields any one of the magic constants $\lambda_1 = 2(n+l+1)+\dfrac{l+1}{2}$; $\lambda_2 = n+2l+3+\dfrac{n+1}{2}$ and $\lambda_3 = 2n+3l+3+\dfrac{n+1}{2}$. Therefore, the (n, l) – dragon is an edge trimagic total labeling when n is odd and l is odd.

Case 4. n is odd and l is even.

Define a bijection $f: V \cup E \rightarrow \{1, 2, \ldots, 2(n+l)\}$ such that

$$f(v_i) = \begin{cases} \dfrac{l+i+3}{2}, \quad 1 \le i \le n, i \text{ is odd} \\ \dfrac{i+2}{2}, \quad 1 \le i \le n, i \text{ is even} \end{cases}$$

$$f(u_j) = \begin{cases} l+\dfrac{j+1}{2}, \ 2 \le j \le n, j \text{ is odd} \\ l+\dfrac{n+j+1}{2}, \ 2 \le j \le n, j \text{ is even} \end{cases}$$

$f(u_1) = 1$; $f(v_iv_{i+1}) = 2(n+l)-i$, $1 \le i \le l-1$; $f(u_1v_1) = 2(n+l)$; $f(u_1u_2) = n+l+1$; $f(u_1u_n) = n+l+2$; $f(u_ju_{j+1}) = 2n+l+2-j$, $2 \le j \le n-1$.

To prove this labeling is an edge trimagic total labeling.

Consider the edges v_iv_{i+1}, $1 \le i \le l-1$.

For odd i, $f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = \frac{l+i+3}{2} + 2(n+l) - i + \frac{i+3}{2} = 2(n+l)+3+\frac{l}{2} = \lambda_1$.

For even i, $f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = \frac{i+2}{2} + 2(n+l) - i + \frac{l+i+4}{2} = 2(n+l)+3+\frac{l}{2} = \lambda_1$.

For the edge u_1v_1, $f(u_1)+f(u_1v_1)+f(v_1) = 1+2(n+l)+\frac{l+4}{2} = 2(n+l)+3+\frac{l}{2} = \lambda_1$.

For the edge u_1u_2, $f(u_1)+f(u_1u_2)+f(u_2) = 1+n+l+1+\frac{n+2l+3}{2} = n+2l+3+\frac{n+1}{2} = \lambda_2$.

For the edge u_1u_n, $f(u_1)+f(u_1u_n)+f(u_n) = 1+n+l+2+\frac{n+2l+1}{2} = n+2l+3+\frac{n+1}{2} = \lambda_2$.

Consider the edges u_ju_{j+1}, $2 \le j \le n-1$.

For odd j,

$f(u_j)+f(u_ju_{j+1})+f(u_{j+1}) = l+\frac{j+1}{2}+2n+l+2-j+l+\frac{n+j+2}{2} = 2n+3l+3+\frac{n+1}{2} = \lambda_3$.

For even j,

$f(u_j)+f(u_ju_{j+1})+f(u_{j+1}) = l+\frac{n+j+1}{2}+2n+l+2-j+l+\frac{j+2}{2} = 2n+3l+3+\frac{n+1}{2} = \lambda_3$.

Hence for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ yields any one of the magic constants $\lambda_1 = 2(n+l)+3+\frac{l}{2}$; $\lambda_2 = n+2l+3+\frac{n+1}{2}$ and $\lambda_3 = 2n+3l+3+\frac{n+1}{2}$. Therefore, the (n, l) – dragon is an edge trimagic total labeling when n is odd and l is even.

Corollary 2.8. The (n, l) – dragon graph is super edge trimagic total labeling.

Proof: We proved that the (n, l) – dragon is an edge trimagic total labeling with $n+l$ vertices. The labeling given in the proof of Theorem 2.7 is as follows:

For even n and even l,

$$f(v_i) = \begin{cases} \frac{l+i+3}{2}, & \text{i is odd and } 1 \le i \le l \\ \frac{i+2}{2}, & \text{i is even and } 1 \le i \le l \end{cases}$$

$f(u_j) = l+j$, $2 \le j \le n$; $f(u_1) = 1$.

For even n and odd l,

$$f(v_i) = \begin{cases} \frac{l+i+2}{2}, & \text{i is odd and } 1 \le i \le l \\ \frac{i+2}{2}, & \text{i is even and } 1 \le i \le l \end{cases}$$

$f(u_j) = l+j$, $2 \le j \le n$; $f(u_1) = 1$.

For odd n and odd l,

$$f(v_i) = \begin{cases} \dfrac{l+i+2}{2}, & i \text{ is odd and } 1 \le i \le l \\ \dfrac{i+2}{2}, & i \text{ is even and } 1 \le i \le l \end{cases}$$

$$f(u_j) = \begin{cases} l + \dfrac{j+1}{2}, & j \text{ is odd and } 2 \le j \le n \\ l + \dfrac{n+j+1}{2}, & j \text{ is even and } 2 \le j \le n \end{cases}$$

$f(u_1) = 1$.

For odd n and even l,

$$f(v_i) = \begin{cases} \dfrac{l+i+3}{2}, & i \text{ is odd and } 1 \le i \le l \\ \dfrac{i+2}{2}, & i \text{ is even and } 1 \le i \le l \end{cases}$$

$$f(u_j) = \begin{cases} l + \dfrac{j+1}{2}, & j \text{ is odd and } 2 \le j \le n \\ l + \dfrac{n+j+1}{2}, & j \text{ is even and } 2 \le j \le n \end{cases}$$

$f(u_1) = 1$.

Hence, the n+l vertices get labels 1, 2, …, n+l. Therefore, the (n, l) – dragon is a super edge trimagic total labeling graphs.

Example 2.9. An edge trimagic total labeling of the (8, 6) – dragon, (6, 5) – dragon, (5, 7) – dragon and (7, 4) – dragon are given in figure 4, figure 5, figure 6 and figure 7 respectively.

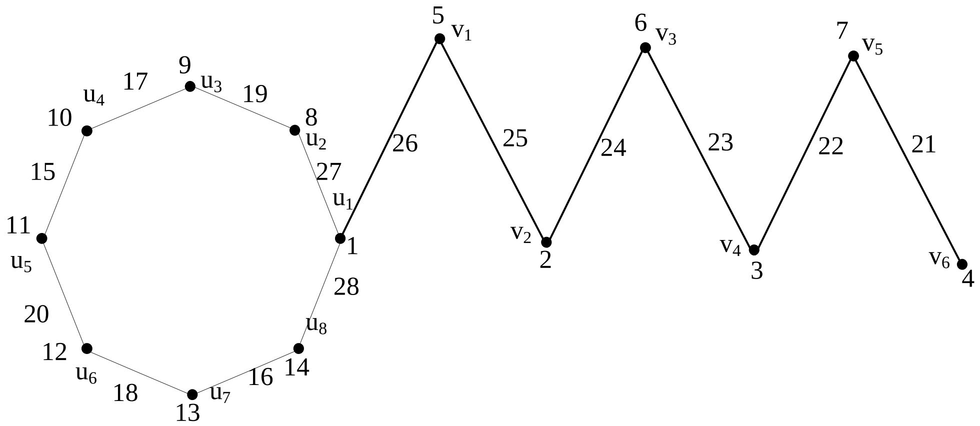

Figure 4: (8, 6) – dragon with $\lambda_1 = 32$, $\lambda_2 = 36$ and $\lambda_3 = 43$

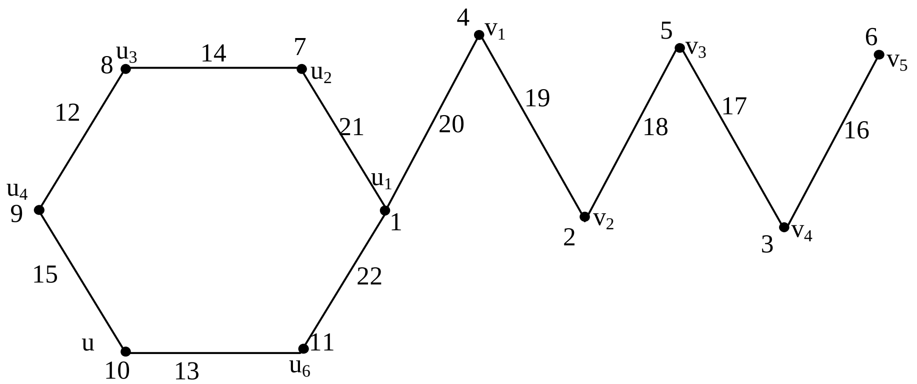

Figure 5: (6, 5) – dragon with $\lambda_1 = 25$, $\lambda_2 = 29$ and $\lambda_3 = 34$

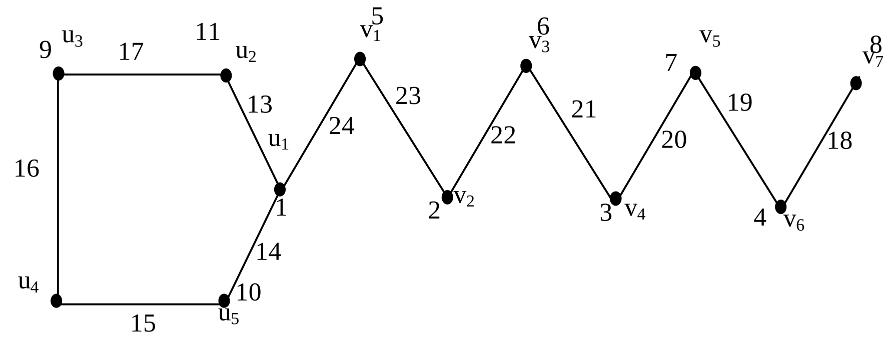

Figure 6: (5, 7) – dragon with $\lambda_1 = 30$, $\lambda_2 = 25$ and $\lambda_3 = 37$

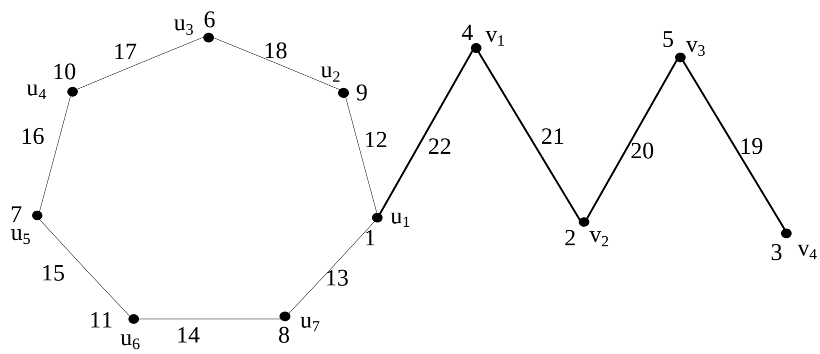

Figure 7: (7, 4) – dragon with $\lambda_1 = 27$, $\lambda_2 = 22$ and $\lambda_3 = 33$

3. Conclusion

In this paper, we proved that the Mobius ladder, Book and (n, *l*) – dragon are edge trimagic total labeling graphs. Theorem 2.1 shows that Mobius ladder M_n is an edge trimagic total labeling for odd n. One can try to prove that the Mobius ladder M_n graph is an edge trimagic for even n.

REFERENCES

1. S. Babith, A. Amarajothi, and J. Basker Babujee, Magic and bimagic labeling for disconnected graphs, *International Journal of Mathematics Trends and Technology* 3(2) (2012) 86-90.
2. J.B.Babujee, On edge bimagic labeling, *Journal of Combinations Information & System Sciences*, 28-29(1-4) (2004) 239-244.
3. L.Girija and A.Elumalai, Edge magic total labeling of the cycle c_n with p_3 chords, *Annals of Pure and Applied Mathematics*, 8(2) (2014) 175-181.
4. F.Harary, Graph theory, Narosa Publishing house, New Delhi (2001).
5. C.Jayasekaran, M.Regees and C.Davidraj, Edge trimagic labeling of some graphs, *Intern. Journal of Combinatorial Graph Theory and Applications*, 6(2) (2013) 175-186
6. C.Jayasekaran and M.Regees, Edge trimagic in digraphs, *Journal of Discrete Mathematical Sciences and Cryptography*, 17(4) (2015) 321-335.
7. C.Jayasekaran and J.Little Flower, On edge trimagic labeling of umbrella, dumb bell and circular ladder graphs, *Annals of Pure and Applied Mathematics*, 13(1) (2017) 73-87.
8. J.A.Gallian, A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, 16 (2013) #DS6.
9. S.R.Kim and J.Y.Park, On super edge-magic graphs, *Ars Combinatoria*, 81 (2001) 113-127.
10. A.Kotzing and A.Rosa, Magic valuations of finite graphs, *Canad. Math. Bull.*, 13 (1970) 415-416.
11. K.M.Koh, D.G.Rogers, H.K.Teo and K.Y.Yap, Graphful graphs: Some further results and problems, *Congr. Numer.*, 29 (1980) 559-571.
12. J.Sedlacek, Problem 27. in Theory of Graphs and its Applications, *Proc. Symposium Smolenice*, June, (1963) 163-167.
13. S.K.Vaidya and N.H.Shah, Some star and bistar related divisor cordial graphs, *Annals of Pure and Applied Mathematics*, 3(1) (2013) 67-77.