Research article

Tree height prediction models for two forest reserves in Nigeria using mixed-effects approach

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[Accepted: 12 April 2019]

Abstract: Height-diameter models for predicting tree height are essential for routine forest inventory. These models can be developed using fixed-or mixed-effects approach. Few studies have applied the mixed-effect approach to developed height prediction model for the natural forest in Nigeria. Therefore, in this study, the mixed-effect modelling approach was used to develop height prediction models for Ikrigon and Cross River South (CRS) Forest Reserves, Nigeria. Data consist of 776 and 438 height-diameter pairs from Ikrigon and CRS Forest Reserves, respectively. Five 2-parameters and five 3-parameters height-diameter models were evaluated including Nalund, Wykoff, Curtis, Meyer, Michaelis-Menten, Chapman-Richards, Ratkowsky, Korf, Logistic and Gompertz. Model fitting was done in two stages: Fixed-effect approach was used in the first stage wherein candidate models were selected and refitted in the second stage using mixed-effect approach. Adjusted coefficient of determination, root mean square error, mean absolute bias, Akaike information and Bayesian information criterion were used to assess the models. The results showed that Gompertz and Meyer models were more consistent. Gompertz and Meyer had adjusted coefficient of determination, root mean square error, mean absolute bias, Akaike information criterion and Bayesian information criterion of 0.642, 4.457, 3.501 and 4591.487, 4638.028; and 0.638, 4.482, 3.541, 4592.008, 4619.933, respectively for Ikrigon and 0.724, 4.076, 3.215, 2536.148 and 2576.970; and 0.711, 4.176, 3.273, 2536.352 and 2560.845, respectively for CRS. The mixed-effect approach improved tree height predicting of the forest stands. These models are recommended for estimating tree height in the forest reserves.

Keywords: Ikrigon - Cross River South forest - Height-diameter models - Gompertz - Meyer - Mixed-effect.

[Cite as: Ogana FN (2019) Tree height prediction models for two forest reserves in Nigeria using mixed-effects approach. *Tropical Plant Research* 6(1): 119–128]

INTRODUCTION

Tree diameter and height are routinely measured in forest inventory because they are the fundamental variables from which other stand characteristics are derived (Ogana 2018). Diameter and height are key components for analysis of forest stand structure, estimation of volume (Gomez-Garcia *et al.* 2014), biomass, basal area, density and may reflect the competitive position of a tree in a stand (West 2015). Also, the height of the tallest trees in forest stand is the basis for assessing site productive capacity (Calama & Montero 2004, West 2015). Tree diameter at breast height (1.3 m above the ground) can easily be measured with diameter or girth tape with accuracy and low cost (Ferraz-Filho *et al.* 2018). However, the measurement of tree height is tedious and relatively complex especially in forest with contiguous canopy and rugged terrain (Krisnawati *et al.* 2010, Mehtätalo *et al.* 2015, Ozcelik *et al.* 2018). In view of this, only subsample of trees is measured for height.

Most studies have relied on the use of height-diameter (h-d) models to estimate the height of trees. Some common h-d models that have been used in quantitative forestry include: Logistic (Peer & Reed 1920), Korf (Lundqvist 1957), Chapman-Richards (Richards 1959), Curtis (Curtis 1967), Wykoff (Wykoff *et al.* 1982), Ratkowsky (Ratkowsky 1990) etc. Many published forestry literatures exist on h-d models (Mehtätalo 2005, Trincado *et al.* 2007, Temesgen *et al.* 2008, Krisnawati *et al.* 2010, Coble & Lee 2011, Mehtätalo *et al.* 2015, Eby *et al.* 2017, Ogana 2018) for plantation and natural forest stand.

Modelling h-d relationship could be achieved either by using a fixed-effect or mixed-effect approach. In fixed-effect approach, the ordinary least square or nonlinear least square technique is used; but the assumption of independence is violated due to hierarchical data structure, that is, tree in plot (Ferraz-Filho *et al.* 2018, Ozcelik *et al.* 2018). Measurement of trees within plot cannot be said to be independent. Such assumption of independence can be readily overcome by mixed-effect model approach. In this approach, both the within and between plot variation of the tree variable of interest are accounted for. Mixed-effect model has fixed and random parameters (factors). One major advantage of mixed-effect model approach is that it “allows prediction of a typical response, using only fixed-effect, and a calibrated response where random effects are predicted and included in the model using measurements of heights from a sample of trees” (Burkhart & Tomé 2012). Different studies including Sharma & Parton (2007), Budhathoki *et al.* (2008), Paulo *et al.* (2011), VanderSchaaf (2014), Mehtätalo *et al.* (2015), Zang *et al.* (2016), Ferraz-Filho *et al.* (2018) and Ozcelik *et al.* (2018) have applied the mixed-effect approach to model h-d relationship especially in even-aged forest plantation. However, few studies have applied the nonlinear mixed-effects modelling approach for constructing h-d models for natural forest stand in Nigeria. Therefore, the main objective of this study was to apply the mixed-effect technique to model the height-diameter relationship in the natural stands of Ikrigon and Cross River South Forest Reserves in Nigeria.

MATERIALS AND METHODS

Data

The data for this study came from Cross River South Forest Reserve (CRS FR) and Ikrigon FR located in Cross River State, Nigeria. CRS FR lies between latitude 5°50.978' to 5°51.029' N and longitude 8°29.833' to 8°29.424' E and occupies an area of 80,534.07 ha. Ikrigon FR lies between latitude 6°17.597' to 6°17.862' N and longitude 8°35.597' to 8°35.276' E and occupies an area of 542.7 ha (Adeniyi 2017). Data were obtained from 20 (10 from each reserve) temporary sample plots (TSPs) of size 0.25 ha using systematic sampling. The plots were laid at alternate pattern along the transect line at 100 m interval. A diameter threshold of ≥ 10 cm was used. Diameter and height were measured to the nearest 0.1 cm and 0.1 m with diameter tape and vertex II, respectively. The descriptive statistics of the inventoried data are presented in table 1. Species composition of the reserves are presented in appendix I and II.

Table 1. Descriptive statistics of the measured variables.

FR	Variables	Statistics			
		Mean	Max	Min	SD
CRS N = 438 Tree species = 63	Diameter	25.3	103.1	10.0	16.4
	Height	20.4	44.1	6.4	7.8
Ikrigon N = 776 Tree species = 74	Diameter	29.9	125.0	10.0	15.9
	Height	24.1	43.8	5.7	7.5

Note: SD, Standard deviation.

Height-Diameter models

Table 2. Height-Diameter Models.

Mode name	H-D Model	References
Naslund	$H = bh + \frac{D^2}{(a+bD)^2}$	Näslund (1937), Mehtätalo <i>et al.</i> (2015)
Wykoff	$H = bh + e^{a+b/(D+1)}$	Wykoff <i>et al.</i> (1982)
Curtis	$H = bh + a(D/(1+D))^b$	Curtis (1967), Mehtätalo <i>et al.</i> (2015)
Meyer	$H = bh + a(1 - e^{-bD})$	Meyer (1940)
Michaelis-Menten (MM)	$H = bh + \frac{aD}{(b+D)}$	Menten & Michaelis (1913), Huang <i>et al.</i> (1992)
Richards	$H = bh + a(1 - e^{-bD})^c$	Richards (1959), Mehtätalo <i>et al.</i> (2015)
Ratkowsky	$H = bh + ae^{-b/(D+c)}$	Ratkowsky (1990), Mehtätalo <i>et al.</i> (2015)
Korf	$H = bh + a(-e^{-bD^{-c}})$	Lundqvist (1957), Krisnawati <i>et al.</i> (2010)
Logistic	$H = bh + \frac{a}{1+be^{-cD}}$	Mehtätalo <i>et al.</i> (2015), Ogana (2018)
Gompertz	$H = 1.3 + ae^{(-be^{(-cD)})}$	Gompertz (1825), Mehtätalo <i>et al.</i> (2015)

Note: a , b , c = model parameters; $bh = 1.3$ (a constant used to account that DBH is measured at 1.3m above the ground); H = height (m); D = diameter (DBH); e , base of the natural logarithm

There are several height-diameter models that have been applied to forestry with varying degree of success. No single height-diameter model is suitable for all data structure. In this study, ten height-diameter models were fitted to the data from Ikrigon and CRS forest reserves. Five 2-parameters and five 3-parameters height-diameter models were evaluated including Nalund, Wykoff, Curtis, Meyer, Michaelis-Menten (MM), Chapman-Richards (Richards), Ratkowsky, Korf, Logistic and Gompertz models (Table 2).

Mixed-effects model

Model fitting was done in two stages. The first stage involved fitting the ten (10) models using fixed-effects modelling approach. In this approach, the between plot-height variation was ignored. The nonlinear least square estimation was used. Conversely, in mixed-effects modelling both the within and between plot-height variation were accounted for. Mixed-effects model has both fixed parameters and random parameters. While the fixed parameter estimates the average height of the stand, the random component provides information on the variation of height across the plots. Mixed-effects could be linear mixed-effects (if the base model is in linear form) or nonlinear mixed-effects (if the base model is nonlinear). In this study, the nonlinear mixed-effects was adopted since all the base models were nonlinear. Only the best two height-diameter models from the first stage that were refitted using mixed-effects approach. The nonlinear mixed-effect was of the form of Mehtätalo *et al.* (2015):

$$h_{ij} = f(d_{ij}; \beta_i) + e_{ij}$$

$$\beta_i = B + b_i$$

Where, h_{ij} is the height of tree j on plot i and corresponding diameter (DBH) represented by d_{ij} . The vector B is the parameters (a , b and c) for a typical plot in the entire population of plots; b_i (a_i , γ_i and λ_i) is the plot effect – it expresses the difference in the parameters of plot i from the typical plot. “Plot effects are assumed to have a common multivariate normal distribution with mean 0 and variance-covariance matrix $\text{var}(b_i) = D$ for all values of i ” (Mehtätalo *et al.* 2015); and e_{ij} is the residual errors which are assumed to be normal, identical, independent with zero mean and constant variance $\text{var}(e_{ij}) = \sigma^2$.

Model assessment was based on adjusted coefficient of determination (R^2_{adj}), root mean square error (RMSE), mean absolute bias (MAB), Akaike information criterion (AIC) and Bayesian information criterion (BIC). To decide on the candidate models, a rank was assigned to each H-D model based on each fit index (Tewari & Singh 2018). The smaller the rank, the better the model. These ranks were thereafter summed up to reach a final fit rank for each model which shows the individual model performance with respect to all fit indices considered in this study. Residual analysis was also used to check if the models violate the assumption of homoscedasticity *i.e.* constant variance. Normality test of the residual was carried out using Shapiro-Wilk at 5% level. All statistical analyses including model fitting, residual analysis and normality test were carried out in R (R Core Team 2017).

RESULT AND DISCUSSION

The models for predicting tree height in Ikrigon and CRS Forest Reserves have been developed and the result for the fixed-effect approach are presented in table 3. The result showed that 2-parameter Meyer and 3-parameter Gompertz h-d models provided the best prediction of tree height in the reserves. In Ikrigon FR, Gompertz h-d model had R^2_{adj} , RMSE, MAB, AIC and BIC of 0.581, 4.823, 3.815, 4652.301 and 4670.918; and Meyer had 0.581, 4.825, 3.815, 4650.780 and 4664.742, respectively. Similarly, in CRS FR Gompertz h-d model had R^2_{adj} , RMSE, MAB, AIC and BIC of 0.668, 4.474, 3.595, 2563.620 and 2579.949; and Meyer had 0.666, 4.490, 3.588, 2564.664 and 2576.910, respectively. Furthermore, Meyer and Gompertz had the smallest rank sum of 8 and 14, respectively in Ikrigon FR. Both functions had 11 in CRS FR data.

The performance of Naslund, Wykoff, Curtis, Michaelis-Menten (MM), Richards, Ratkowsky, Korf and Logistic h-d models were comparable to Gompertz and Meyer. In fact, only marginal differences exist in the fit indices of the h-d models. As a rule of thumb, two models are said to be the same if the difference in their AICs (ΔAIC) values is less than 2 (Tewari & Singh 2018, Ogana 2018). The difference in AIC values for all models were less than two in both Ikrigon and CRS forest reserves. Furthermore, Shapiro-Wilk (S-W) test of normality showed the ten h-d models did not violate the assumption of normality at 5% level of significance. All models had $p\text{-value} > 0.05$. Also, the models followed the expected h-d curve of monotomic increment, inflection point and asymptotic value for both reserves (Fig. 1).

Mehtätalo *et al.* (2015) also found these models to be efficient in predicting tree height for a wide geographic region. However, the authors concluded that the 2-parameter Curtis and Naslund were more

consistent than others. In this study, the Curtis and Naslund models performed relatively well. Though all models were good in this study, but Gompertz and Meyer were selected as the candidate h-d models for Ikrigon and CRS forest reserves. This is because there were wide differences in the rank sum of Gompertz and Meyer models compared to the other function considered in this study. And as such, they were further fitted with nonlinear mixed-effect modelling approach. So that between plot variation can be accounted for.

Table 3. Adjusted coefficient of determination (R^2_{adj}), root mean square error (RMSE), mean absolute bias (MAB), Akaike information criterion (AIC) and Bayesian information criterion (BIC) of the Fitted height-diameter fixed-effect models.

FR	Model	Parameters			R^2_{adj}	RMSE	MAB	AIC	BIC	$\Sigma Rank$	S-W p-value
		a	b	c							
Ikrigon	Naslund	1.387	0.155		0.579(5)	4.835(5)	3.821(7)	4654.219(5)	4668.182(3)	25	0.599
	Wykoff	3.669	-14.158		0.578(6)	4.839(8)	3.814(1)	4655.288(7)	4669.251(5)	27	0.554
	Curtis	38.8	13.57		0.578(6)	4.841(9)	3.815(4)	4656.130(8)	4670.092(6)	33	0.542
	Meyer	33.792	0.044		0.581(1)	4.825(2)	3.815(3)	4650.780(1)	4664.742(1)	8	0.515
	MM	45.69	26.42		0.576(10)	4.854(10)	3.852(10)	4660.348(10)	4674.310(9)	49	0.724
	Richards	34.449	0.04	0.945	0.580(2)	4.827(3)	3.819(6)	4651.561(2)	4665.523(2)	15	0.518
	Ratkowsky	42.25	19.03	5.000	0.578(6)	4.837(6)	3.827(9)	4654.861(6)	4668.824(4)	31	0.625
	Korf	42.618	8.87	0.819	0.578(6)	4.837(7)	3.817(5)	4656.914(9)	4675.530(10)	37	0.599
CRSFR	Logistic	31.739	3.387	0.083	0.580(2)	4.827(4)	3.824(8)	4653.439(4)	4672.056(8)	26	0.406
	Gompertz	32.422	1.822	0.064	0.581(1)	4.823(1)	3.815(2)	4652.301(3)	4670.918(7)	14	0.438
	Naslund	1.687	0.151		0.662(7)	4.518(8)	3.590(4)	2570.230(8)	2582.477(7)	34	0.908
	Wykoff	3.694	-16.317		0.653(9)	4.579(9)	3.615(9)	2581.891(9)	2594.138(9)	45	0.847
	Curtis	39.67	15.59		0.651(10)	4.593(10)	3.623(10)	2584.558(10)	2596.805(10)	50	0.833
	Meyer	36.866	0.033		0.666(3)	4.490(3)	3.588(1)	2564.664(3)	2576.910(1)	11	0.914
	MM	52.08	38.35		0.666(3)	4.494(4)	3.596(6)	2565.451(4)	2577.698(2)	19	0.957
	Richards	34.962	0.04	1.113	0.663(6)	4.506(6)	3.590(3)	2567.772(5)	2580.018(4)	24	0.894
CRSFR	Ratkowsky	44.15	22.27	5.000	0.662(7)	4.512(7)	3.589(2)	2569.058(7)	2581.305(6)	29	0.926
	Korf	74.399	5.694	0.463	0.664(5)	4.501(5)	3.596(7)	2568.934(6)	2585.262(8)	31	0.956
	Logistic	34.284	3.854	0.069	0.668(1)	4.476(2)	3.606(8)	2563.902(2)	2580.231(5)	18	0.767
	Gompertz	35.607	1.857	0.048	0.668(1)	4.474(1)	3.595(5)	2563.620(1)	2579.949(3)	11	0.875

Note: MM, Michaelis-Menten; S-W, Shapiro-Wilk; Values in the parentheses are ranks assigned based on the values of the fit indices.

The result of the nonlinear mixed modelling approach for Gompertz and Meyer are presented in table 4. The result showed that there was improvement in tree height prediction for Gompertz and Meyer in both reserves. The estimated variance-covariance matrices of Gompertz and Meyer models were positive definite. The R^2_{adj} increased considerably while the RMSE, MAB, AIC and BIC for the candidate h-d models decreased compared to the fixed approach. For example, the RMSE of Gompertz decreased by 0.366 m and 0.398 m while Meyer decreased by 0.343 and 0.314 in Ikrigon and CRS FR, respectively (the values were obtained by subtracting the models' fit indices in table 3 and 4). This gives an indication of the efficiency of the mixed-effect modelling

approach. This improvement is not far-fetched from the fact that this approach was able to capture both within and between plot variations in tree height.

Figure 1. Fitted height-diameter curves of the ten models for (A) Ikrigon and (B) CRS forest reserves.

Temesgen *et al.* (2008) used RMSE and bias to evaluate the fixed-effect prediction part of a mixed-effects model and the traditional fixed-effect model. The authors observed relatively poor predictive performance with the fixed-effect part of a mixed-effects model. This is acceptable if estimation of the mean height of trees is of importance; though fixed effect models does not give plot-specific estimation for a plot of interest (Mehtätalo *et al.* 2015). The plot-specific prediction is improved by random-effect part of the mixed-effect models. Also, Ozcelik *et al.* (2018) found the mixed-effect model for Chapman-Richards to be better than fixed-effect and quantile regression for two Turkish tree species. In this study, the Chapman-Richards function was not further considered because its rank sum based on the fit indices was relatively large.

Table 4. Fitted nonlinear mixed-effects height-diameter models and the fit indices.

FR	Model	Parameter	Estimates	R^2_{adj}	RMSE	MAB	AIC	BIC
Ikrigon	Meyer	a	34.662	0.638	4.482	3.541	4592.008	4619.933
		b	0.042					
		sd (α_i)	2.948					
		sd (γ_i)	0.008					
		corr (α_i, γ_i)	-0.67					
		σ^2	4.525					
	Gompertz	a	32.666	0.642	4.457	3.501	4591.487	4638.028
		b	1.896					
		c	0.063					
		sd (α_i)	2.297					
		sd (γ_i)	0.404					
		sd (λ_i)	0.005					
		corr (α_i, γ_i)	-0.204					
		corr (α_i, λ_i)	-0.619					
		corr (γ_i, λ_i)	0.892					
		σ^2	4.501					
CRS	Meyer	a	34.315	0.711	4.176	3.273	2536.352	2560.845
		b	0.038					
		sd (α_i)	5.826					
		sd (γ_i)	0.006					
		corr (α_i, γ_i)	-1.000					
		σ^2	4.217					
	Gompertz	a	34.482	0.724	4.076	3.215	2536.148	2576.970
		b	1.760					
		c	0.048					
		sd (α_i)	4.845					
		sd (γ_i)	0.257					
		sd (λ_i)	0.006					

$\text{corr}(\alpha_i, \gamma_i)$	0.203
$\text{corr}(\alpha_i, \lambda_i)$	-0.584
$\text{corr}(\gamma_i, \lambda_i)$	0.676
σ^2	4.14

The trend in residual was analysed by plotting the model residual against the predicted height in 10 classes in the data set for Meyer and Gompertz h-d model. Thin and thick lines were added to the class-specific means. The thin line corresponds to the class-specific standard deviations while the thick line corresponds to the 95% confidence intervals (CIs) of class mean. Thick lines that do not intersect the horizontal line at y equal 0 *i.e.*, $y = 0$ are highlighted in red colour (Fig. 2 & 3). This was used to ascertain the model accuracy and the assumption of homoscedasticity. The graphs show that the Gompertz Meyer models for predicting tree height in the reserves had all 95% CIs for means that is, the thick lines intersected the x-axis except for Ikrigon. The Meyer h-d predicted poorly in the first class of Ikrigon data (highlighted in red colour). The implication is that Gompertz h-d model fit the Ikrigon and CRS forest reserve data relatively well compared to Meyer h-d. Also, the vertical thin lines that is, class-specific standard deviations did not show increasing variance as a function of tree height in the two reserves; which implies that the assumption of constant variance (homoscedasticity) was not violated.

Figure 2. Residual plots of the (A) Meyer h-d model and (B) Gompertz h-d model in the data set from Ikrigon Forest Reserve.

Figure 3. Residual plots of the (A) Meyer h-d model and (B) Gompertz h-d model in the data set from CRS Forest Reserve.

CONCLUSION

The application of nonlinear mixed models for fitting height-diameter remains an effective and efficient technique for improving the approximation of tree height. This technique improved the tree height prediction in Ikrigon and CRS forest reserve. The 3-parameter Gompertz h-d and 2-parameter h-d model had the best prediction. One major limitation of this study is that the fitted functions do not account for species variation. Nevertheless, the functions can be used as proxy for estimation of tree height in Ikrigon and Cross River South forest reserves.

ACKNOWLEDGEMENTS

The author is thankful to Ogana T.E. for making her data available for this study. Special thanks to the anonymous reviewers for their helpful comments which improved the quality of the paper.

REFERENCES

- Adeniyi TE (2017) Potential of joint forest management for restoring degraded Forest Reserves in Nigeria as a REDD+ strategy option, (M.Sc. Thesis). School of Agriculture, Forestry and Food Science, Bern University of Applied Sciences, Switzerland, 106 p.
- Budhathoki CB, Lynch TB & Guldin JM (2008) A mixed-effects model for the dbh-height relationship of shortleaf pine (*Pinus echinate* Mill.). *Southern Journal of Applied Forestry* 32: 5–11.
- Burkhardt HE & Tomé M (2012) *Modeling Forest Trees and Stands*, (2nd edition). Springer Dordrecht Heidelberg New York, 271 p.
- Calama R & Montero G (2004) Interregional nonlinear height-diameter model with random coefficients for tone pine in Spain. *Canadian Journal of Forest Research* 34: 150–163.
- Coble DW & Lee YJ (2011) A mixed-effects height-diameter model for individual loblolly and slash trees in east Texas. *Southern Journal of Applied Forestry* 35(1): 12–17.
- Curtis RO (1967) Height-diameter and height-diameter-age equations for second-growth Douglas-fir. *Forest Science* 13(4): 365–375.
- Eby WM, Oyamakin SO & Chukwu AU (2017) A new nonlinear model applied to the height-DBH relationship in *Gmelina arborea*. *Forest Ecology and Management* 397: 139–149.
- Ferraz-Filho AC, Mola-Yudego B, Ribeiro A, Scolforo JRS, Loos RA & Scolforo HF (2018) Height-diameter models for *Eucalyptus* sp. Plantations in Brazil. *Cerne* 24(1): 9–17.
- Gomez-Garcia E, Dieguez-Aranda U, Castedo-Dorado F & Crecente-Campo F (2014) A comparison of model forms for the development of height-diameter relationships in even-aged stands. *Forest Science* 60: 560–568.
- Gompertz B (1825) On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies. *Philosophical Transactions of the Royal Society B: Biological Sciences* 115: 513–585.
- Huang S, Titus SJ & Weins DP (1992) Comparison of nonlinear height-diameter functions for major Alberta tree species. *Canadian Journal of Forest Research* 22: 1297–1304.
- Krisnawati H, Wang Y & Ades PK (2010) Generalized height-diameter models for *Acacia mangium* Willd Plantations in South Sumatra. *Journal of Forest Research* 7(1): 1–19.
- Lundqvist B (1957) On the height growth in cultivated stands for pine and spruce in Northern Sweden. *Medd. Frstatens skogforsk* 133
- Mehtätalo L (2005) Height-diameter models for Scots pine and birch in Finland. *Silva Fennica* 39(1): 55–66.
- Mehtätalo L, de-Miguel S & Gregoire TG (2015) Modelling height-diameter curves for prediction. *Canadian Journal of Forest Research* 45: 826–837.
- Menten L & Michaelis MI (1913) Die Kinetik der Invertinwirkung. *Biochemical Journal* 49: 333–369.
- Meyer W (1940) A mathematical expression for height curves. *Journal of Forestry* 38: 415–420.
- Näslund M (1937) *Skogsförsöksanstaltens gallringsförsök I tallskog (Forest research institute's thinning experiments in Scots pine forests)*. Meddelanden frstatens skogsförsöksanstalt Häfte 29 (In Swedish).
- Ogana FN (2018) Comparison of a modified Log-Logistic distribution with established models for tree height prediction. *Journal of Research in Forestry, Wildlife and Environment* 10(2): 49–55.
- Ozcelik R, Cao QV, Trincado G & Nilsum G (2018) Predicting tree height from tree diameter and dominant height using mixed-effect and quantile regression models for two species in Turkey. *Forest Ecology and Management* 419(420): 240–248.
- Paulo JA, Tomé J & Tomé M (2011) Nonlinear fixed and random generalized height-diameter models for Portuguese cork oak stands. *Annals of Forest Science* 68: 295–309.
- Pearl R & Reed LJ (1920) On the rate of growth of the population of the United States since 1790 and its mathematical representation. *Proceedings of the National Academy of Sciences of the United States of America* 6: 275–288.
- R Core Team (2017) *R: A language and environment for statistical computing*. R Foundation for Statistical Computing, Vienna, Austria.
- Ratkowsky DA (1990) *Handbook of nonlinear regression*. Marcel Dekker, Inc., New York.

- Richards FJ (1959) A flexible growth function for empirical use. *Journal of Experimental Biology* 10: 290–300.
- Sharma M & Parton J (2007) Height-diameter equations for boreal tree species in Ontario using a mixed-effects modelling approach. *Forest Ecology and Management* 249: 187–198.
- Temesgen H, Monleon VJ & Hann DW (2008) Analysis and comparison of nonlinear tree height prediction strategies for douglas-fir forests. *Canadian Journal of Forest Research* 38(3): 553–565.
- Tewari VP & Singh B (2018) Total wood volume equation for *Tectona grandis* Linn F. stands in Gujarat, India. *Journal of Forest and Environmental Science* 34(4): 313–320.
- Trincado G, VanderSchaaf CL & Burkhart HE (2007) Regional mixed-effects height-diameter models for loblolly pine (*Pinus taeda* L.) plantations. *European Journal of Forest Research* 126: 253–262.
- VanderSchaaf CL (2014) Mixed-effect height-diameter models for ten conifers in the inland Northwest, USA. *Southern Forests: A Journal of Forest Science* 76: 1–9.
- West PW (2015) *Tree and forest measurement, (3rd edition)*. Springer Cham Heidelberg New York Dordrecht London, 218 p.
- Wykoff WR, Crookston NL & Stage AR (1982) *User's guide to the stand prognosis model*. USDA Forest Service, General Technical Report, INT-133.
- Zang H, Lei X & Zeng W (2016) Height-diameter equations for larch plantations in northern and northeastern China: a comparison of the mixed-effects, quantile regression and generalized additive models. *Forestry: An International Journal of Forest Research* 89: 434–445.

Appendix I: Tree species composition of Cross River South Forest Reserve.

S.N.	Species	S.N.	Species
1	<i>Afzelia bipindensis</i>	38	<i>Irvingia gabonensis</i>
2	<i>Albizia gummifera</i>	39	<i>Irvingia wombulu</i>
3	<i>Albizia zygia</i>	40	<i>Khaya ivorensis</i>
4	<i>Allanblackia floribunda</i>	41	<i>Klainedoxa gabonensis</i>
5	<i>Amphimas pterocarpoides</i>	42	<i>Lannae welwitschii</i>
6	<i>Baillonella toxisperma</i>	43	<i>Lecanodiscus cupanoides</i>
7	<i>Baphia nitida</i>	44	<i>Lophira alata</i>
8	<i>Barteria nigritana</i>	45	<i>Lovoa trichilioides</i>
9	<i>Berlinia confusa</i>	46	<i>Mammea africana</i>
10	<i>Blighia sapida</i>	47	<i>Mitregiana inermis</i>
11	<i>Brachystegia nigerica</i>	48	<i>Musanga cecropioides</i>
12	<i>Bridelia ferruginea</i>	49	<i>Myrianthus arboreus</i>
13	<i>Canarium schweinfurthii</i>	50	<i>Parkia bicolor</i>
14	<i>Canthium palma</i>	51	<i>Pausinystalia talbotii</i>
15	<i>Carapa procera</i>	52	<i>Pentaclethra macrophylla</i>
16	<i>Celtis zenkeri</i>	53	<i>Pentadesma butyracea</i>
17	<i>Chrysophyllum giganteum</i>	54	<i>Piptadeniastrum africanum</i>
18	<i>Cleistopholis patens</i>	55	<i>Pterocarpus mibreadii</i>
19	<i>Coelocaryon preussii</i>	56	<i>Pterocarpus osun</i>
20	<i>Cola acuminata</i>	57	<i>Pterocarpus santalinoides</i>
21	<i>Cola millenii</i>	58	<i>Pterocarpus soyauxi</i>
22	<i>Combretodendron macrocarpum</i>	59	<i>Pterygota macrocarpa</i>
23	<i>Cylicodiscus gabunensis</i>	60	<i>Pycnanthus angolensis</i>
24	<i>Dacryodes klaineana</i>	61	<i>Ricinodendron heudelotii</i>
25	<i>Daniella ogea</i>	62	<i>Santiria trimera</i>
26	<i>Dialium africana</i>	63	<i>Spathodea campanulata</i>
27	<i>Dialium guineense</i>	64	<i>Staudtia stipitata</i>
28	<i>Diospyros crassiflora</i>	65	<i>Sterculia setigera</i>
29	<i>Dracaena mannii</i>	66	<i>Strombosia pustulata</i>
30	<i>Drypetes staudtii</i>	67	<i>Symphonia globulifera</i>
31	<i>Enantia chlorontha</i>	68	<i>Tetrapleura tetraptera</i>
32	<i>Entandrophragma cylindricum</i>	69	<i>Treculia obovoidea</i>
33	<i>Entandrophragma utile</i>	70	<i>Uapaca guineensis</i>
34	<i>Funtumia africana</i>	71	<i>Uapaca staudtii</i>
35	<i>Funtumia elastica</i>	72	<i>Vitex grandifolia</i>
36	<i>Garcinia kola</i>	73	<i>Xylopia aethiopica</i>
37	<i>Hylocodendron gabunense</i>	74	<i>Zanthoxylum zanthoxyloides</i>

Appendix II: Tree species composition of Ikrigon Forest Reserve.

S.N.	Species	S.N.	Species
1	<i>Azela africana</i>	33	<i>Irvingia wombulu</i>
2	<i>Albizia gummifera</i>	34	<i>Khaya grandifoliola</i>
3	<i>Albizia lebeck</i>	35	<i>Lannea welwitschii</i>
4	<i>Albizia parinari</i>	36	<i>Lecanodiscus cupanoides</i>
5	<i>Albizia zygia</i>	37	<i>Lovoa trichilioides</i>
6	<i>Alstonia boonei</i>	38	<i>Margaritaria discoidea</i>
7	<i>Anthocleista vogelii</i>	39	<i>Milicia excelsa</i>
8	<i>Antiaris toxicaria</i>	40	<i>Millettia thonningii</i>
9	<i>Baphia nitida</i>	41	<i>Olex subscorpioidea</i>
10	<i>Blighia sapida</i>	42	<i>Parinari hypochrysea</i>
11	<i>Borassus aethiopicum</i>	43	<i>Parkia bicolor</i>
12	<i>Bridelia ferruginea</i>	44	<i>Pentaclethra macrophylla</i>
13	<i>Carapa procera</i>	45	<i>Pentadesma butyracea</i>
14	<i>Ceiba petandra</i>	46	<i>Pierreodendron africanum</i>
15	<i>Chrysophyllum giganteum</i>	47	<i>Pseudospondias microcarpa</i>
16	<i>Cleistopholis patens</i>	48	<i>Pterocarpus osun</i>
17	<i>Cola acuminata</i>	49	<i>Pterocarpus santalinoides</i>
18	<i>Cola gigantea</i>	50	<i>Pterocarpus soyauxi</i>
19	<i>Cola millenii</i>	51	<i>Pterygota macrocarpa</i>
20	<i>Cola nitida</i>	52	<i>Pycnanthus angolense</i>
21	<i>Combretodendron macrocarpum</i>	53	<i>Pycnanthus angolensis</i>
22	<i>Dacryodes klaineana</i>	54	<i>Rauvolfia vomitoria</i>
23	<i>Dialium africana</i>	55	<i>Ricinodendron heudelotii</i>
24	<i>Dialium guineense</i>	56	<i>Statodi campanulata</i>
25	<i>Diospyros crassiflora</i>	57	<i>Sterculia setigera</i>
26	<i>Distemonanthus benthamianus</i>	58	<i>Symphonia globulifera</i>
27	<i>Dracaena mannii</i>	59	<i>Tectona grandis</i>
28	<i>Ficus exasperata</i>	60	<i>Terminalia superba</i>
29	<i>Funtimia africana</i>	61	<i>Tetrapleura tetraptera</i>
30	<i>Gmelina arborea</i>	62	<i>Trema guineensis</i>
31	<i>Gossweilerodendron balsamiferum</i>	63	<i>Vitex doniana</i>
32	<i>Grewia mollis</i>		