Camera Pose Measurement from 2D-3D Correspondences of Three Z Shaped Lines

Chang Liu^{1,2,3,4*}, Feng Zhu^{1,4}, Jinjun Ou^{1,4}, Yong Yu⁵, Renbo Xia^{1,4}

¹ *Shenyang Institute of Automation, Chinese Academy of Sciences, Shenyang 110016, China*

² *School of Information Science & Engineering, Shenyang Ligong University, Shenyang 110159, China*

³ *Graduate School of the Chinese Academy of Sciences, Beijing 100039, China*

⁴ *Key Laboratory of Opto-Electronic Information Processing, CAS, Shenyang 110016, China*

⁵ *Broadcom Corporation, Andover, MA 01810, U.S.A.*

* *Corresponding author's Email: syliuch@sia.cn*

Abstract: Pose measurement is an important problem in computer vision. This paper presents a method for finding the closed form solutions to camera pose measurement problem by using 2D-3D correspondences of three Z shaped lines. This problem is a special case of Perspective-three-Line (P3L) problem. By introducing two undetermined parameters, we translate the problem into solving a quadratic equation. We also prove the problem's solution number depends on the relative position between the camera's optical center and the three Z shaped lines. A unique solution exists when the optical center locates inside two right circular cones. The experiments show the method is efficient. The method provides a new way using line features and monocular vision to pose measurement and can be applied to many vision systems.

Keywords: monocular vision; pose measurement; Perspective-three-Line (P3L); line feature; closed form solution

1. Introduction

Computer vision based pose measurement has been used in many fields such as aviation, robot navigation, industry automatic assembling, etc. If only one camera is used in a vision system, it is called monocular vision. Given some features of a target object, the position and orientation of the three-dimensional object can be estimated from the correspondences of features on the image with only one perspective view [1]. There are a variety of features which can be used such as points, lines, planes, corners, etc. Previous researchers attached much importance to point features. One of the most famous problems is Perspective-n-Point (PnP) problem presented in [2] which n control points and correspondences for pose estimation are used. Many researchers studied this problem and

their results have been used in many engineering applications successfully [3].

In real world, straight line is another important feature for many target objects. Pose measurement from line correspondences can also be applied to monocular vision systems. Perspective-n-Line (PnL) problem needs to solve the problem of how to calculate position and orientation parameters from 2D-3D correspondences of n lines. When the number of line correspondences is three, it is a P3L problem. Dhome et al. [4] researched a general case of the P3L problem and deduced an eight-degree polynomial equation. The P3L problem's solutions are the roots of the polynomial equation. By their methods, we can obtain eight possible solutions at most. It needs further conditions to judge which one is the real solution.

Chen [5] updated Dhome's method and pointed out that in some cases the maximum number of real solutions to the problem is eight. Liu et al. [6] pointed out P3L problem can be solved by nonlinear approach and presented an iterative algorithm.

Generally, P3L problem has multiple solutions which make it difficult to use in real applications and unique solution conditions have drawn much attention in recent studies. It is found that if three lines on the target object have special geometric relationship such as parallelity or orthogonality, the problem has further properties. Shi et al. [7] studied a case that three lines intersect at one point. This is also a case of corner correspondence. They proved the existence of unique solution if given mutual angles among three lines and another point correspondence. Ying et al. [8] studied a case of parallel lines correspondence and applied to distinguish crosswalks and staircases aiding for the partially sighted. Frank [9] and Wang et al. [10] studied a case that four lines form a parallelogram and obtained a unique solution, but their problems both belong to P4L. Qin et al. [11, 12] studied a case that three lines intersect at two points and proved the maximum solution number is four in this case. Liu et al. [13] studied a case that three lines are shaped like a door and proved if the camera's optical center does not locates on a special plane, the problem has a unique solution.

In this paper, we study another special case of P3L that three lines on the object are Z shaped, which means two of three lines are parallel and the third line intersects with them at two points. If the camera's optical center has a special position in 3D region, the problem has a unique solution. This result makes it possible to estimate the pose parameters of a camera from a single perspective view and 2D to 3D correspondences of three Z shaped lines in monocular vision systems.

2. Problem statement

2.1 Camera model and parameters

In this paper we adopt pin-hole model for a camera. Suppose the camera has been calibrated. The camera's intrinsic parameters include: the principal point (u_o, v_o) , the focal ratio f_u and f_v . The camera coordinate system is defined as follow: the origin is the camera's optical center; the X axis and axis are respectively parallel to the X axis and axis of the image; the Z axis is the camera's optical axis which is perpendicular to the image plane. The object coordinates (x_w, y_w, z_w) and the camera coordinates (x_c, y_c, z_c)

Figure 1 Z shaped P3L problem formulation.

of a point satisfying

$$\begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix} = R \begin{pmatrix} x_w \\ y_w \\ z_w \end{pmatrix} + T, \quad (1)$$

In equations 1,

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \quad (2)$$

R is the rotation matrix and $T = (T_x, T_y, T_z)^T$ is the translation vector.

Rotation matrix R can be expressed by three rotation angles around three axes. In this paper let the angle around X axis be ψ ; the angle around Y axis be ϕ ; the angle around Z axis be θ . The rotation order is X, Y, Z . We have

$$\begin{cases} r_{11} = \cos \theta \cos \phi \\ r_{12} = \sin \theta \cos \phi \\ r_{13} = -\sin \theta \\ r_{21} = \cos \theta \sin \phi \sin \psi - \sin \theta \cos \psi \\ r_{22} = \sin \theta \sin \phi \sin \psi + \cos \theta \cos \psi \\ r_{23} = \cos \phi \sin \psi \\ r_{31} = \cos \theta \sin \phi \cos \psi + \sin \theta \sin \psi \\ r_{32} = \sin \theta \sin \phi \cos \psi - \cos \theta \sin \psi \\ r_{33} = \cos \phi \cos \psi \end{cases} \quad (3)$$

From (3), it is not difficult to parse three rotation angles from a given rotation matrix R .

2.2 Z shaped P3L problem formulation

Figure.1 depicts the Z shaped P3L problem. We have several requirements as follows:

1) Three lines L_1, L_2 and L_3 are coplanar and placed as Z shaped. L_1 and L_3 are rays and parallel to each other; L_2 intersects with L_1 and L_3 at P_1 and P_2 ; the angle between L_1 and L_2 is ω . Because [13] has studied the case that L_2 is perpendicular to L_1 and L_3 , here we

only discuss the case $0 < \omega < \pi/2$. l_1, l_2 and l_3 on the image plane correspond to lines L_1, L_2 and L_3 . q_1 and q_2 on the image plane correspond to the intersection points P_1 and P_2 .

2) The distance between P_1 and P_2 is d ($d > 0$).

3) In the camera coordinate system, the straight line equations of l_1, l_2 and l_3 on the image plane are given: $a_i x + b_i y + c_i = 0$ ($i = 1, 2, 3$). Here we only study the nonsingular case that the images of three lines are also three lines, they are not coincident or do not degenerate to points.

4) The line equations of L_1, L_2 and L_3 in the object coordinate system are given. Let $\vec{V}$ denotes a vector or the direction of a ray V . Here we suppose the object coordinate system is defined as: the origin is P_1 ; the X axis is L_1 and its direction is $\vec{L}_1$; the Y axis is on the plane which contains L_1 and L_2 , perpendicular to L_1 and directs to L_3 ; the Z axis is determined by the X and Y axes according to the right-hand rule.

5) All intrinsic parameters of the camera are given.

The question is to find out the transformation (R and T in (1)) between the object and the camera coordinates. The problem is equivalent to finding the coordinates description of three lines L_1, L_2 and L_3 in the camera coordinate system from their images.

Qin et al. [12] studied a similar problem and presented a method to obtain the problem's closed form solutions. They also pointed out that the problem has two different solutions. In [12], and L_1 and L_3 are supposed to be straight lines, while in this paper we suppose they are rays. Under this assumption, we find out the conditions that unique solution exists. Unique solution is important for pose determination in computer vision systems because we need not distinguish the real solutions from many possible ones.

3. Closed form solutions to the problem

3.1 Direction of the X axis of the object coordinate system

If the camera's image plane is not parallel to L_1 and L_3 , l_1 is not parallel to l_3 . Suppose the intersection point of l_1 and l_3 is q_3 . q_3 is the vanishing point of L_1 and L_3 . Regarding the vanishing point, we have the following well-known lemma [14].

Lemma 1 : The line through the camera optical center and the vanishing point is parallel to the lines in the scene which have given rise to the vanishing point.

From Lemma 1 we have the ray $\vec{O_C q_3}$ which is parallel to L_1 . Because L_1 is a ray, its image l_1 is also a ray. We can use the following method to obtain its

direction: If the vanishing point q_3 is on $\vec{l_1}$, the direction of L_1 is $\vec{O_C q_3}$; if q_3 is on the oppositely elongated line of $\vec{l_1}$, the direction of L_1 is $\vec{q_3 O_C}$.

If the camera's image plane is parallel to L_1 and L_3 , then $l_1 // l_3$. The X axis of the object coordinate system has a same direction with $\vec{l_1}$. Let q_3 be an arbitrary point on $\vec{l_1}$, then $\vec{q_1 q_3}$ is the direction of the X axis.

3.2 Method for closed form solutions

Let f be the focal length of the camera. If the camera's image plane is not parallel to L_1 and L_3 , the three intersection points are $q_1(x_1, y_1, f)$, $q_2(x_2, y_2, f)$, $q_3(x_3, y_3, f)$, where

$$\begin{cases} x_1 = \frac{b_1 c_2 - b_2 c_1}{a_1 b_2 - a_2 b_1} & y_1 = \frac{a_2 c_1 - a_1 c_2}{a_1 b_2 - a_2 b_1} \\ x_2 = \frac{b_2 c_3 - b_3 c_2}{a_2 b_3 - a_3 b_2} & y_2 = \frac{a_3 c_2 - a_2 c_3}{a_2 b_3 - a_3 b_2} \\ x_3 = \frac{b_1 c_3 - b_3 c_1}{a_1 b_3 - a_3 b_1} & y_3 = \frac{a_3 c_1 - a_1 c_3}{a_1 b_3 - a_3 b_1} \end{cases} \quad (4)$$

Using the method in section 3.1, the direction of the X axis of the object coordinate system is

$$\vec{L}_1 = (x_3, y_3, f) \quad (5)$$

or

$$\vec{L}_1 = (-x_3, -y_3, -f). \quad (6)$$

If the camera's image plane is parallel to L_1 and L_3 , suppose (x_0, y_0) is an arbitrary point on the ray l_1 , and let $x_3 = x_0 - x_1$, $y_3 = y_0 - y_1$, the direction of X axis is

$$\vec{L}_1 = (x_3, y_3, 0). \quad (7)$$

To find solutions to the above problem, here we introduce two undetermined parameters. In the camera coordinate system, suppose $\vec{O_C P_i} = k \cdot \vec{O_C q_i}$ ($i = 1, 2$) where k_i is the undetermined parameter whose geometric meaning is the ratio between the distance from P_i to O_C and the distance from q_i to O_C . The direction of L_2 can be expressed as

$$\begin{aligned} \vec{L}_2 = \vec{P_1 P_2} &= k_2 \cdot \vec{O_C q_2} - k_1 \cdot \vec{O_C q_1} \\ &= (k_2 x_2 - k_1 x_1, k_2 y_2 - k_1 y_1, (k_2 - k_1)f). \end{aligned} \quad (8)$$

The angle between L_1 and L_2 is ω , then

$$\begin{aligned} \cos \omega &= \frac{\|\vec{L}_1 \cdot \vec{L}_2\|}{\|\vec{L}_1\|^{1/2} \|\vec{L}_2\|^{1/2}} \\ &= \frac{\|\vec{O_C q_3} \cdot \vec{P_1 P_2}\|}{\|\vec{O_C q_3}\|^{1/2} \|\vec{P_1 P_2}\|^{1/2}}. \end{aligned} \quad (9)$$

The distance between P_1 and P_2 is d , then

$$\sqrt{(k_2 x_2 - k_1 x_1)^2 + (k_2 y_2 - k_1 y_1)^2 + f^2 (k_2 - k_1)^2} = d.$$

(10)

If the direction of L_1 is (5), from (9) and (10) we have

$$(x_3x_2 + y_3y_2 + f^2)k_2 = (x_3x_1 + y_3y_1 + f^2)k_1 + d \cos \omega \sqrt{x_3^2 + y_3^2 + f^2}. \quad (11)$$

There are following two possible cases.

(a) If $x_3x_2 + y_3y_2 + f^2 \neq 0$, let

$$\begin{cases} s = \frac{x_3x_1 + y_3y_1 + f^2}{x_3x_2 + y_3y_2 + f^2} \\ r = \frac{d \cos \omega \sqrt{x_3^2 + y_3^2 + f^2}}{x_3x_2 + y_3y_2 + f^2} \end{cases} \quad (12)$$

then (11) is rewritten into

$$k_2 = sk_1 + r \quad (13)$$

(13) depicts the linear relationship between k_2 and k_1 . Put (13) into (10), we have

$$((sk_1 + r)x_2 - k_1x_1)^2 + ((sk_1 + r)y_2 - k_1y_1)^2 + f^2((sk_1 + r) - k_1)^2 = d^2 \quad (14)$$

Let

$$\begin{cases} a = (sx_2 - x_1)^2 + (sy_2 - y_1)^2 + (s - 1)^2 f^2 \\ b = 2r[(sx_2 - x_1)x_2 + (sy_2 - y_1)y_2 + (s - 1)^2 f^2] \\ c = r^2(x_2^2 + y_2^2 + f^2) - d^2 \end{cases} \quad (15)$$

(14) can be rewritten as

$$ak_1^2 + bk_1 + c = 0. \quad (16)$$

Because q_1 and q_2 are not coincident, we have $(x_1, y_1) \neq (x_2, y_2)$, then $a > 0$. (16) has two solutions

$$k_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \quad (17)$$

From (13), we can obtain two solutions of k_2 according to k_1 . From the geometric meaning of k_1 and k_2 , they must satisfy

$$k_1 > 0 \text{ and } k_2 > 0. \quad (18)$$

From (18), we can judge which one is the real solution to the problem.

(b) If $x_3x_2 + y_3y_1 + f^2 = 0$, (11) is rewritten into

$$(x_3x_1 + y_3y_1 + f^2)k_1 + d \cos \omega \sqrt{x_3^2 + y_3^2 + f^2} = 0. \quad (19)$$

From $d > 0$, $\omega \neq \pi/2$, and $\|\overrightarrow{Ocq_3}\| \neq 0$, it must have $x_3x_1 + y_3y_1 + f^2 \neq 0$. Then we have

$$k_1 = -\frac{d \cos \omega \sqrt{x_3^2 + y_3^2 + f^2}}{x_3x_1 + y_3y_1 + f^2}. \quad (20)$$

From (10), we have

$$(x_2^2 + y_2^2 + f^2)k_2^2 - 2k(x_1x_2 + y_1y_2 + f^2)k_2 + k_1^2(x_1^2 + y_1^2 + f^2) - d^2 = 0 \quad (21)$$

This is a quadratic equation with respect to k_2 . let

$$\begin{cases} a' = x_2^2 + y_2^2 + f^2 \\ b' = 2k(x_1x_2 + y_1y_2 + f^2) \\ c' = k_1^2(x_1^2 + y_1^2 + f^2) - d^2 \end{cases} \quad (22)$$

We can obtain two solutions of (21) as

$$k_2 = \frac{-b' \pm \sqrt{b'^2 - 4a'c'}}{2a'}. \quad (23)$$

Similarly, if the direction of L_1 is (6), from (9) and (10), we have

$$(x_3x_2 + y_3y_2 + f^2)k_2^2 = (x_3x_1 + y_3y_1 + f^2)k_1 - d \cos \omega \sqrt{x_3^2 + y_3^2 + f^2}. \quad (24)$$

If the direction of L_1 is (7), from (9) and (10), we have

$$(x_3x_2 + y_3y_2)k_2^2 = (x_3x_1 + y_3y_1)k_1 + d \cos \omega \sqrt{x_3^2 + y_3^2}. \quad (25)$$

We can obtain similar results in both cases. The above deductions justify the existence of two possible solutions at most. From criteria (18), we can distinguish the real solution to the problem.

Using the above method, we can obtain the coordinates of in the camera coordinate system, that is (k_1x_1, k_1y_1, k_1f) . As P_1 is the origin of the object coordinate system, its object coordinates are $(0, 0, 0)$. Then the translation vector is

$$T = (T_X, T_Y, T_Z)^T = (k_1x_1, k_1y_1, k_1f)^T. \quad (26)$$

We can also obtain the direction of $\overrightarrow{L_2}$ from (8) after we get the camera coordinates of P_1 and P_2 . It is not difficult to obtain the directions of the three axes of the object coordinate system and the rotation matrix R . From the above method, the problem's closed form solutions are resolved.

Figure 2 Illustration of Theorem 1.

4. Problem's solution number

Unique solution conditions are important for P3L in real vision applications. Regarding the problem solution's properties, we have the following theorems.

Theorem 1: Let the plane which contains L_1, L_2 and L_3 be π_1 ; the plane which is perpendicular to π_1 and passes L_2 be π_2 . If the camera's optical center locates on π_2 , the problem has a unique solution.

4.1 Z shaped P3L problem formulation

Proof: As shown in Figure.2, assuming that L_1, L_2 and L_3 (L_3 is not shown in Figure. 2) are the real solution lines of the problem, whose intersection points are P_1 and P_2 , from the given conditions, the angle between L_1 and L_2 is ω , and $|P_1P_2| = d$. Make a right circular cone A ; let the vertex of A be P_1 , the axis of A be L_1 , and the vertex angle of A be 2ω , L_2 must locate on the surface of the cone A . From the definition of π_2 , the cone A is tangent with π_2 and the tangent line between A and π_2 is L_2 .

Similarly, assuming that L'_1, L'_2 and L'_3 (L'_3 is not shown in Figure.2) are also solution lines of the problem. Their intersection points are P'_1 and P'_2 . Because we have obtained the direction of L_1 in section 3.1, it must have $L_1 \parallel L'_1$. From the given conditions, P'_1 is on the ray $\overrightarrow{O_C P_1}$; P'_2 is on the ray $\overrightarrow{O_C P_2}$; $|P'_1 P'_2| = d$; the angle between L'_1 and L'_2 is also ω . Make another right circular cone A' ; let the vertex of A' be P'_1 , the axis of A' be L'_1 , and the vertex angle of A' be 2ω . Then L'_2 must locate on the surface of the cone A' ; the cone A' must be tangent with π_2 ; and the tangent line between A' and π_2 is L'_2 .

Regarding two triangles $O_C P_1 P_2$ and $O_C P'_1 P'_2$, we have $P_1 P_2 \parallel P'_1 P'_2$. Because the real solution must satisfy $|P_1 P_2| = |P'_1 P'_2| = d$, the two triangles $O_C P_1 P_2$ and $O_C P'_1 P'_2$ must be coincident. Then the two solutions are coincident. The problem has a unique solution. This proves Theorem 1.

Because a plane is not a 3D space, Theorem 1 only

Figure 3 Illustration of Theorem 2.

has theoretical meaning. It shows π_2 is an important reference plane for the problem.

Theorem 2: Let the angle between L_1 and $O_C q_1$ be an acute angle. If the angle is less than ω , the problem has a unique solution.

Proof: As shown in Figure.3, assuming that L_1, L_2 and L_3 (L_3 is not shown in Figure. 3) are the real solution lines of the problem whose intersection points are P_1 and P_2 , from the given conditions, P_1 is on the ray $\overrightarrow{O_C q_1}$; P_2 is on the ray $\overrightarrow{O_C q_2}$; $|P_1 P_2| = d$ and the angle of L_1 and L_2 is ω . Make a right circular cone A , let the vertex of A be P_1 , the axis of A be L_1 ; the vertex angle of A be 2ω , L_2 must locate on the surface of the cone A . Because the angle of L_1 and $\overrightarrow{O_C P_1}$ is less than ω , there is only one intersection point of $\overrightarrow{O_C q_2}$ and the surface of the cone A , let it be P_2 . Make a plane π , let it be perpendicular to L_1 and pass P_2 . Suppose the cone A intersects with π at circle C , let the intersection point of L_1 and π be S , the intersection point of the ray $\overrightarrow{O_C q_3}$ and π be U , the intersection point of $\overrightarrow{O_C q_1}$ and π be R . Obviously, the three points R, S and U are collinear.

Similarly, assuming that L'_1, L'_2 and L'_3 is not shown in Figure.3) are also solution lines of the problem. Their intersection points are P'_1 and P'_2 . From the given conditions we have P'_1 on $\overrightarrow{O_C q_1}$, P'_2 on $\overrightarrow{O_C q_2}$; the angle between L'_1 and L'_2 is also ω ; $|P'_1 P'_2| = d$; $L_1 \parallel L'_1 \parallel \overrightarrow{O_C q_3}$. Make another right circular cone A' ; let the vertex of A' be P'_1 , the axis of A' be L'_1 , and the vertex angle of A' be 2ω , L'_2 must locate on the surface of the cone A' . Because the angle of L'_1 and $\overrightarrow{O_C P'_1}$ is less than ω , there is also only one intersection point of $\overrightarrow{O_C q_2}$ and the surface of the cone A' , let it be P'_2 . Make a plane π' , let it be perpendicular to L'_1 and pass P'_2 . Suppose the cone A' intersects with π' at circle C' . Let the intersection point of L'_1 and π' be S' ; the intersection point

Figure 4 The unique solution region to the problem.

of $\overrightarrow{OCq_3}$ and π' is U' ; the intersection point of $\overrightarrow{OCq_1}$ and π' is R' . It is clear that three points R' , S' and U' are collinear.

The intersection line of plane OCq_1q_2 and π is RP_2 , the intersection line of plane OCq_1q_2 and π' is $R'P'_2$, and $\pi' // \pi$, then we have $RP_2 // R'P'_2$.

Make $P_1T \perp P_2R$ at T , $P'_1T' \perp P'_2R'$ at T' . From $RP_2 // R'P'_2$ we have $P_1T // P'_1T'$. From $P_1S // OCq_3 // P'_1S'$ we have $\angle TP_1S = \angle T'P'_1S'$. From $P_1S \perp ST$ and $P'_1S' \perp S'T'$ we have triangle P_1ST is similar to triangle $P'_1S'T'$, then we have

$$\frac{P_1T}{P'_1T'} = \frac{P_1S}{P'_1S'} \quad (27)$$

where P_2 and P'_2 are on the surface of A and A' respectively, and the vertex angle of A and A' are both 2ω , we have $\angle P_2P_1S = \angle P'_2P'_1S'$. From $P_1S \perp SP_2$ and $P'_1S' \perp S'P'_2$, we have triangle P_1SP_2 is similar to triangle $P'_1S'P'_2$, then

$$\frac{P_1P_2}{P'_1P'_2} = \frac{P_1S}{P'_1S'} = \frac{P_1T}{P'_1T'} \quad (28)$$

where P_1TP_2 and $P'_1T'P'_2$ are both right triangles, they are similar to each other. Then $\angle TP_2P_1 = \angle T'P'_2P'_1$. From $RP_2 // R'P'_2$, and as RP_2 , $R'P'_2$, P_1P_2 , $P'_1P'_2$ four lines are coplanar, we have $P_1P_2 // P'_1P'_2$.

Regarding two triangles $OC P_1P_2$ and $OC P'_1P'_2$, because $P_1P_2 // P'_1P'_2$ and $|P_1P_2| = |P'_1P'_2| = d$, we conclude they must be coincident. Then the two solutions of the problem are same. The problem has a unique solution. This proves Theorem 2.

Theorem 3: Let the angle between L_1 and OCq_2 be an acute angle. If the angle is less than ω , the problem has a unique solution. The proof of Theorem 3 is similar to that of Theorem 2.

Theorem 2 and Theorem 3 define a 3D region constituted of four right circular cones, whose vertexes

Figure 5 Illustration of Theorem 4.

are P_1 and P_2 , vertex angles are 2ω and axes are L_1 and L_3 respectively. Regarding these four cones, it is not difficult to prove that two of them are included by the other two cones. As shown in Figure.4, if the optical center locates inside the two cones, the problem has a unique solution.

Theorem 4: If the optical center does not locate into the region described in Theorem 1, Theorem 2 or Theorem 3, the problem has two solutions.

Proof: As shown in Figure.5, make two right circular cone A and A' , let their vertexes be P_1 and P'_1 which are two points on the ray $\overrightarrow{OCq_1}$; their axes be parallel to $\overrightarrow{OCq_3}$ which we obtain from section 3.1; their vertex angles be both 2ω . If the optical center does not locate in the region described before, there are two intersection points of $\overrightarrow{OCq_2}$ and each cone. Let the intersection points of $\overrightarrow{OCq_2}$ and A be $P_2^{(1)}$ and $P_2^{(2)}$; the intersection points of $\overrightarrow{OCq_2}$ and A' be $P_2^{(1)'} and $P_2^{(2)'}.$ Similar to the proof of Theorem 2, triangle P_1ST is similar to $P'_1S'T'$; $P_1SP_2^{(2)}$ is similar to $P'_1S'P_2^{(2)'}$, then$

$$\frac{P_1P_2^{(2)}}{P'_1P_2^{(2)'}} = \frac{P_1S}{P'_1S'} = \frac{P_1T}{P'_1T'}. \quad (29)$$

Then two right triangles $P_1TP_2^{(2)}$ and $P'_1T'P_2^{(2)'}$ are similar to each other and $\angle TP_2^{(2)}P_1 = \angle T'P_2^{(2)'}P'_1$. From $RP_2^{(2)} // R'P_2^{(2)'}$ and $RP_2^{(2)}$, $R'P_2^{(2)'}$, $P_1P_2^{(2)}$, $P'_1P_2^{(2)'}$ are coplanar, we have

$$P_1P_2^{(2)} // P'_1P_2^{(2)'}. \quad (30)$$

Similarly, we have

$$P_1P_2^{(1)} // P'_1P_2^{(1)'}. \quad (31)$$

From Equations (30) and (31), there are only two possible directions of P_1P_2 which may satisfy the problem's conditions on angle constraints. To find out the location of L_2 , we need to move P_1 on the ray $\overrightarrow{O_Cq_1}$ from its end point O_C , according to the direction $P_1P_2^{(1)}$ or $P_1P_2^{(2)}$, for the segment P_1P_2 satisfied $|P_1P_2| = d$. Obviously there is only one possible segment P_1P_2 on each direction satisfied the condition. Then the problem has two solutions. This proves Theorem 4.

The above four theorems show the problem's solution number depends on the relative position between the optical center and the three lines. For real applications, if we control the camera's optical center into the 3D region described in Figure.4, we can obtain the unique solution of the problem.

5. Experimental results and analysis

5.1 Simulations

Experiment 1: Select three Z shaped lines satisfying $\omega = 45^\circ$ and $d = 141.4mm$. Suppose the image size is 512×512 ; the camera's focal ratio at two directions are both 2000; the principle point is $(256, 256)$. Let the translation vector be $(T_X, T_Y, T_Z) = (0mm, 0mm, 2000mm)$, and three rotation angles be $\psi = 0^\circ$, $\phi = 50^\circ$, and $\theta = 0^\circ$. Suppose there is no error on image and lines detection. The experimental results of pose parameters are shown in Table 1. For the first solution, $k_1 > 0$ and $k_2 > 0$, it is a correct solution. For the second solution, $k_1 < 0$, which does not satisfy (18), so it is not a reasonable solution. There is only one solution for the case. In this experiment, the angle $\angle q_1O_Cq_3 = 40^\circ$, and $\angle q_2O_Cq_3 = 38.3^\circ$. The camera's optical center locates inside the circular cones in Figure.4. This experiment verifies Theorem 2 and Theorem 3.

Table 1 Results of Experiment 1

NO.	k_1	k_2	T_X	T_Y	T_Z	ψ	ϕ	θ
1	125	129.8	0	0	2000	0	50	0
2	-31.3	-22.5	-	-	-	-	-	-

Experiment 2: The experiment conditions are similar to that of Experiment 1. Let The translation vector is also $(T_X, T_Y, T_Z) = (0mm, 0mm, 2000mm)$ and three rotation angles be $\psi = 0^\circ$, $\phi = 40^\circ$, and $\theta = 0^\circ$. The experimental results are shown in Table 2. Both solutions have $k_1 > 0$ and $k_2 > 0$, so they are both possible solutions. In this experiment, $\angle q_1O_Cq_3 = 50^\circ$, and $\angle q_2O_Cq_3 = 47.9^\circ$. The optical center is outside the circular cones in Figure.4. This experiment verifies

Figure 6 Experiment system of Experiment 3.

Theorem 4.

Table 2 Results of Experiment 2

NO.	k_1	k_2	T_X	T_Y	T_Z	ψ	ϕ	θ
1	125	129	0	0	2000	0	40	0
2	13.7	22.5	0	0	218.7	97.7	173.6	140.4

5.2 Real data

Experiment 3: The experiment system is shown in Figure.6. We fix a camera on a tripod. The camera we used in the experiment is Pylon piA640-210gm produced by Basler. The image size is 640×480 . A model with three Z shaped edge lines is put in front of the camera which is calibrated in advance. Here $\omega = 65.43^\circ$ and $d = 73.38mm$. The pose parameters, which are expressed by rotation angles and translation vector, between the model and the camera are measured by a Leica AT901-LR absolute tracker in this experiment. The measured data is exact enough to be considered as reference standard in data verification. We take a photo of the model by the camera, while keeping the position and attitude between the model and the camera unchanged. Figure.7(a) is the photo of the model taken by the camera. We first detect the edge points of the image by Canny operator and then detect the straight lines on the image by Hough transform algorithm. Generally, there are many possible lines detected, here we select three as the Z shaped edge lines of the model by ourselves. Figure.7(b) is the edge image after Canny detection and we also mark the three lines selected for calculation in Figure.7(a). After we obtain the equations of three lines on the image, we calculate the rotation matrix and translation vector by the proposed method.

The results measured by the laser tracker and calculated from the image are listed in Table 3. From Table

Figure 7 Images in Experiment 3.

3, the two results are close to each other. The maximum error of the elements of the translation vector is less than 5%, and the maximum error of the rotation angles is less than 2° .

Table 3 Results of Experiment 3

	measured results	calculated results
ψ	163.445°	165.299°
ϕ	51.512°	51.972°
θ	356.085°	357.302°
T_X	-45.612mm	-44.713mm
T_Y	35.594mm	33.982mm
T_Z	508.295mm	489.577mm

Experiment 4: The objective of this experiment is to measure the relative position between two cards (see Figure.8). Two cards K and Q are placed on a table and we take a photo with a camera calibrated in advance. The ruler-measured distance between two corner points of cards is 90.0mm. As shown in Figure.8, we set two object coordinate systems (x_1, y_1, z_1) and (x_2, y_2, z_2) . Suppose that

$$\begin{pmatrix} x_C \\ y_C \\ z_C \end{pmatrix} = R_1 \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + T_1 \text{ and } \begin{pmatrix} x_C \\ y_C \\ z_C \end{pmatrix} = R_2 \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} + T_2$$

then we have

$$\begin{aligned} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} &= R_2^{-1} \begin{pmatrix} x_C \\ y_C \\ z_C \end{pmatrix} - R_2^{-1} T_2 \\ &= R_2^{-1} R_1 \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + R_2^{-1} (T_1 - T_2) \end{aligned}$$

Using the frame lines of cards as control lines, we calculate the transformation between two coordinate systems. The results are

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} 0.364 & 0.931 & 0.010 \\ 0.931 & -0.364 & 0.007 \\ 0.010 & 0.007 & -1.000 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{pmatrix} -55.452 \\ 71.826 \\ -6.226 \end{pmatrix}$$

After parsing from the rotation matrix, three rotation angles are $\psi = 179.61^\circ$, $\phi = -0.59^\circ$, $\theta = 68.66^\circ$. The

Figure 8 Image of Experiment 4.

distance between O_1 and O_2 is

$$\sqrt{(-55.452)^2 + 71.826^2 + (-6.226)^2} = 90.954(mm)$$

This result approximates the real values. It shows the method in this paper is efficient.

6. Conclusion

By geometric deduction, we proved that the solution number of Z shaped P3L problem depends on the relative position between the camera's optical center and the three Z shaped lines. The main conclusions include:

1) If optical center locates inside the 3D region of two right circular cones, as shown in Figure.4, whose vertex angles are both 2ω , vertexes are P_1 and P_2 , and axes are L_1 and L_3 respectively, the problem has a unique solution. The space inside two cones extends with the increase of vertex angle, which is advantageous to pose measurement.

2) If optical center locates on the specific plane described in Theorem 1, the problem also has a unique solution. But because a plane is not a 3D space, the plane is only an important reference plane.

3) If optical center locates on the plane which contains L_1 , L_2 and L_3 , the problem is degenerated because the images of three lines are coincident. We did not discuss this case in the paper.

4) If optical center has other locations, the problem has two solutions. It needs further criteria to distinguish which one is the real solution.

To summarize, it is possible to use three Z shaped lines to pose measurement between the camera and the target object. The problem's solutions can also be resolved by a closed form method presented in this paper. Its performance is more effective than iterative

methods. Experiment results show the efficiency of the method. It provides us a new pose measurement method in computer vision systems.

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