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Measure of Modified Rotatability for Second Order Response Surface Designs using Pairwise Balanced Designs

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Abstract: In this paper, measure of modified rotatability for second order response surface designs using pairwise balanced design is suggested which enables us to assess the degree of modified rotatability for a given response surface design. It is observed that this method sometimes leads to designs with lesser number of design points.
Keywords and phrases: Response surface designs, modified rotatable designs, pairwise balanced designs, measure of rotatability for second order response surface designs.

I. INTRODUCTION

Response surface methodology is a statistical technique that is very useful in design and analysis of scientific experiments. In many experimental situations, the experimenter is concerned with explaining certain aspects of a functional relationship $Y = f(x_1, x_2, ..., x_v) + \varepsilon$, where Y is the response; $x_1, x_2, ..., x_v$ are the levels of v-quantitative variables or factors; and ε is the random error. Response surface methods are useful where several independent variables influence a dependent variable. The independent variables are assumed to be continuous and controlled by the experimenter. The response is assumed to be as random variable. For example, if a chemical engineer wishes to find the temperature (x_1) and pressure (x_2) that maximizes the yield (response) of his process, the observed response Y may be written as a function of the levels of the temperature (x_1) and pressure (x_2) as $Y = f(x_1, x_2) + \varepsilon$.

The concept of rotatability, which is very important in response surface designs, was proposed by Box and Hunter (1957). Das and Narasimham (1962) constructed rotatable designs through balanced incomplete block designs (BIBD). Tyagi (1964) constructed second order rotatable designs (SORD) using pairwise balanced designs (PBD). If the circumstances are such that exact rotatability is unattainable, it is still a good idea to make the design nearly rotatable. Thus, it is important of know if a particular design is rotatable or, if is not, to know how rotatable it is. Park et al. (1993) introduced measure of rotatability for second order response surface designs. Victorbabu and Surekha (2012, 2013, 2015) studied measure of rotatability for second order response surface designs using CCD, incomplete block designs and BIBD respectively. Das et al. (1999) studied response surface designs, symmetrical and asymmetrical, rotatable and modified. Victorbabu and Vasundharadevi (2005) studied modified second order response surface designs, rotatable designs using BIBD. Victorbabu et al. (2006) suggested modified second order response surface designs, rotatable designs using pairwise balanced designs. Victorbabu et al. (2008) suggested modified rotatable central composite designs. Victorbabu and Vasundharadevi (2008) studied modified second order response surface designs, rotatable designs using symmetrical unequal block arrangements (SUBA) with two unequal block sizes. Jyostna et al. (2020) constructed measure of modified rotatability for second order response surface designs. Jyostna and Victorbabu (2020) studied measure of modified rotatability for second order response surface designs using BIBD.

II. CONDITIONS FOR SECOND ORDER ROTATABLE DESIGNS

Suppose we want to use the second order response surface design D= $((x_{iu}))$ to fit the surface,

$$Y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum_{i<j}\sum b_{ij} x_{iu} x_{ju} + e_u \qquad (1)$$

where x_{iu} denotes the level of the i^{th} factor (i =1,2,…,v) in the u^{th} run (u=1,2,…,N) of the experiment, e_u's are uncorrelated random errors with mean zero and variance σ^2. D is said to be second order rotatable design (SORD), if the variance of the estimate of first order partial derivative of $Y_u(x_1, x_2, ..., x_v)$ with respect to each of independent variables (x_i) is only a function

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of the distance ($d^2=\sum_{i=1}^{v} x_i^2$) of the point ($x_1,x_2, \ldots,x_v$) from the origin (centre) of the design. Such a spherical variance function for estimation of second order response surface is achieved if the design points satisfy the following conditions [cf. Box and Hunter (1957), Das and Narasimham (1962)].

1) $\sum x_{iu}=0$, $\sum x_{iu}x_{ju}=0$, $\sum x_{iu}x_{ju}^2=0$, $\sum x_{iu}x_{ju}x_{ku}=0$, $\sum x_{iu}^3=0$, $\sum x_{iu}x_{ju}^3=0$, $\sum x_{iu}x_{ju}x_{ku}^2=0$, $\sum x_{iu}x_{ju}x_{ku}x_{lu}=0$; for $i \neq j \neq k \neq l$; (2)
2) (i) $\sum x_{iu}^2=$ constant $= N\lambda_2$;
3) $\sum x_{iu}^4=$ constant $= cN\lambda_4$; for all i (3)
4) $\sum x_{iu}^2 x_{ju}^2=$ constant $= N\lambda_4$; for $i \neq j$ (4)
5) $\sum x_{iu}^4 = c\sum x_{iu}^2 x_{ju}^2$ (5)
6) $\dfrac{\lambda_4}{\lambda_2^2} > \dfrac{v}{(c+v-1)}$ (6)

where c, λ_2 and λ_4 are constants and the summation is over the design points.

If the above mentioned conditions are satisfied, the variances and covariances of the estimated parameters become,

$$V(\hat{b}_0)=\frac{\lambda_4(c+v-1)\sigma^2}{N\left[\lambda_4(c+v-1)-v\lambda_2^2\right]},$$

$$V(\hat{b}_i)=\frac{\sigma^2}{N\lambda_2},$$

$$V(\hat{b}_{ij})=\frac{\sigma^2}{N\lambda_4},$$

$$V(\hat{b}_{ii})=\frac{\sigma^2}{(c-1)N\lambda_4}\left[\frac{\lambda_4(c+v-2)-(v-1)\lambda_2^2}{\lambda_4(c+v-1)-v\lambda_2^2}\right],$$

$$Cov(\hat{b}_0,\hat{b}_{ii})=\frac{-\lambda_2\sigma^2}{N[\lambda_4(c+v-1)-v\lambda_2^2]},$$

$$Cov(\hat{b}_{ii},\hat{b}_{jj})=\frac{(\lambda_2^2-\lambda_4)\sigma^2}{(c-1)N\lambda_4[\lambda_4(c+v-1)-v\lambda_2^2]} \qquad (7)$$

and other covariances are zero.

III. CONDITIONS FOR MODIFIED SECOND ORDER ROTATABLE DESIGNS

Let $(v,b,r,k_1,k_2,\ldots k_p,\lambda)$, be an equi-replicated PBD and k= sup($k_1,k_2,\ldots, k_p$). Then $2^{t(k)}$ denotes a resolution V fractional factorial of 2^k in ±1 levels, such that no interaction with less than five factors is confounded. n_0 denotes the number of central points in the design. Let $[1-(v,b,r,k_1,k_2,\ldots k_p,\lambda)]$ denotes the design points generated from the transpose of incidence matrix of PBD, $[1-(v,b,r,k_1,k_2,\ldots k_p,\lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from PBD by multiplication (cf. Raghavarao, 1971). $(\beta,0,0,...,0)2^1$ denotes the design points generated from $(\beta,0,0,...,0)$ point set, and U denotes combination of the design points generated from different sets of points. The usual method of construction of SORD is to take combinations with unknown constants, associate a 2^v factorial combinations or a suitable fraction of it with factors each at ± 1 levels to make the level codes equidistant.

All such combinations form a design. Generally, SORD need at least five levels (suitably coded) at $0,\pm 1,\pm\beta$ for all factors ($(0,0,...0)$)- chosen centre of the design, unknown level 'β' are to be chosen suitably to satisfy the conditions of the rotatability) generation of design points this way ensures satisfaction of all the conditions even though the design points contain unknown levels. Alternatively, by putting some restrictions indicating some relation among $\sum x_{iu}^2, \sum x_{iu}^4$ and $\sum x_{iu}^2 x_{ju}^2$ some equations involving the unknowns are obtained and their solution gives the unknown levels. In SORD the restriction used is $\sum x_{iu}^4 = 3\sum x_{iu}^2 x_{ju}^2$, i.e., $c=3$. Other restrictions are also possible through, it seems, not exploited well. Das et al (1999) proposed the restriction $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ i.e., $\lambda_2^2=\lambda_4$ to get another series of symmetrical second order response surface designs, which provide more precise estimates of response at specific points of interest than what is available from the corresponding existing designs. Further, the variances and covariances of the estimated parameters are,

$$V(\hat{b}_0) = \frac{(c+v-1)\sigma^2}{N(c-1)}$$

$$V(\hat{b}_i) = \frac{\sigma^2}{N\sqrt{\lambda_4}}$$

$$V(\hat{b}_{ij}) = \frac{\sigma^2}{N\lambda_4}$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2}{(c-1)N\lambda_4}$$

$$Cov(\hat{b}_0, \hat{b}_{ii}) = \frac{-\sigma^2}{N\sqrt{\lambda_4}(c-1)} \qquad (8)$$

and other covariances are zero. These modifications of the variances and covariances affect the variance of the estimated response at specific points considerably.

Using these variances and covariances, variance of estimated response at any point can be obtained. Let $\hat{y}_u$ denote the estimated response at the point $(x_{1u}, x_{2u}, \ldots x_{vu})$. Then,

$$V(\hat{Y}_u) = V(\hat{b}_0) + d^2[V(\hat{b}_i) + 2cov(\hat{b}_0, \hat{b}_{ii})] + d^4 V(\hat{b}_{ii}) + (\sum x_{iu}^2 x_{ju}^2)[(c-3)\sigma^2/(c-1)N\lambda_4]$$

Construction of modified response surface designs is the same as for SORD except that instead of taking $c=3$ the restriction $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ is to be used and this condition will provide different values of the unknowns involved. (cf. Das et al. 1999).

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IV. CONDITIONS FOR MEASURE OF ROTATABILITY FOR SECOND ORDER RESPONSE SURFACE DESIGNS

Following Box and Hunter (1957), Das and Narasimham (1962), Park et al (1993), conditions (2) to (6) and (7) give the necessary and sufficient conditions for a measure of rotatability for any general second order response surface designs. Further we have,

$V(b_i)$ are equal for i,

$V(b_{ii})$ are equal for i,

$V(b_{ij})$ are equal for i, j, where $i \neq j$,

$$Cov(b_i,b_{ii})=Cov(b_i,b_{ij})=Cov(b_{ii},b_{ij})=Cov(b_{ij},b_{il})=0 \text{ for all } i \neq j, j \neq l, l \neq i. \quad (9)$$

Park et al. (1993) suggested that if the conditions in (2) to (6) together with (7) and (9) are met, then the following measure $(P_v(D))$ given below can be used to assess the degree of rotatability for any general second order response surface design (cf. Park et al., 1993).

$$P_v(D)=\frac{1}{1+R_v(D)}, \quad (10)$$

where

$$R_v(D)=\left[\frac{N}{\sigma^2}\right]^2 \frac{6v\left[V(\hat{b}_{ij})+2\,cov(\hat{b}_{ii},\hat{b}_{jj})-2V(\hat{b}_{ii})\right]^2(v-1)}{(v+2)^2(v+4)(v+6)(v+8)g^8} \quad (11)$$

and g is the scaling factor.

On simplification, numerator of (11), $[V(\hat{b}_{ij})+2\,cov(\hat{b}_{ii},\hat{b}_{jj})-2V(\hat{b}_{ii})]$ using (7) becomes $(c-3)\sigma^2/(c-1)N\lambda_4$. Thus $R_v(D)$ becomes

$$R_v(D)=\left[\frac{N}{\sigma^2}\right]^2\left(\frac{6v[(c-3)\sigma^2]^2(v-1)}{[(c-1)N\lambda_4]^2(v+2)^2(v+4)(v+6)(v+8)g^8}\right) \quad (12)$$

Note: For SORD, we have $c=3$. Substituting the value of 'c' in (12) and on simplification we get $R_v(D)$ is zero. Hence from (10), we get $P_v(D)$ is one if and only if a design is rotatable and less than one then it is nearly rotatable design.

V. MODIFIED ROTATABILITY FOR SECOND ORDER RESPONSE SURFACE DESIGNS USING PBD (CF. VICTORBABU ET AL (2006))

The arrangement of v-treatments in b blocks will be called a PBD of index λ and type $(v, k_1, k_2, \ldots, k_m)$ if each block contains $k_1,k_2,\ldots k_m$ that are all distinct treatments occurs in exactly λ blocks of the design. If b_i is the number of blocks of size k_i = (i = 1, 2,…,m), then $b=\sum_{i=1}^{m} b_i$ and $\lambda v(v-1)=\sum_{i=1}^{m} b_i k_i(k_i-1)$.

Let $(v,b,r,k_1,k_2,\ldots k_p,\lambda)$, be a PBD. Then $2^{t(k)}$ denotes a resolution V fractional factorial of 2^k in ± 1 levels, such that no interaction with less than five factors is confounded. Let $[1-(v,b,r,k_1,k_2,\ldots k_p,\lambda)]$ denotes the design points generated from the transpose of incidence matrix of PBD, $[1-(v,b,r,k_1,k_2,...k_p,\lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from PBD by multiplication (cf. Raghavarao, 1971). Repeat these design points y_1 times. $(\beta,0,0,...,0)2^1$ denotes the design points generated from $(\beta,0,0,...,0)$ point set, and repeat this set of additional design points say y_2 times and n_0 denotes the number of central points.

The design points, $y_1[1\text{-}(v,b,r,k,\lambda)]2^{t(k)}$ U $y_2(\beta,0,0,....0)2^1$ U (n_0) will give a v dimensional modified SORD in $N = \frac{(y_1 r2^{t(k)} + 2y_2\beta^2)^2}{y_1\lambda 2^{t(k)}}$ design points if,

$$\beta^4 = \frac{(3\lambda\text{-}r)y_1 2^{t(k)-1}}{y_2},$$

$$n_0 = \frac{(y_1 r2^{t(k)} + 2y_2\beta^2)^2}{y_1\lambda 2^{t(k)}} - [y_1 b2^{t(k)} + y_2 2v]$$ and n_0 turns out to be an integer.

VI. MEASURE OF ROTATABILITY FOR SECOND ORDER RESPONSE SURFACE DESIGNS USING PBD (CF. VICTORBABU AND SUREKHA (2013))

Let $(v,b,r,k_1,k_2,...k_p,\lambda)$ denote a PBD. For the design points, $y_1[1\text{-}(v,b,r,k,\lambda)]2^{t(k)}$ U $y_2(\beta,0,0,....0)2^1$ U (n_0) will give a measure of rotatability for second order response surface designs using PBD in $N= y_1 b2^{t(k)} + 2vy_2 + n_0$ design points with level 'β' prefixed and $c = \frac{r2^{t(k)}y_1 + 2y_2\beta^4}{\lambda 2^{t(k)} y_1}$.

We can obtain the measure of rotatability values for second order response surface designs using PBD. We have

$$R_v(D) = \left[\frac{(c\text{-}3)}{(c\text{-}1)}\right]^2 \frac{6v(v\text{-}1)}{\lambda_4^2 (v+2)^2 (v+4)(v+6)(v+8) g^8}$$

where

$$g = \begin{cases} \frac{1}{\beta}, & \text{if } \beta < \sqrt{\frac{2^{t(k)-1}(b\text{-}r)y_1}{y_2} + v} \\ \frac{1}{\sqrt{\frac{2^{t(k)-1}(b\text{-}r)y_1}{y_2} + v}}, & \text{if } \beta > \sqrt{\frac{2^{t(k)-1}(b\text{-}r)y_1}{y_2} + v} \end{cases}$$

$$P_v(D) = \frac{1}{1+R_v(D)}$$

If $P_v(D)$ is 1 if and only if the design is rotatable, and it is smaller than one for a non-rotatable designs.

VII.MEASURE OF MODIFIED ROTATABILITY FOR SECOND ORDER RESPONSE SURFACE DESIGNS USING PAIRWISE BALANCED DESIGN

In this section the proposed new method of measure of modified rotatability for second order response surface designs is suggested below:

Let $(v, b, r, k_1, k_2, ...k_p, \lambda)$, be a PBD. $2^{t(k)}$ denotes a resolution V fractional factorial of 2^k in ± 1 levels, such that no interaction with less than five factors is confounded. $[1-(v, b, r, k_1, k_2, ...k_p, \lambda)]$ denotes the design points generated from the transpose of incidence matrix of PBD, $[1-(v, b, r, k_1, k_2, ...k_p, \lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from PBD by multiplication. Repeat these design points y_1 times. Let $(\beta, 0, 0, ..., 0)2^1$ denote the design points generated from $(\beta, 0, 0, ..., 0)$ point set. Repeat this set of additional design points say y_2 times and n_0 is the number of central points.

Consider the design points, $y_1[1\text{-}(v,b,r,k,\lambda)]2^{t(k)}$ U $y_2(\beta,0,0,....0)2^1$ U (n_0) generated from PBD, we have,

$$\sum x_{iu}^2 = y_1 r 2^{t(k)} + y_2 2\beta^2 = N\lambda_2 \qquad (13)$$

$$\sum x_{iu}^4 = y_1 r 2^{t(k)} + y_2 2\beta^4 = cN\lambda_4 \qquad (14)$$

$$\sum x_{iu}^2 x_{ju}^2 = y_1 \lambda 2^{t(k)} = N\lambda_4 \qquad (15)$$

To make the design rotatable, we take $c = 3$. From equations (14) and (15), we have

$$\beta^4 = \frac{y_1(3\lambda\text{-}r)2^{t(k)-1}}{y_2},$$

The modified condition $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ leads to N which is given by

$$N = \frac{(y_1 r 2^{t(k)} + y_2 2\beta^2)^2}{y_1 \lambda 2^{t(k)}}$$ alternatively N may be obtained directly as $y_1 b 2^{t(k)} + y_2 2v + n_0$, where n_0 is given by

$$n_0 = \frac{(y_1 r 2^{t(k)} + 2\beta^2 y_2)^2}{y_1 \lambda 2^{t(k)}} - [y_1 b 2^{t(k)} + 2y_2 v]$$ and n_0 turns out to be an integer. From equations (13) and (15) and on simplification we get $\lambda_2 = \frac{y_1 r 2^{t(k)} + 2y_2\beta^2}{N}$ and $\lambda_4 = \frac{y_1 \lambda 2^{t(k)}}{N}$.

To obtain measure of modified rotatability for second order response surface designs using PBD, we have

$$P_v(D) = \frac{1}{1+R_v(D)}$$

$$R_v(D) = \left[\frac{(c\text{-}3)}{(c\text{-}1)}\right]^2 \frac{6v(v\text{-}1)}{\lambda_4^2 (v+2)^2 (v+4)(v+6)(v+8)g^8},$$

where g is a scaling factor

$$g = \begin{cases} \frac{1}{\beta}, & \text{if } \beta < \sqrt{\frac{y_1 2^{t(k)-1}(b-r)}{y_2} + v} \\ \frac{1}{\sqrt{\frac{y_1 2^{t(k)-1}(b-r)}{y_2} + v}} & \text{otherwise} \end{cases}$$

The following table gives the values of a measure of modified rotatability for second order response surface designs using PBD. It can be verified that $P_v(D)$ is 1 if and only if the design is modified rotatable, and it is smaller than one for nearly modified rotatable designs.

Example: We illustrate the measure of modified rotatability for second order response surface designs for $v=10$ factors with the help of a PBD $(v=10,b=11,r=5,k_1=5,k_2=4,\lambda=2)$ The design points,

$y_1[1\text{-}(v=10,b=11,r=5,k_1=5,k_2=4,\lambda=2)]2^4$ U $y_2(\pm\beta,0,0,....0)2^1$ U (n_0) will give a measure of modified rotatability for second order response surface designs in N=242 design points. From (13), (14) and (15), we have

$$\sum x_{iu}^2 = y_1 80 + y_2 2\beta^2 = N\lambda_2 \tag{16}$$

$$\sum x_{iu}^4 = y_1 80 + y_2 2\beta^4 = cN\lambda_4 \tag{17}$$

$$\sum x_{iu}^2 x_{ju}^2 = y_1 32 = N\lambda_4 \tag{18}$$

From equations (17) and (18) with rotatability value $c=3$, $y_1=1$ and $y_2=2$, we get $a^4=4 \Rightarrow a^2=2 \Rightarrow a=1.414214$. From equations (16) and (18) using the modified condition with $(\lambda_2^2=\lambda_4)$ along with $a^2=2$, $y_1=1$ and $y_2=2$, we get $N=242$, $n_0=26$. For modified SORD we get $P_v(D)=1$ by taking $a=1.414214$ and scaling factor $g=0.7071$. Then the design is modified SORD using PBD.

Instead of taking $a=1.414214$ if we take $a=2.5$ for the above PBD $(v=10,b=11,r=5,k_1=5,k_2=4,\lambda=2)$ from equations (17) and (18), we get $c=7.3828$. The scaling factor $g=0.4$, $R_v(D)=38.2687$ and $P_v(D)=0.0255$. Here $P_v(D)$ becomes smaller it deviates from modified rotatability.

Here we may point out that this measure of modified rotatability for second order response surface designs using PBD for v=9 and v=10 factors has only 242 design points, whereas the corresponding measure of modified rotatability for second order response surface design using BIBD obtained by Jyostna and Victorbabu (2020) need 726 and 441 design points respectively. Thus the new method leads to a 9-factor and 10-factor measure of modified second order response surface designs using PBD in less number of design points than the corresponding measure of modified rotatability for second order response surface designs using BIBD.

Table 1 gives the values of measure of modified rotatability $P_v(D)$ for second order response surface designs using PBD, at different values of 'β' for $9 \le v \le 12$. It can be verified that $P_v(D)$ is 1, if and only if a design 'D' is modified rotatable. $P_v(D)$ becomes smaller as 'D' deviates from a modified rotatable design.

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Table1. Measure of modified rotatability for second order response surface designs using PBD

$(9,11,5,5,4,3,2), N = 242, \beta = 1.414214, y_1 = 1, y_2 = 2, n_0 = 30$				
β	c	g	$R_v(D)$	$P_v(D)$
1.0	2.625	1	3.2802	0.9967
1.3	2.85701	0.76923	0.003	0.997
*1.414214	3	0.70711	0	1
1.6	3.3192	0.625	0.0316	0.9693
1.9	4.129	0.5263	1.3619	0.4234
2.2	5.4282	0.4545	10.1634	0.0896
2.5	7.3828	0.4	44.3147	0.0221
2.8	10.1832	0.3571	142.3824	0.007
3.1	14.044	0.3226	376.5976	0.0026

$(10,11,5,5,4,2), N = 242, \beta = 1.414214, y_1 = 1, y_2 = 2, n_0 = 26$				
β	c	g	$R_v(D)$	$P_v(D)$
1.0	2.625	1	0.0028	0.9972
1.3	2.85701	0.76923	0.0026	0.9974
*1.414214	3	0.70711	0	1
1.6	3.3192	0.625	0.0433	0.9585
1.9	4.129	0.5263	1.1761	0.4595
2.2	5.4282	0.4545	105.3227	0.0094
2.5	7.3828	0.4	38.2687	0.0255
2.8	10.1832	0.3571	122.9573	0.0081
3.1	14.044	0.3226	325.2123	0.0031

$(13,15,7,7,6,5,3), N = 3364, \beta = 2.828427, y_1 = 3, y_2 = 3, n_0 = 406$				
β	c	g	$R_v(D)$	$P_v(D)$
1.0	2.3438	1	0.005	0.995
1.3	2.3631	0.7692	0.0372	0.9641
1.6	2.4016	0.625	0.1638	0.8593
1.9	2.4691	0.5263	0.464	0.6831
2.2	2.5774	0.4545	0.8242	0.5482
2.5	2.7402	0.4	0.7112	0.5844
2.8	2.9736	0.3571	0.2121	0.825
*2.828427	3	0.3536	0	1
3.1	3.2953	0.3226	2.9537	0.2529

$(14,15,7,7,6,3)$, $N = 3364, \beta = 2.828427, y_1 = 3, y_2 = 3, n_0 = 400$				
β	c	g	$R_v(D)$	$P_v(D)$
1.0	2.3438	1	0.0044	0.9956
1.3	2.3631	0.7692	0.0327	0.9683
1.6	2.4016	0.625	0.1438	0.8743
1.9	2.4691	0.5263	0.4075	0.7105
2.2	2.5774	0.4545	0.7238	0.5801
2.5	2.7402	0.4	0.6245	0.6156
2.8	2.9736	0.3571	0.0164	0.9838
*2.828427	3	0.3536	0	1
3.1	3.2953	0.3226	2.5939	0.2783

*indicates modified rotatability value using BIBD. (cf. Victorbabu et al. (2006))

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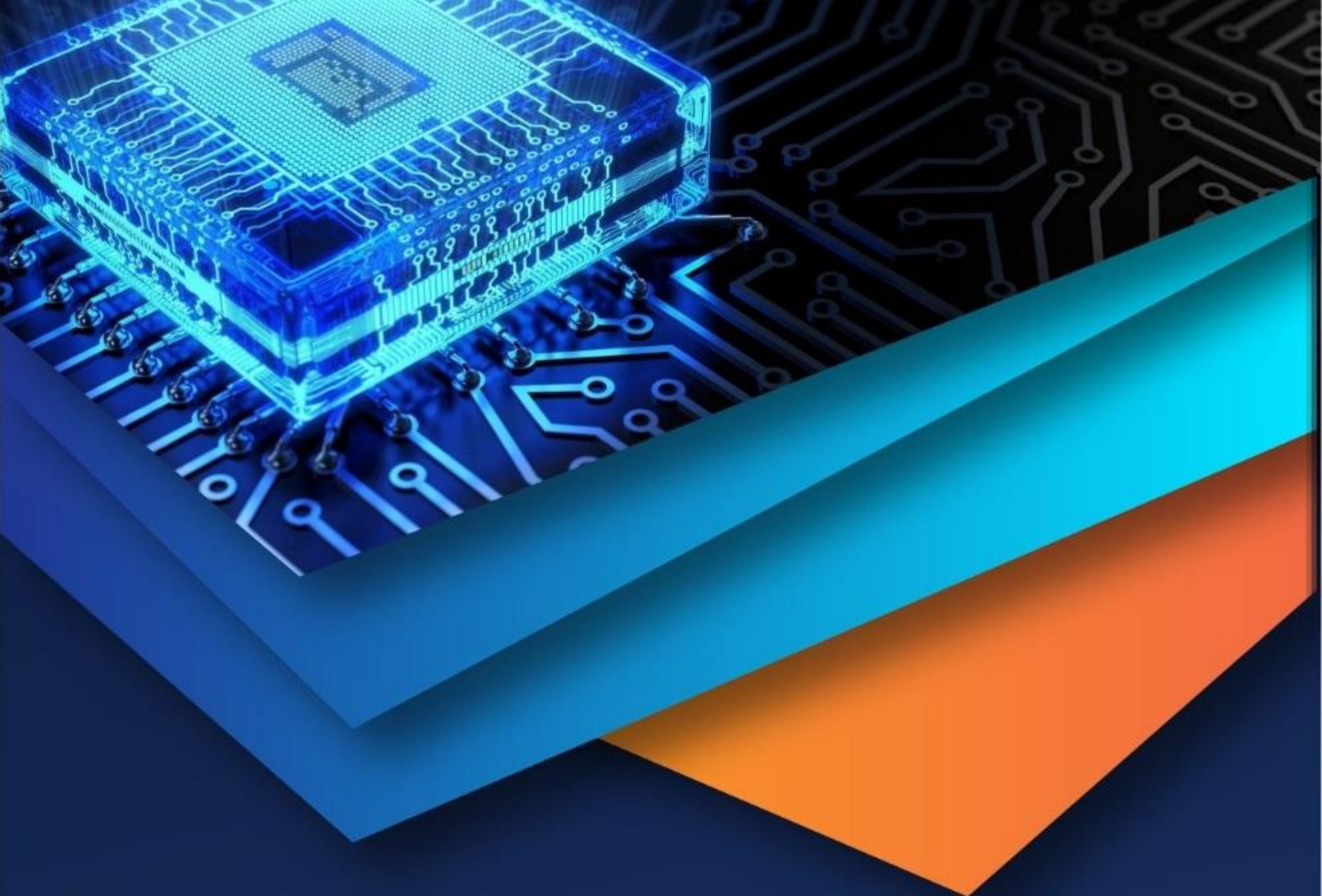

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IMPACT FACTOR:
7.129

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