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On New Functions in Soft Topological Space

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Abstract: Some newly functions like contra almost soft feebly regular irresolute, almost soft F.reg. irresolute are introduced and also further works are soft feebly regular set-connected with separation axioms in it.

Keywords : soft feebly open, soft feebly closed, soft feebly regular open, soft feebly regular closed, soft feebly regular T_1 , weakly soft feebly Hausdrouff, ultra soft feebly regular Hausdrouff space, soft feebly regular T_2 .

I. PRELIMINARIES

- 1) **Definition 1.1 [5]:** Let X be an initial universe set and let E be the set of all possible parameters with respect to X . Let $P(X)$ denote the power set of X . Let A be a nonempty subset of E . A pair (F, A) is called soft set over X , where F is a mapping given by $F: A \rightarrow P(X)$. A soft set (F, A) on the universe X is defined by the set of ordered pairs $(F, A) = \{(x, f_A(x)) : x \in E, f_A(x) \in P(X)\}$ where $f_A: E \rightarrow P(X)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here f_A is called an approximate function of the soft set (F, A) . The collection of soft set (F, A) over a universe X and the parameter set A is a family of soft sets denoted by $SS(x)_A$.
- 2) **Definition 1.2[4]:** A set set (F, A) over X is said to be null soft set denoted by $\emptyset$ if for all $e \in A$, $F(e) = \emptyset$. A soft set (F, A) over X is said to be an absolute soft set denoted by A if all $e \in A$, $F(e) = X$.
- 3) **Definition 1.3[6]:** Let Y be a nonempty subset of X , then Y denotes the soft set (Y, E) over X for which $Y(e) = Y$, for all $e \in E$. In particular, (X, E) will be denoted by X .
- 4) **Definition 1.4 [6]:** Let τ be the collection of soft sets over X , then τ is said to be a soft topology on X if (i) $\emptyset, X \in \tau$ (ii) If $(F, E), (G, E) \in \tau$ then $(F, E) \cap (G, E) \in \tau$ (iii) If $\{(F_i, E)\}_{i \in I} \in \tau$ then $\bigcup_{i \in I} (F_i, E) \in \tau$. The pair (X, τ, E) is called a soft topological space. Every member of τ is called a soft open set. A soft set (F, E) is called soft closed in X if $(F, E)^c \in \tau$.
- 5) **Definition 1.5:** Let (X, τ, E) be a soft topological space over X and let (A, E) be a soft set over X
 - (i) the soft interior[8] of (A, E) is the soft set $\widetilde{int}(A, E) = \bigcup \{(O, E) : (O, E) \text{ which is soft open and } (O, E) \subseteq (A, E)\}$
 - (ii) the soft closure[6] of (A, E) is the soft set $\widetilde{cl}(A, E) = \bigcap \{(F, E) : (F, E) \text{ which is soft closed and } (A, E) \subseteq (F, E)\}$. Clearly $\widetilde{cl}(A, E)$ is the smallest soft closed set over X which contains (A, E) and $\widetilde{int}(A, E)$ is the largest soft open set over X which is contained in (A, E) .
- 6) **Definition 1.6 [3]:** In a soft topological space (X, τ, E) , a soft set
 - a) (A, E) is said to be soft feebly-open set if $(A, E) \subseteq_s \widetilde{cl}(\widetilde{int}(A, E))$.
 - b) (A, E) is said to be soft feebly-closed set if $s\widetilde{int}(\widetilde{cl}(A, E)) \subseteq (A, E)$.

It is said to be soft feebly-clopen if it is both soft feebly-open and soft feebly-closed.
- 7) **Definition 1.7 [3]:** Let (X, τ, E) be a soft topological spaces and let (A, E) be a soft set over X .
 - a) Soft feebly-closure of a soft set (A, E) in X is denoted by $f\widetilde{cl}(A, E) = \bigcap \{(F, E) : (F, E) \text{ which is a soft feebly-closed set and } (A, E) \subseteq (F, E)\}$.
 - b) Soft feebly-interior of a soft set (A, E) in X is denoted by $f\widetilde{int}(A, E) = \bigcup \{(O, E) : (O, E) \text{ which is a soft feebly-open set and } (O, E) \subseteq (A, E)\}$. Clearly $f\widetilde{cl}(A, E)$ is the smallest soft feebly-closed set over X which contains (A, E) and $f\widetilde{int}(A, E)$ is the largest soft feebly-open set over X which is contained in (A, E) .
- 8) **Definition 1.8 ([5],[6],[1],[7]):** For a soft (F, E) over the universe U , the relative complement of (F, E) is denoted by $(F, E)'$ and is defined by $(F, E)' = (F', E)$, where (F', E) , where $F' : E \rightarrow P(U)$ is a mapping defined by $F'(e) = U - F(e)$ for all $e \in E$.
- 9) **Definition 1.9 ([2]):** A subset (A, E) of soft topological space (X, τ, E) is said to be soft feebly regular open (briefly soft F.reg.open) if $(A, E) = f\widetilde{int}(f\widetilde{cl}(A, E))$ where soft feebly interior and soft feebly closure are denoted by $f\widetilde{int}$ and $f\widetilde{cl}$. Here, always soft feebly regular open set is analyzed in the way if (A, E) is both soft feebly open and soft feebly closed.
- 10) **Definition 1.10 ([2]):** A subset (A, E) of soft topological space (X, τ, E) is said to be soft feebly regularly closed if $(A, E) = f\widetilde{cl}(f\widetilde{int}(A, E))$ (briefly soft F.reg.closed).

- 11) *Definition 1.11 ([2]):* A subset (A, E) of soft topological space (X, τ, E) is said to be soft feebly regular clopen if $(A, E) = f\widetilde{int}(f\widetilde{cl}(f\widetilde{int}(A, E)))$. On the other hand, if and only if (A, E) is soft F.reg.open and soft F.reg.closed.
- 12) *Definition 1.12 ([2]):* Let (A, E) be subset of soft topological space (X, τ, E) . The soft feebly regular closure of (A, E) (briefly soft F.reg. $\widetilde{cl}(A, E)$) is the intersection of all soft feebly regular closed set containing (A, E) and the soft feebly regular interior of (A, E) (briefly soft F.reg. $\widetilde{int}(A, E)$) is the union of all soft feebly regular open sets contained in (A, E) . The complement of soft feebly regular open set is soft feebly regular closed.
- 13) *Definition 1.13 ([2]):* A function $f: (X, \tau, E) \rightarrow (Y, \tau, E)$ is said to be soft F.reg.open (resp. soft F.reg.closed) if the image of every soft open set (soft closed set) in X is soft F.reg.open (soft F.reg.closed) in Y .

II. SOFT FEEBLY REGULAR SET-CONNECTED FUNCTION

- 1) *Definition 2.1:* A function $f: (X, \tau, E) \rightarrow (Y, \tau, E)$ is said to be soft set- connected if $f^{-1}(V, E)$ is soft clopen in X for every (V, E) is soft clopen subset of Y .
- 2) *Definition 2.2:* A function $f: (X, \tau, E) \rightarrow (Y, \tau, E)$ is said to be soft feebly regular set- connected (abbr. soft F.reg.set-connected) if $f^{-1}(V, E)$ is soft clopen in X for every (V, E) is soft F.reg.clopen(Y).
- 3) *Theorem 2.3:* Let (X, τ, E) and (Y, τ, E) be soft topological space $f: (X, \tau, E) \rightarrow (Y, \tau, E)$
 - a) f is soft F.reg.set-connected
 - b) $f^{-1}(f\widetilde{int}(f\widetilde{cl}(G, E)))$ is soft clopen for every soft F.reg.open subset (G, E) of Y .
- i) *Proof:*
- ii) (i) $\Rightarrow$ (ii) Let (G, E) be any soft F.reg.open subset of Y . Since $f\widetilde{int}(f\widetilde{cl}(G, E))$ is soft F.reg.open, by (i) it follows that $f^{-1}(f\widetilde{int}(f\widetilde{cl}(G, E)))$ is soft clopen.
- (ii) $\Rightarrow$ (i) Let (V, E) be soft F.reg.open in Y . By (ii) $f^{-1}(f\widetilde{int}(f\widetilde{cl}(V, E)))$ is soft clopen in X and hence f is soft F.reg.set-connected.
- 4) *Remark 2.4*
 - a) A subset (A, E) of soft topological space (X, τ, E) is said to be soft clopen if (A, E) is soft open and soft closed and .
 - b) A soft topological space X is said to be soft clopen T_1 space if for any pair of distinct points x and y , there exists the soft clopen sets (G, E) and (H, E) such that $x \in (G, E)$, $y \notin (G, E)$ and $x \notin (H, E)$, $y \in (H, E)$.
- 5) *Theorem 2.5:* If $f: (X, \tau, E) \rightarrow (Y, \tau, E)$ is soft F.reg.set-connected function and (A, E) is any subset of X , then the restriction $f|(A, E) : (A, E) \rightarrow (Y, \tau, E)$ is soft F.reg.set-connected function.
- a) *Proof:* Let (V, E) be a soft F.reg.open set in Y . By hypothesis $f^{-1}(V, E)$ is soft clopen in X . We have $f^{-1}(V, E) \cap (A, E) = (f|(A, E))^{-1}(V, E)$ is soft clopen in (A, E) . Hence $f|(A, E)$ is soft F.reg.set-connected function.
- 6) *Theorem 2.6:* Let $f: (X, \tau, E) \rightarrow (Y, \tau, E)$ be soft set-connected and $g: (Y, \tau, E) \rightarrow (Z, \tau, E)$ be soft F.reg.set-connected. Then $g \circ f: (X, \tau, E) \rightarrow (Z, \tau, E)$ is soft F.reg.set-connected function.
- a) *Proof:* Let (V, E) be soft F.reg.open in Z . Since g is soft F.reg.set-connected, $g^{-1}(V, E)$ is soft clopen in Y . Since f is soft set-connected, $f^{-1}(g^{-1}(V, E))$ is soft clopen in X . Hence $g \circ f$ is soft F.reg.set-connected.
- 7) *Theorem 2.7:* If $f: (X, \tau, E) \rightarrow (Y, \tau, E)$ is a surjective soft F.reg.open and soft F.reg.closed function and $g: (Y, \tau, E) \rightarrow (Z, \tau, E)$ is a function such that $g \circ f: (X, \tau, E) \rightarrow (Z, \tau, E)$ is soft F.reg.set-connected, then g is soft F.reg.set-connected.
- a) *Proof:* Let (V, E) be soft F.reg.open in Z . $(g \circ f)^{-1}(V, E)$ is soft clopen in X . That is $f^{-1}(g^{-1}(V, E))$ is soft clopen in X . Since f is surjective soft F.reg.open and soft F.reg.closed, $f(f^{-1}(g^{-1}(V, E))) = g^{-1}(V, E)$ is soft clopen. Therefore g is soft F.reg.set-connected.
- 8) *Definition 2.8:* A function $f: (X, \tau, E) \rightarrow (Y, \tau, E)$ is said to be
 - a) Soft F.reg.irresolute if the inverse image of every soft F.reg.open set in Y is soft F.reg.open in X .
 - b) Contra soft F.reg.irresolute if the inverse image of every soft F.reg.open set in Y is soft F.reg.closed in X .
 - c) Almost soft F.reg.irresolute if the inverse image of soft F.reg.open in Y is soft feebly open set in X .
 - d) Contra almost soft F.reg.irresolute continuous if the inverse image soft F.reg.open in Y is soft feebly closed in X .
- 9) *Theorem 2.9:* Every contra soft F.reg.irresolute is contra almost soft F.reg.irresolute function.
- a) *Proof:* Suppose $f: (X, \tau, E) \rightarrow (Y, \tau, E)$ is a contra soft F.reg.irresolute function and (A, E) be any soft F.reg.open set in Y then $f^{-1}(A, E)$ is soft F.reg.closed in X . Thus the inverse image of each soft F.reg.open set in Y is soft feebly closed in X . Therefore f is contra almost soft F.reg. irresolute function.

10) *Theorem 2.10:* The followings are equivalent for a function $f: (X, \tau, E) \rightarrow (Y, \tau, E)$

- F is contra almost soft F .reg.irresolute for every soft F .reg.closed set (F, E) of Y , $f^{-1}(F, E)$ is soft feebly open set of X .
 - For each $x \in X$ and each soft F .reg.closed set (F, E) of Y containing $f(x)$, there exists soft feebly open set (U, E) containing x such that $f(U, E) \subseteq (F, E)$.
 - For each $x \in X$ and each soft F .reg.open set (V, E) of Y not containing $f(x)$, there exists feebly closed set K not containing x such that $f^{-1}(V, E) \subseteq (K, E)$.
- i) *Proof:* (1) $\Rightarrow$ (2): Let (F, E) be a soft F .reg.closed set in Y , then $Y - (F, E)$ is a soft F .reg.open set in Y . By (1), $f^{-1}(Y - (F, E)) = X - f^{-1}(F, E)$ is soft feebly closed set in X . This implies $f^{-1}(F, E)$ is soft feebly open set in X .
- (2) $\Rightarrow$ (1): Let (G, E) be a soft F .reg.open set of Y , then $Y - (G, E)$ is a soft F .reg.closed set in Y . By (2), $f^{-1}(Y - (G, E))$ is soft feebly open set in X . This implies $X - f^{-1}(G, E)$ is soft feebly open set in X , which implies $f^{-1}(G, E)$ is feebly closed set in X . Therefore, (1) holds.
- (2) $\Rightarrow$ (3): Let (F, E) be a soft F .reg.closed set in Y containing $f(x)$, which implies $x \in f^{-1}(F, E)$. By (2), $f^{-1}(F, E)$ is soft feebly open in X containing x . Set $(U, E) = f^{-1}(F, E)$, which implies (U, E) is feebly open in X containing x and $f(U, E) = f(f^{-1}(F, E)) \subseteq (F, E)$. Therefore (3) holds.
- (3) $\Rightarrow$ (2): Let (F, E) be a soft F .reg.closed set in Y containing $f(x)$, which implies $x \in f^{-1}(F, E)$. From (3), there exists feebly open $(U, E)_x$ in X containing x such that $f((U, E)_x) \subseteq (F, E)$. That is $(U, E)_x \subseteq f^{-1}(F, E)$. Thus $f^{-1}(F, E) = \bigcup \{(U, E)_x : x \in f^{-1}(F, E)\}$, which is the union of soft feebly open sets. Therefore, $f^{-1}(F, E)$ is soft feebly open set of X .
- (3) $\Rightarrow$ (4): Let (V, E) be a soft F .reg.open set in Y not containing $f(x)$. Then $Y - (V, E)$ is a soft F .reg.closed set in Y containing $f(x)$. From (3), there exists a feebly open set (U, E) in X containing x such that $f(U, E) \subseteq Y - (V, E)$. This implies $(U, E) \subseteq f^{-1}(Y - (V, E)) = X - f^{-1}(V, E)$.
- (4) $\Rightarrow$ (3): Let (F, E) be a soft F .reg.closed set in Y containing $f(x)$. Then $Y - (F, E)$ is a soft F .reg.open set in Y not containing $f(x)$. From (4), there exists soft feebly closed set (K, E) in X not containing x such that $f^{-1}(Y - (F, E)) \subseteq (K, E)$. This implies $X - f^{-1}(F, E) \subseteq (K, E)$. Hence, $X - (K, E) \subseteq f^{-1}(F, E)$, that is $f(X - (K, E)) \subseteq (F, E)$. Set $(U, E) = X - (K, E)$, then (U, E) is soft feebly open set containing x in X such that $f(U, E) \subseteq (F, E)$.

11) *Theorem 2.11:* The following are equivalent for a function $f: (X, \tau, E) \rightarrow (Y, \tau, E)$ f is contra almost soft F .reg.irresolute.

- $f^{-1}(\text{soft } F\text{.reg.}\widetilde{\text{int}}(\text{soft } F\text{.reg.}\widetilde{\text{cl}}(G, E)))$ is soft feebly closed set in X for every soft F .reg.opensubset (G, E) of Y .
 - $f^{-1}(\text{soft } F\text{.reg.}\widetilde{\text{cl}}(F\text{.reg.}\widetilde{\text{int}}(F, E)))$ is soft feebly open set in X for every soft F .reg.closedsupset (F, E) of Y .
- i) *Proof:* (1) $\Rightarrow$ (2): Let (G, E) be a soft F .reg.open set in Y . Then soft F .reg. $\widetilde{\text{int}}(\text{soft } F\text{.reg.}\widetilde{\text{cl}}(G, E))$ is soft F .reg.open set in Y . By (1) $f^{-1}(\text{soft } F\text{.reg.}\widetilde{\text{int}}(\text{soft } F\text{.reg.}\widetilde{\text{cl}}(G, E)))$ belongs to soft feebly closed set of X .
- (2) $\Rightarrow$ (1): Obvious.
- (1) $\Rightarrow$ (3): Let (F, E) be a soft F .reg.closed in Y . Then soft F .reg. $\widetilde{\text{cl}}(\text{soft } F\text{.reg.}\widetilde{\text{int}}(G, E))$ is soft F .reg.closed set in Y . By (1), $f^{-1}(\text{soft } F\text{.reg.}\widetilde{\text{cl}}(\text{soft } F\text{.reg.}\widetilde{\text{int}}(G, E)))$ belongs to soft feebly open set of X .
- (3) $\Rightarrow$ (1): Obvious.

III. SEPARATION AXIOMS IN SOFT TOPOLOGICAL SPACE

- Definition 3.1:* A soft topological space (X, τ, E) is said to be soft feebly regular T_1 (briefly soft F .reg. T_1) space if for any pair of distinct points x and y , there exists the soft F .reg.open sets (G, E) and (H, E) such that $x \in (G, E)$, $y \notin (G, E)$ and $x \notin (H, E)$, $y \in (H, E)$.
- Definition 3.2:* A space X is said to be weakly soft feebly Hausdorff if each elements of X is an intersection of soft F .reg.closed sets.
- Definition 3.3:* A soft topological space X is called ultra soft F .reg.Hausdorff space, if for every pair of disjoint points x and y in X , there exist disjoint soft F .reg.clopen sets (U, E) and (V, E) in X containing x and y , respectively.
- Definition 3.4:* A soft topological space X is said to be soft feebly regular T_2 (briefly soft F .reg. T_2) space if for any pair of disjoint points x and y , there exists disjoint soft F .reg.open sets (G, E) and (H, E) such that $x \in (G, E)$ and $y \in (H, E)$.
- Theorem 3.5:* If $f: (X, \tau, E) \rightarrow (Y, \tau, E)$ is a contra almost soft F .reg. irresolute injection and Y is weakly soft feebly Hausdorff then X is soft F .reg. T_1 .

- a) *Proof:* Suppose Y is weakly soft feebly Hausdorff. For any distinct points x and y in X , there exist soft F.reg.closed sets (V, E) and (W, E) in Y such that $f(x) \in (V, E)$, $f(y) \notin (V, E)$, $f(y) \in (W, E)$ and $f(x) \notin (W, E)$. Since f is contra almost soft F.reg.irresolute, $f^{-1}(V, E)$ and $f^{-1}(W, E)$ are soft F.reg.open subsets of X such that $x \in f^{-1}(V, E)$, $y \notin f^{-1}(V, E)$, $y \in f^{-1}(W, E)$ and $x \notin f^{-1}(W, E)$. This shows that X is soft F.reg. T_1 .
- 6) *Theorem 3.6:* If $f: (X, \tau, E) \rightarrow (Y, \tau, E)$ is a soft F.reg.set-connected injection and Y is soft F.reg. T_1 , then X is soft clopen T_1 .
- a) *Proof:* Since Y is soft F.reg. T_1 for any disjoint points x and y in X , there exist (V, E) , (W, E) are soft F.reg.open(Y) such that $f(x) \in (V, E)$, $f(y) \notin (V, E)$, $f(x) \notin (W, E)$, $f(y) \in (W, E)$. Since f is soft F.reg.set-connected, $f^{-1}(V, E)$ and $f^{-1}(W, E)$ are soft clopen in X . Furthermore $y \notin f^{-1}(V, E)$ and $x \notin f^{-1}(W, E)$. This shows that X is soft clopen T_1 .
- 7) *Theorem 3.8:* If $f: (X, \tau, E) \rightarrow (Y, \tau, E)$ and $g: (X, \tau, E) \rightarrow (Y, \tau, E)$ be soft F.reg. set-connected function and Y is soft F.reg. Hausdorff, then $(F, E) = \{x \in X : f(x) = g(x)\}$ is soft F.reg.closed in X .
- a) *Proof:* If $x \in X - (F, E)$ then it follows that $f(x) \neq g(x)$. Since Y is soft F.reg.Hausdorff, there exist soft F.reg.open sets (V, E) and (W, E) such that $f(x) \in (V, E)$, $g(x) \in (W, E)$ and $(V, E) \cap (W, E) \neq \emptyset$. Since f and g are soft F.reg.set-connected, $f^{-1}(f \widetilde{\text{int}}(f \widetilde{\text{cl}}(V, E)))$ and $g^{-1}(f \widetilde{\text{int}}(f \widetilde{\text{cl}}(W, E)))$ are soft clopen in X with $x \in f^{-1}(f \widetilde{\text{int}}(f \widetilde{\text{cl}}(V, E)))$ and $x \in g^{-1}(f \widetilde{\text{int}}(f \widetilde{\text{cl}}(W, E)))$.
- 8) *Theorem 3.9:* Iff $f: (X, \tau, E) \rightarrow (Y, \tau, E)$ is a contra almost soft F.reg. irresolute injective function and Y is ultra soft F.reg. Hausdorff space, then X is soft F.reg. T_2 .
- a) *Proof:* Let x and y be any two distinct points in X . Since f is injective $f(x) \neq f(y)$ and Y is ultra soft F.reg.Hausdorff space, there exist disjoint soft F.reg.clopen sets (U, E) and (V, E) of Y containing $f(x)$ and $f(y)$, respectively. Then $x \in f^{-1}(U, E)$ and $y \in f^{-1}(V, E)$, where $f^{-1}(U, E)$ and $f^{-1}(V, E)$ are disjoint soft feebly open sets in X . Therefore X is soft F.reg. T_2 .

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