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A Non-Newtonian Power Law Model for Two Phase Cerebral Blood Flow in Human Artery during Bacterial Meningitis

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Abstract: In this paper we discuss the Power law model during bacterial meningitis. Clinical evaluation of data collected in a hospital is also presented graphically. This study aims at providing the considerable role of red blood cells in two phase blood flow. The role of hematocrit is explicit in bacterial meningitis. The solution technique adopted is analytical as well as numerical. The clinical data based on empirical ground is analyzed with the help of mathematical interpretation.

Keyword: Power law model, bacterial meningitis, hematocrit.

I. INTRODUCTION

Brain blood flow is the movement of blood through the network of cerebral arteries and veins supplying the brain. Large arteries that have thick walls or within the microcirculation within which flow is non-Newtonian. The rate of cerebral blood flow within the adult is usually 750 milliliters per minute, representing 15-20% of the flow rate. This equates to an average of 50 to 54 milliliters of blood per a hundred grams of brain tissue per minute. CBF is tightly regulated to fulfill the brain's metabolic demands. The arteries deliver oxygenated blood, glucose, and alternative nutrients to the brain, and also the veins carry deoxygenated blood back to the heart, removing carbon dioxide, lactic compound, and different metabolic products. Since the brain is incredibly at risk of compromises in its blood supply, the cerebral vascular system has several safeguards as well as the biological process of the blood vessels and also the failure of this safeguard may end up intend and another disease like bacterial meningitis. The amount of blood that the cerebral circulation carries is known as cerebral blood flow (CBF).

A. Blood Flow Distribution In Cerebral Arteries

The major artery carrying recently aerated blood away from the center is that the arterial blood vessel. The terribly first branches off the arterial blood vessel provide the center with nutrients and oxygen. Succeeding branches produce to the carotid arteries, that additional branch into the interior arterial arteries. The external carotid artery arteries provide blood to the tissues on the surface of the brainpan. The bases of the carotid artery contain stretch receptors that now reply to the visiting force per unit area upon standing. The orthostatic reflex could be a reaction to the present modification in body position, in order that force per unit area is maintained against the increasing impact of gravity.

Figure1. Structure and function of cerebral arteries

The internal artery enters the cranium through the carotid canal within the bone. The second set of vessels that offer the system square measure the bone arteries, that square measure protected they meet up with the neck region by the traverse foramina of the cervical vertebrae. The bone arteries enter the cranium through the foramen of the membrane bone. Branches off the left and right bone arteries merge into the anterior spinal artery supply the anterior facet of the spinal cord, found on the anterior median fissure. The two bone arteries then merge into the aorta, which supplies rise to branches to the brain stem and neural structure. The left and right internal arteria arteries and branches to the aorta all become the circle of Wills, a confluence of arteries that may maintain the introduction of the brain even though narrowing or an oversized limits flow through one half. Nearly four hundred years past, Thomas Wills describes the blood vessel ring at the bottom of the brain (the circle of Wills, CW) and recognized it as an offsetting system within the case of blood vessel occlusion. This theory remains accepted.

Understanding the balance of the blood flow to the brain, i.e., its distribution and factors affecting the blood flow rate (BFR), can give important new information to be used for evaluating patients with cerebral vascular disease. Although total cerebral blood flow (tCBF) has been reported to be 616 to 781 ml/min in healthy subjects, very few have investigated BFR in the cerebral vascular tree; i.e. the BFR in the circle of Wills(CW) and in more peripheral cerebral arteries. Until now the most commonly used method to study BFR in arteries in clinical practice and for research proposes has been color duplex and Doppler ultrasonography. This method cannot give information about BFR in distal cerebral arteries. However, phase-contrast magnetic resonance imaging (PCMRI) enables measurement of BFR in milliliters per minute (ml/min) and is currently the only method that can measure BFR within cerebral and extra cerebral arteries down to a diameter ~1.5 mm.

B. Constitution of Blood

Human blood is composed of blood cells consists blood plasma. Plasma, that constitutes 55% of blood fluid, is mostly water 92% by volume and contains dissipated proteins, glucose, mineral, hormones and blood cells themselves.

Figure2. Constitution of blood

The blood cell is especially red blood cells (also referred to as RBCs or erythrocytes) and white blood cells, including leukocytes and platelets. The average volume of an erythrocyte represents approximately 40 to 45% and more than 99% of all blood cells. The volume percentage (vol%) of red blood cells (RBC) in the blood is called Hematocrit. The hematocrit of men average about 42, while that of women averages about 38. The red blood cells are semisolid particles, increase the velocity of blood and can have an effect on the behavior of a fluid. The viscosity of normal blood is about three times as great as the viscosity of water. It has been pointed out that plasma behaves Newtonian fluid wearers whole blood show non-Newtonian character.

Platelets are a vital component of the blood clotting mechanism. The entire volume concentration of leukocytes and platelets only concerned with 1% (N. Bessonve et al., 2016). Then we have considered only two phases of blood. Which is one of the red blood cells and another phase is plasma.

C. Description Of Bacterial Meningitis

Meningitis is an inflammation of the meninges, the covering of the brain and spinal cord. It is most often caused by infection (bacterial, viral or fungal), but can also be produced by chemical irritation, subarachnoid hemorrhage, cancer and other condition [8]. Bacterial meningitis is common in children and old age people. Additionally, people with a weak immune system are more at risk for bacterial meningitis.

This disease mostly spreads in communities/societies that living in crowded areas close quarters (e.g. police staff, police cells, college students, military staff, hostels and prisons).

The symptom associated with bacterial meningitis can be fever, headache, body aches, fatigue & sleepiness. Later symptoms that may occur are nausea, vomiting, confusion, stiff neck, and sensitivity of light.

In babies, the symptoms include fever, fussiness, refusal to eat, difficulty waking up, and swelling of the soft spot on the baby's head. Infection from bacterial meningitis can cause permanent disabilities such as brain damage, hearing loss, and learning disabilities. The primary test for meningitis is a lumbar puncture. Bacterial meningitis need shut observation within the hospital and treatment with medicine. Additionally, ventilator assistant, kidney dialysis or different supportive treatment could also be required.

II. REAL MODEL

A. Choice of Frame of Reference

In mathematical modeling selection of choice frame of reference depend on the difficulty of blood flow. Here we discussed three dimensional 3-dim Euclidean spaces which are denoted by E^3 . Blood flow related qualities are interpreted in the tensorial form more realistic and comparatively.

The biophysical laws thus expressed fully hold good in any coordinate system, which is a compulsion for the truthfulness of the law now, let the co-ordinate axes be OX^i where O is origin and superscript $i=1,2,3$ let X^i is the co-ordinate of any point P in space. The mathematical description of the state if moving blood is effected by mean of function which gives the distribution of the blood velocity $V^k = V^k(X^i, t)$, $k=1,2,3$ and of pertaining of the blood any two thermodynamic quantities.

B. Two Phase Description

Blood is a complex fluid consisting of particulate corpuscles suspended in a non-Newtonian fluid. The particulate solids are red blood cells (RBCs) White blood cells (WBCs) and platelets. 55% of the plasma and 45% of the blood cells in whole blood and approximately 99% of RBCs in 45% of blood cells and there are few parts (<1%) of other cells. Which are ignorable, so one phase of the blood plasma and another phase of blood is RBCs.

C. Constitute Equations

Generally, blood is a non-homogeneous mixture of plasma and blood cells. Through for practical purpose, it may be considered to be a homogeneous two-phase mixture of plasma and blood cells, the constitutive equation proposed for whole blood mixture is as follows.

$$T^{ij} = -pg^{ij} + \eta_m (e^{ij})^n = -pg^{ij} + T'^{ij} \quad (3.1)$$

Where T^{ij} is stress tensor and T'^{ij} is shearing stress tensor.

1) Newtonian Power Law Equation

$\tau = \eta e^n$ When $n=1$ then the nature of the fluid is Newtonian^[12]

2) The non-Newtonian Power Law Equation

$$\tau = \eta e^n$$

This is found to be comfortable for strain rate 5 to 200 per second

Where η is the velocity coefficient. This is found to hold good in blood flow vessels where there is low hematocrit. It holds well when blood shows yield stress. We notice that the yield stress arises because of blood cells from aggregate in the form of rouleaux at a low strain rate.

D. Boundary Conditions are as follows

- 1) The velocity of blood flow on the axis of arteries at $r=0$ will be maximum and finite, say V^0 =maximum velocity, $V=V^0$ then $A=0$
- 2) The velocity of the blood flow on the wall of a cerebral artery at $r=R$, where R is the radius of the cerebral artery, will be zero. This condition is well known as a no-slip condition, $V=0$ at $r=R$

E. Mathematical Modeling/Formulation

The equation of continuity for power law flow will be as follows:

$$1/\sqrt{g}(\sqrt{g} v^i)_{,i} = 0 \quad (3.2)$$

Again the equation of motion is extended as follows:

$$\rho_m \frac{\partial v^i}{\partial t} + \rho_m v^i v_{,j}^j = T_j^{ij} \quad (3.3)$$

Where T_j^{ij} is taken from the constitutive equation of power law flow (3.1). $\rho_m = X\rho_c + (1-X)\rho_p$ Density of blood and $\eta_m = X\eta_c + (1-X)\rho_p$ is the viscosity of a mixture of blood.

$X = Ht/100$ is a volume ratio of blood cells. Ht is a hematocrit. Other symbols have their usual meanings.

Since the blood vessels are cylindrical, the above governing equations have to be transformed into cylindrical coordinates. As we know earlier:

$$x^1 = r, \quad x^2 = \theta, \quad x^3 = z,$$

Matrix of the metric tensor in cylindrical coordinates is as follows:

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

While the matrix of the conjugate metric tensor is as follows:

$$[g^{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Whereas the Christoffel's symbols of 2nd kind are as follows:

$$\left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\} = -r, \quad \left\{ \begin{matrix} 2 \\ 2 \end{matrix} \right\} = \left\{ \begin{matrix} 2 \\ 1 \end{matrix} \right\} = \frac{1}{r}, \text{ remaining other is zero.}$$

The Relation between contravariant and physical components of the velocity of cerebral blood flow will be as follows:

$$\sqrt{g_{11}} v^1 = v_r \Rightarrow v_r = v^1$$

$$\sqrt{g_{22}} v^2 = v_\theta \Rightarrow v_\theta = r v^2$$

$$\text{And } \sqrt{g_{33}} v^3 = v_z \Rightarrow v_z = v^3$$

Again the physical components of $-p_j g^{ij}$ are $-\sqrt{g_{ii}} p_j g^{ij}$

The matrix of physical components of shearing stress-tensor $T'^{ij} = \eta_m (e^{ij})^n = \eta_m (g^{ik} v_{,k}^i + g^{jk} v_{,k}^j)^n$ will be as follows:

$$\begin{bmatrix} 0 & 0 & \eta_m (dv/dr)^n \\ 0 & 0 & 0 \\ \eta_m (dv/dr)^n & 0 & 0 \end{bmatrix}$$

The covariant derivative of T'^{ij} is

$$T_j'^{ij} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} (\sqrt{g} T'^{ij}) + \left\{ \begin{matrix} i \\ j \end{matrix} \right\} T'^{kj}$$

Keeping in view the above facts, the governing Tensorial equation can be transformed into a cylindrical form which is as follows:

1) Equation of Continuity

$$\frac{\partial v}{\partial z} = 0 \quad (3.4)$$

2) Equation of Motion

a) r - component

$$-\frac{\partial p}{\partial r} = 0 \quad (3.5)$$

b) θ – component

$$0=0 \quad (3.6)$$

c) z - component

$$0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r} \frac{\partial}{\partial r} \left[r \left(\frac{\partial v_z}{\partial r} \right)^n \right] \quad (3.7)$$

Here this fact has been taken in view that the blood flow is axially symmetric in arteries concerned, i.e. $v_\theta = 0$, and v_r , v_z and p do not depend upon θ . also the blood flows steadily, i.e.

$$\frac{\partial p}{\partial t} = \frac{\partial v_r}{\partial t} = \frac{\partial v_\theta}{\partial t} = \frac{\partial v_z}{\partial t} = 0$$

Solution: On integrating equation (3.4) we get

$$v_z = v(r) \text{ Because } v \text{ does not depend upon } \theta. \quad (3.8)$$

The integration of the equation of motion (3.5) yields:

$$P=p(z), \text{ since } p \text{ does not depend upon } \theta. \quad (3.9)$$

Now, with the help of equations (3.8) and (3.9), the equation (3.7) converts in the following form:

$$0 = -\frac{dp}{dz} + \frac{\eta_m}{r} \frac{d}{dr} \left[r \left(\frac{dv}{dr} \right)^n \right] \quad (3.10)$$

The pressure gradient $-(dp/dz) = p$ of blood flow in the arteries remote from heart can be supposed to be constant and hence the equation (3.10) takes the following form:

$$\frac{d}{dr} \left[r \left(\frac{dv}{dr} \right)^n \right] = -\frac{pr}{\eta_m} \quad (3.11)$$

On integrating the equation (3.11), we get

$$r \left(\frac{dv}{dr} \right)^n = -\frac{pr^2}{2\eta_m} + A \quad (3.12)$$

We know that the velocity of blood flow on the axis of cylindrical arteries is maximum and constant. So that we apply the boundary condition: at $r = 0, v = V_0$ (constant), on equation (3.12) to get the arbitrary constant $A = 0$. Hence the equation (3.12) takes the following form

$$r \left(\frac{dv}{dr} \right)^n = -\frac{pr^2}{2\eta_m} \Rightarrow -\frac{dv}{dr} = \left(\frac{pr}{2\eta_m} \right)^{1/n} \quad (3.13)$$

The equation (3.13) is integrating once again, we get

$$v = -\left(\frac{p}{2\eta_m} \right)^{1/n} \frac{r^{\frac{1}{n}+1}}{(n+1)/n} + B \quad (3.14)$$

To determine the arbitrary constant B, we apply the no-slip condition on the inner wall of the arteries: at $r = R, v = 0$, where R = radius of vessels, on equation (3.14) so as to get

$$B = \left(\frac{p}{2\eta_m} \right)^{1/n} \frac{R^{\frac{1}{n}+1}}{(n+1)}$$

Hence the equation (3.14) takes the following from:

$$v = \left(\frac{p}{2\eta_m} \right)^{1/n} \frac{n}{(n+1)} \left(R^{\frac{1}{n}+1} - r^{\frac{1}{n}+1} \right) \quad (3.15)$$

Equation 3.15 determines the velocity of blood flow in the arteries remote from the heart where P is a gradient of blood pressure and η_m is the viscosity of blood mixture.

III. RESULT AND DISCUSSION

Table1. Clinical Data of cerebral artery during Bacterial Meningitis.
Blood pressure and hemoglobin during bacterial meningitis, 2018-19

S. No.	Date DDMMYY	Hemoglobin(HB) gm/dl	Hematocrit(3×HB) kg/m ³	Blood pressure mm Hg	Artery Pressure Drop $\left(\frac{S+D}{2}\right) - S$ mm Hg
1.	220119	11.7	35.1	100/70	-15
2.	250119	10.6	31.8	110/60	-25
3.	280119	09.8	29.4	140/60	-40
4.	310119	10.1	30.3	100/70	-15
5.	040219	10.3	30.9	110/80	-15
6.	110219	10.3	33.9	120/70	-25
7.	180219	10.5	31.5	100/60	-20

Source: Sanjay Gandhi Medical and Hospital Rewa (M.P.)

1) *Observation:* Hematocrit v/s blood pressure drop during bacterial meningitis.

Clinical data was collected when the patient Jairam Kol age of 30 years was suffering from bacterial meningitis. When the patient admitted to hospital he was apathy healthy till two days and symptoms are seen in the patient mainly headaches and vomiting, and their BP 100/70 and hemoglobin is 11.7.

The flow flux of two phase blood flow in arteries is-

$$\begin{aligned}
 F &= \int_0^R v \cdot 2\pi r \, dr = \int_0^R \left(\frac{p}{2\eta_m} \right)^{1/n} \frac{n}{(n+1)} \left(R^{\frac{1}{n}+1} - r^{\frac{1}{n}+1} \right) 2\pi r \, dr \\
 &= \left(\frac{p}{2\eta_m} \right)^{1/n} \frac{n \cdot 2\pi}{(n+1)} \left(\frac{R^{\frac{1}{n}+3}}{2} - \frac{n r^{\frac{1}{n}+1}}{3n+1} \right) \Bigg|_0^R \\
 &= \left(\frac{p}{2\eta_m} \right)^{1/n} \frac{n \cdot 2\pi}{(n+1)} \frac{(n+1) R^{\frac{1}{n}+3}}{2(3n+1)} \\
 &= \left(\frac{p}{2\eta_m} \right)^{1/n} \frac{\pi n R^{\frac{1}{n}+3}}{(3n+1)}
 \end{aligned} \tag{3.16}$$

Hematocrit (Ht) = 30.9

$\eta_m = 0.0039 \text{ Pa.s}^{[3,4]}$

$\eta_p = 0.00149 \text{ Pa.s}^{[4]}$

We know that

$\eta_m = \eta_c X + \eta_p (1 - X)$, where $X = \text{Ht}/100$ (3.17)

$0.0039 = \eta_c (0.309) + 0.00149 (1 - 0.309) \Rightarrow \eta_c = 0.012288159 \text{ Pa.s}$

Again using the relation and change in to the hematocrit

$\eta_m = \eta_c X + \eta_p (1 - X) \Rightarrow \eta_m = 0.000122882 \text{ Ht} + 0.00102959$

From equation (3.16)

$P = -dp/dz$, we get

$$F = \left(\frac{\Delta P}{2\eta_m \Delta Z} \right)^{1/n} \frac{\pi n R^{\frac{1}{n}+3}}{3n+1} \tag{3.18}$$

$F = 698 \text{ ml/m} = 0.011633 \text{ m}^3/\text{s}^{[15]}$

Length of cerebral artery $\Delta Z = 9.0 \text{ cm} = 0.09 \text{ m}^{[6]}$

Radius of cerebral artery $R = 2.55 \text{ mm} = 0.00255 \text{ m}^{[6]}$

Pressure drop $\Delta P = \left(\frac{S_p + D_p}{2} \right) - S_p = 3328.30 \text{ Pa.s}$

Put the value of ΔP , ΔZ and R in equation (3.18)

$$0.011633 = \left(\frac{3328.30}{2 \times 0.0039 \times 0.09} \right)^{\frac{1}{n}} \frac{n \times 3.14 \times (0.00255)^{\frac{1}{n}+3}}{3n+1}$$

$$0.011633 = (4741168.09)^{\frac{1}{n}} \left(\frac{n}{3n+1} \right) 3.14 \times (0.00255)^{\frac{1}{n}+3}$$

By using trial and error method we get the value of n

$$n = 0.706$$

Again using from equation (3.18)

$$F = \left(\frac{\Delta P}{2\eta_m \Delta Z} \right)^{1/n} \frac{\pi n R^{\frac{1}{n}+3}}{3n+1}$$

$$\left(\frac{\Delta P}{2\eta_m \Delta Z} \right)^{1/n} = F \frac{3n+1}{\pi n R^{\frac{1}{n}+3}}$$

$$\left(\frac{\Delta P}{2\eta_m \Delta Z} \right) = F^n \frac{(3n+1)^n}{(\pi n)^n R^{1+3n}}$$

$$\Delta P = \eta_m 2\Delta Z F^n \frac{(3n+1)^n}{(\pi n)^n R^{1+3n}}$$

$$\Delta P = \eta_m (0.09) (0.011633)^{0.706} \frac{[3(0.706) + 1]^{0.706}}{[3.14(0.706)]^{0.706} 0.00255^{(1+3(0.706))}}$$

$$\Delta P = \eta_m 0.09 \times 0.025 \frac{(3.118)^{0.706}}{(0.26)^{0.706} 0.00255^{(3.49)}}$$

$$\Delta P = \eta_m 0.00255 \frac{2.82}{0.3267 \times 0.00000000088}$$

$$\Delta P = \eta_m 0.00255 \frac{2.82}{2.87496E-10}$$

$$\Delta P = \eta_m 2512521.91$$

$$\Delta P = (0.000122882 \text{Ht} + 0.00102959) \times 2512521.91$$

Table2. Hematocrit (Ht) v/s Blood Pressure Drop (BPD)

Date	220119	250119	280119	310119	040219	110219	180219
Ht (kg/m ³)	35.1	31.8	29.4	30.3	30.9	33.9	31.5
BPD (Pa.s)	13423.77	12404.91	11633.93	11941.80	12127.04	13053.27	12312.29

Figure3. (Bio-physical interpretation of clinical data of cerebral artery in case of bacterial meningitis)

IV. CONCLUSION

Figure3 (Table1, 2) Shows from 22/01/19 to 28/01/19 shows when hematocrit is decrease blood pressure drop is also decreased other is that of increased.

In this piece of research, we've concluded that when the nature of the graph is increasing reference to hematocrit v/s blood pressure drop then we cannot suggest a high dose of anesthesia other is that of, we prefer a dose of anesthesia as per demand. According to the steepness of slopes in figure3 at different conditions, suggested for successful operation when clinical data was collected from duration of the declared operation.

V. ACKNOWLEDGEMENT

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