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Some New Graphs on k-Super Mean Labeling

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Abstract: Let G be a (p, q) graph and $f: V(G) \rightarrow \{1, 2, 3, \dots, p + q + k - 1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called k - Super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1, 2, 3, \dots, p + q + k - 1\}$. A graph that admits a k -Super mean labeling is called k -Super mean graph.

In this paper we investigate k -super mean labeling of $\langle C_m, K_{1,n} \rangle$ and $\langle C_m * K_{1,n} \rangle$

Keywords: k -Super mean labeling, k -Super mean graph, $Q_n \odot K_1$, $[P_n: D(T_2)]$, $T(C_n)$ AMS Subject Classification--- 05C78

I. INTRODUCTION

All graphs in this thesis are finite, simple and undirected. Terms not defined here are used in the sense of Harary [7]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G . Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967. Labeled graphs serve as useful models for a broad range of applications such as X-ray, crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. Particularly interesting applications of graph theory can be found in [1-4]. The concept of mean labeling was introduced and studied by S. Somasundaram and R. Ponraj [12]. The concept of super mean labeling was introduced and studied by D. Ramya et al [11]. Further some results on super mean graphs are discussed in [8,9,10,13,15]. B. Gayathri and M. Tamilselvi [5-6, 14] extended super mean labeling to k -super mean labeling. In this paper we investigate k -super mean labeling of $S(Q_n \odot K_1)$, $[P_n: D(T_2)]$, $T(C_n)$ and $(C_3 \times P_n) \cup D(T_m)$. Here k denoted as any positive integer greater than or equal to 1.

II. MAIN RESULTS

A. Definition 2.1:

Let G be a (p, q) graph and $f: V(G) \rightarrow \{1, 2, 3, \dots, p + q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1, 2, 3, \dots, p + q\}$. A graph that admits a super mean labeling is called super mean graph.

B. Definition 2.2

Let G be a (p, q) graph and $f: V(G) \rightarrow \{k, k + 1, k + 2, \dots, p + q + k - 1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called k -super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k + 1, k + 2, \dots, p + q + k - 1\}$. A graph that admits a k -Super mean labeling is called k -Super mean graph.

C. Definition 2.3

A subdivision of a graph G is a graph resulting from the subdivision of each edge by a new vertex.

D. Definition 2.4

A quadrilateral snake (Q_n) is obtained from a path $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} to a new vertices v_i and w_i respectively and then joining v_i and w_i for $1 \leq i \leq n$.

E. Definition 2.5

A triangular snake (T_n) is obtained from a path by identifying each edge of the path with an edge of the cycle C_3 .

F. Definition 2.6

A double triangular snake $D(T_n)$ consists of two triangular snake that have a common path. That is, a double triangular snake is obtained from a path $v_1, v_2, \dots, v_n$ by joining v_i and v_{i+1} to a new vertices w_i for $i = 1, 2, \dots, n-1$ and to a new vertices u_i for $i = 1, 2, \dots, n-1$.

G. Definition 2.7

The corona of Q_n with K_1 , $Q_n \odot K_1$ is the graph obtained by taking one copy of Q_n and n copies of K_1 and joining the i th vertex of Q_n with an edge to every vertex in the i th copy of K_1 .

H. Theorem 2.8

The graph $S(Q_n \odot K_1)$ is a k -Super mean graph for all $n \geq 2$.

Proof:

Let $V(S(Q_n \odot K_1)) = \{u_i, v_i, v'_i; 1 \leq i \leq n\} \cup \{u'_i, w_i, w'_i, s_i, s'_i, x_i, x'_i, y_i, y'_i, z_i; 1 \leq i \leq n-1\}$ and $E(S(Q_n \odot K_1)) = \{e'' = (u_i, v'_i), e''' = (v'_i, v_i); 1 \leq i \leq n\} \cup \{e_i = (u_i, u'_i), e'_i = (u'_i, u_{i+1}), e^{iv} = (u_i, w'_i), e^v = (w'_i, w_i), e^{vi} = (w_i, z_i), e^{vii} = (z_i, s_i), e^{viii} = (w_i, x'_i), e^{ix} = (x'_i, x_i), e^x = (y'_i, y_i), e^{xi} = (s_i, y'_i), e^{xii} = (s'_i, s_i), e^{xiii} = (u_{i+1}, s'_i); 1 \leq i \leq n\}$ be the vertices and edges of $S(Q_n \odot K_1)$ respectively.

Define $f: V(S(Q_n \odot K_1)) \rightarrow \{k, k+1, k+2, \dots, 27n+k-23\}$ by

$$f(u_i) = k + 27i - 23; 1 \leq i \leq n$$

$$f(v_i) = k + 27i - 27; 1 \leq i \leq n$$

$$f(v'_1) = k + 2$$

$$f(v'_i) = k + 27i - 29; 2 \leq i \leq n$$

$$f(w_i) = k + 27i - 18; 1 \leq i \leq n-1$$

$$f(w'_i) = k + 27i - 21; 1 \leq i \leq n-1$$

$$f(x_i) = k + 27i - 13; 1 \leq i \leq n-1$$

$$f(x'_i) = k + 27i - 15; 1 \leq i \leq n-1$$

$$f(z_i) = k + 27i - 5; 1 \leq i \leq n-1$$

$$f(y_i) = k + 27i - 4; 1 \leq i \leq n-1$$

$$f(y'_i) = k + 27i - 12; 1 \leq i \leq n-1$$

$$f(s_i) = k + 27i - 9; 1 \leq i \leq n-1$$

$$f(s'_i) = k + 27i + 2; 1 \leq i \leq n-1$$

$$f(u'_i) = k + 27i - 17; 1 \leq i \leq n-1$$

Now the induced edge labels are

$$f^*(e_i) = k + 27i - 20; 1 \leq i \leq n-1$$

$$f^*(e'_i) = k + 27i - 6; 1 \leq i \leq n-1$$

$$f^*(e''_1) = k + 3$$

$$f^*(e''_i) = k + 27i - 26; 2 \leq i \leq n$$

$$f^*(e'''_1) = k + 1$$

$$f^*(e'''_i) = k + 27i - 28; 2 \leq i \leq n$$

$$f^*(e^{iv}_i) = k + 27i - 22; 1 \leq i \leq n-1$$

$$f^*(e^v_i) = k + 27i - 19; 1 \leq i \leq n-1$$

$$f^*(e^{vi}_i) = k + 27i - 11; 1 \leq i \leq n-1$$

$$f^*(e^{vii}_i) = k + 27i - 7; 1 \leq i \leq n-1$$

$$f^*(e^{viii}_i) = k + 27i - 16; 1 \leq i \leq n-1$$

$$f^*(e^{ix}_i) = k + 27i - 14; 1 \leq i \leq n-1$$

$$f^*(e^x_i) = k + 27i - 8; 1 \leq i \leq n-1$$

$$f^*(e^{xi}_i) = k + 27i - 10; 1 \leq i \leq n-1$$

$$f^*(e^{xii}_i) = k + 27i - 3; 1 \leq i \leq n-1$$

$$f^*(e_i^{xiii}) = k + 27i + 3; 1 \leq i \leq n - 1$$

Here $p = 13n - 10$, $q = 14n - 12$ and $p + q = 27n - 22$.

Clearly, $f(V) \cup \{f^*(e): e \in E(S(Q_n \odot K_1))\} = \{k, k + 1, \dots, k + 27n - 23\}$.

So f is a k - Super mean labeling.

Hence $S(Q_n \odot K_1)$ is a k - Super mean graph for all $n \geq 2$.

I. Example 2.9

100 - Super mean labeling of $S(Q_4 \odot K_1)$ is given in figure 1:

Figure 1: 100 -Super mean labeling of $S(Q_4 \odot K_1)$

J. Definition 2.10

Let G be a graph with fixed vertex v , and let $[P_m; G]$ be the graph obtained from m copies of G by joining v_i and v_{i+1} by means of an edge for some j and $1 \leq i \leq m - 1$.

K. Theorem 2.11

The graph $[P_n; D(T_2)]$ is a k -Super mean graph for all $n \geq 2$.

Proof:

Let $V([P_n; D(T_2)]) = \{u_i, v_i, w_i, z_i; 1 \leq i \leq n\}$ and $E([P_n; D(T_2)]) = \{e_i = (u_i, u_{i+1}), e'_i = (u_i, v_i), e''_i = (u_i, w_i), e'''_i = (v_i, z_i), e^{iv}_i = (w_i, z_i), e^v_i = (v_i, w_i); 1 \leq i \leq n\}$ be the vertices and edges of $[P_n; D(T_2)]$ respectively.

Define $f: V([P_n; D(T_2)]) \rightarrow \{k, k + 1, k + 2, \dots, k + 10n - 2\}$ by

$$f(u_1) = k + 2$$

$$f(u_i) = k + 10i - 4; 2 \leq i \leq n$$

$$f(v_i) = k + 10i - 10; 1 \leq i \leq n$$

$$f(w_i) = k + 10i - 2; 1 \leq i \leq n$$

$$f(z_1) = k + 6$$

$$f(z_i) = k + 10i - 8; 2 \leq i \leq n$$

Now the induced edge labels are

$$f^*(e_i) = k + 10i - 1; 1 \leq i \leq n - 1$$

$$f^*(e'_1) = k + 1$$

$$f^*(e'_i) = k + 10i - 7; 2 \leq i \leq n$$

$$f^*(e''_1) = k + 5$$

$$f^*(e_i'') = k + 10i - 3; 2 \leq i \leq n$$

$$f^*(e_1''') = k + 3$$

$$f^*(e_i''') = k + 10i - 9; 2 \leq i \leq n$$

$$f^*(e_1^{iv}) = k + 7$$

$$f^*(e_i^{iv}) = k + 10i - 5; 2 \leq i \leq n$$

$$f^*(e_i^v) = k + 10i - 6; 1 \leq i \leq n$$

Here $p = 4n$, $q = 6n - 1$ and $p + q = 10n - 1$.

Clearly, $f(V) \cup \{f^*(e) : e \in E([P_n : D(T_2)])\} = \{k, k + 1, \dots, k + 10n - 2\}$.

So f is a k – Super mean labeling.

Hence $[P_n : D(T_2)]$ is a k – Super mean graph for all $n \geq 2$.

L. Example 2.12

425 – Super mean labeling of $[P_4 : D(T_2)]$ is given in figure 2:

Figure 2: 425 –Super mean labeling of $[P_4 : D(T_2)]$.

M. Definition 2.13

A total graph $T(G)$ of a graph G is a graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent in $T(G)$ if they are adjacent or incident in G .

N. Theorem 2.14

The graph $T(C_n)$ is a k -Super mean graph, if n is odd and for all $n \geq 3$.

Proof:

Let $V(T(C_n)) = \{v_i, v_i'; 1 \leq i \leq n\}$ and $E(T(C_n)) = \{e_i = (v_i, v_{i+1}), e_i' = (v_i, v_i'), e_i^{ii} = (v_{i+1}, v_i'), e_i''' = (v_i', v_{i+1}'); 1 \leq i \leq n\}$ be the vertices and edges of $T(C_n)$ respectively.

Let $n = 2l + 1$.

Define $f: V(T(C_n)) \rightarrow \{k, k + 1, k + 2, \dots, k + 6n - 1\}$ by

$$f(v_i) = k + 2i - 2; 1 \leq i \leq l + 1$$

$$f(v_{l+1+i}) = k + 2(l + 1) + 2i - 1; 1 \leq i \leq l$$

$$f(v_i') = k + 4n + 2i - 2; 1 \leq i \leq l + 1$$

$$f(v_{l+1+i}') = k + 4n + 2(l + 1) + 2i - 1; 1 \leq i \leq l$$

Now the induced edge labels are

$$f^*(e_i) = k + 2i - 1; 1 \leq i \leq l$$

$$f^*(e_{l+i}) = k + 2l + 2i; 1 \leq i \leq l$$

$$f^*(e_n) = k + n$$

$$f^*(e_i') = k + 2n + 2i - 2; 1 \leq i \leq l + 1$$

$$f^*(e'_{l+i+1}) = k + 2n + 2(l + 1) + 2i - 1; 1 \leq i \leq l$$

$$f^*(e''_i) = k + 2n + 2i - 1; 1 \leq i \leq l$$

$$f^*(e''_{l+1+i}) = k + 2n + 2(l + 1) + 2i - 2; 1 \leq i \leq l$$

$$f^*(e''_n) = k + 3n$$

$$f^*(e'''_i) = k + 4n + 2i - 1; 1 \leq i \leq l$$

$$f^*(e'''_{l+1+i}) = k + 4n + 2l + 2i; 1 \leq i \leq l$$

$$f^*(e'''_n) = k + 5n$$

Here $p = 2n$, $q = 4n$ and $p + q = 6n$.

Clearly, $f(V) \cup \{f^*(e): e \in E(T(C_n))\} = \{k, k + 1, \dots, k + 6n - 1\}$.

So f is a k - Super mean labeling.

Hence $T(C_n)$ is a k - Super mean graph if n is odd and for all $n \geq 3$.

O. Example 2.15

99 - Super mean labeling of $T(C_7)$ is given in figure 3:

Figure 3: 149 - Super mean labeling of $T(C_7)$

P. Theorem 2.16

The graph $(C_3 \times P_n) \cup D(T_m)$ is a k -Super mean graph for all $m, n \geq 2$.

Proof:

Let us denote $(C_3 \times P_n) \cup D(T_m)$ by G .

Let $V(G) = \{u_i, v_i, w_i; 1 \leq i \leq n\} \cup \{x_i; 1 \leq i \leq m\}$ and $E(G) = \{e_i = (u_i, w_i), e'_i = (u_i, v_i), e''_i = (v_i, w_i); 1 \leq i \leq n\} \cup \{e'''_i = (u_i, u_{i+1}), e^{iv}_i = (v_i, v_{i+1}), e^v_i = (w_i, w_{i+1}); 1 \leq i \leq n-1\} \cup \{e^{vi}_i = (x_i, x_{i+1}), e^{vii}_i = (x_i, y_i), e^{viii}_i = (x_{i+1}, y_i), e^{ix}_i = ((x_i, z_i), e^x_i = (x_{i+1}, z_i); 1 \leq i \leq m-1\}$ be the vertices and edges of G respectively.

Define $f: V(G) \rightarrow \{k, k + 1, k + 2, \dots, k + 9n + 7m - 10\}$ by

$$f(u_i) = \begin{cases} k + 9i - 7; & \text{if } i \text{ is odd, } 1 \leq i \leq n \\ k + 9i - 4; & \text{if } i \text{ is even, } 1 \leq i \leq n \end{cases}$$

$$f(v_i) = \begin{cases} k + 9i - 9; & \text{if } i \text{ is odd, } 1 \leq i \leq n \\ k + 9i - 7; & \text{if } i \text{ is even, } 1 \leq i \leq n \end{cases}$$

$$f(w_i) = \begin{cases} k + 9i - 4; & \text{if } i \text{ is odd, } 1 \leq i \leq n \\ k + 9i - 9; & \text{if } i \text{ is even, } 1 \leq i \leq n \end{cases}$$

$$f(x_1) = \begin{cases} f(w_n) + 1; & \text{if } i \text{ is odd} \\ f(u_n) + 1; & \text{if } i \text{ is even} \end{cases}$$

$$f(x_i) = \begin{cases} f(w_n) + 8i - 7; & \text{if } i \text{ is odd}, 2 \leq i \leq m \\ f(u_n) + 8i - 7; & \text{if } i \text{ is even}, 2 \leq i \leq m \end{cases}$$

$$f(y_i) = \begin{cases} f(w_n) + 8i - 5; & \text{if } i \text{ is odd}, 1 \leq i \leq m - 1 \\ f(u_n) + 8i - 5; & \text{if } i \text{ is even}, 1 \leq i \leq m - 1 \end{cases}$$

$$f(z_i) = \begin{cases} f(w_n) + 8i - 1; & \text{if } i \text{ is odd}, 1 \leq i \leq m - 1 \\ f(u_n) + 8i - 1; & \text{if } i \text{ is even}, 1 \leq i \leq m - 1 \end{cases}$$

Now the induced edge labels are

$$f^*(e_i) = \begin{cases} k + 9i - 5; & \text{if } i \text{ is odd}, 1 \leq i \leq n \\ k + 9i - 6; & \text{if } i \text{ is even}, 1 \leq i \leq n \end{cases}$$

$$f^*(e'_i) = \begin{cases} k + 9i - 8; & \text{if } i \text{ is odd}, 1 \leq i \leq n \\ k + 9i - 5; & \text{if } i \text{ is even}, 1 \leq i \leq n \end{cases}$$

$$f^*(e''_i) = \begin{cases} k + 9i - 6; & \text{if } i \text{ is odd}, 1 \leq i \leq n \\ k + 9i - 8; & \text{if } i \text{ is even}, 1 \leq i \leq n \end{cases}$$

$$f^*(e'''_i) = k + 9i - 1; 1 \leq i \leq n - 1$$

$$f^*(e^{iv}_i) = k + 9i - 3; 1 \leq i \leq n - 1$$

$$f^*(e^v_i) = k + 9i - 2; 1 \leq i \leq n - 1$$

$$f^*(e^{vi}_i) = \begin{cases} f(w_n) + 8i - 3; & \text{if } i \text{ is odd}, 1 \leq i \leq m - 1 \\ f(u_n) + 8i - 3; & \text{if } i \text{ is even}, 1 \leq i \leq m - 1 \end{cases}$$

$$f^*(e^{vii}_i) = \begin{cases} f(w_n) + 8i - 6; & \text{if } i \text{ is odd}, 1 \leq i \leq m - 1 \\ f(u_n) + 8i - 6; & \text{if } i \text{ is even}, 1 \leq i \leq m - 1 \end{cases}$$

$$f^*(e^{viii}_i) = \begin{cases} f(w_n) + 8i - 2; & \text{if } i \text{ is odd}, 1 \leq i \leq m - 1 \\ f(u_n) + 8i - 2; & \text{if } i \text{ is even}, 1 \leq i \leq m - 1 \end{cases}$$

$$f^*(e^{ix}_i) = \begin{cases} f(w_n) + 8i - 4; & \text{if } i \text{ is odd}, 1 \leq i \leq m - 1 \\ f(u_n) + 8i - 4; & \text{if } i \text{ is even}, 1 \leq i \leq m - 1 \end{cases}$$

$$f^*(e^x_i) = \begin{cases} f(w_n) + 8i; & \text{if } i \text{ is odd}, 1 \leq i \leq m - 1 \\ f(u_n) + 8i; & \text{if } i \text{ is even}, 1 \leq i \leq m - 1 \end{cases}$$

Here $p = 3n + 3m - 2$, $q = 6n + 4m - 7$ and $p + q = 9n + 7m - 9$.

Clearly, $f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k + 1, \dots, k + 9n + 7m - 10\}$.

So f is a k - Super mean labeling.

Hence $(C_3 \times P_n) \cup D(T_m)$ is a k -Super mean graph for all $m, n \geq 2$.

Q. Example 2.17

20 – Super mean labeling of $(C_3 \times P_2) \cup D(T_3)$ is given in figure 4:

Figure 4: 20 – Super mean labeling of $(C_3 \times P_2) \cup D(T_3)$

III.CONCLUSIONS

Graph labeling has its own applications in communication networks and astronomy. so, enormous types of labeling have grown. Towards this, k-super mean labeling is also a kind of labeling which is an extension of super mean labeling. we discussed k-super mean labeling of the graphs $S(Q_n \odot K_1)$, $[P_n : D(T_2)]$, $T(C_n)$, $(C_3 \times P_n) \cup D(T_m)$.

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