iJRASET

International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 Issue: IV Month of publication: April 2018

DOI: <http://doi.org/10.22214/ijraset.2018.4384>

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Near Mean Labeling of Path Related Splitted Graphs

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Abstract: The concept of near mean Graph was introduced in [9]. A function f is called a near mean Labeling of graph G if $f: V(G) \rightarrow \{0, 1, 2, \dots, q-1, q+1\}$ is injective and the induced function $f^*: E(G) \rightarrow \{1, 2, \dots, q\}$ defined as

$$f^*(e = uv) = \begin{cases} \frac{f(u) + f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \frac{f(u) + f(v) + 1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

is bijective. The graph which admits near mean labeling is called a near mean Graph. In this paper, we proved that $S(H_n)$: $(n: \text{odd}), S(H_n)$ $(n: \text{even}), S(P_n^+)$ are near mean graphs.

Keywords: Near mean Labeling, Near mean Graph.

2000 mathematics Subject classification: 05C78

I. INTRODUCTION

We begin with simple, finite, connected and undirected graph $G = (V(G), E(G))$ with p vertices and q edges. For standard terminology and notations we follow (Harary, F., 1972). We will provide brief summary of definitions and other information which are prerequisites for the present investigations.

II. PRELIMINARIES

A function f is called a near mean labeling of graph G if $f: V(G) \rightarrow \{0, 1, 2, \dots, q\}$ is injective and the induced function $f^*: E(G) \rightarrow \{1, 2, \dots, q\}$ defined as

$$f^*(e = uv) = \begin{cases} \frac{f(u) + f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \frac{f(u) + f(v) + 1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases} \text{ is bijective. The graph which admits near mean labeling is called a near mean graph.}$$

- 1) **Definition 2.1.** Let G be a graph. For each vertex u of a graph G , take a new vertex v . Join v to those vertices of G adjacent to u . The graph thus obtained is called the *splitting graph* of G . It is denoted by $S(G)$. For a graph G , the splitting graph S of G is obtained by adding a new vertex v corresponding to each vertex u of G such that $N(u) = N(v)$ and it is denoted by $S(G)$.
- 2) **Definition 2.2.** Let H_n –graph of a path P_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, v_3, \dots, v_n$ and $u_1, u_2, u_3, \dots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ by means of an edge if n is odd and vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even.
- 3) **Definition 2.3.** $G_1 \Theta G_2$ of two graphs G_1 and G_2 is obtained by taking one copy of G_1 (with p vertices) and p copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2 . When $G_1 = P_n$ and $G_2 = mK_1$ we obtain $P_n \Theta mk_1$.

III. MAIN RESULTS

Theorem 3.1: $S(H_n)$: $(n: \text{odd})$ is Mean Graph.

Proof: Let $V[S(H_n)] = \{u_i, v_i, u_i^l, v_i^l : 1 \leq i \leq n\}$

$$E[S(H_n)] = \{[(u_i, u_{i+1}) \cup (u_i^l, u_{i+1}^l) : 1 \leq i \leq n-1] \cup [(v_i, v_{i+1}) \cup (v_i^l, v_{i+1}^l) : 1 \leq i \leq n-1] \cup (u_{(n+1)/2}, u_{(n+1)/2}^l) \cup (v_{(n+1)/2}, v_{(n+1)/2}^l) \cup (u_{(n+1)/2}, v_{(n+1)/2}^l)\}$$

Let $f: V[S(H_n)] \rightarrow \{0, 1, 2, \dots, q\}$ by

$$\begin{aligned}
 f(u_{2i-1}) &= 2(i-1) & 1 \leq i \leq [n+1]/2 \\
 f(u_{2i}) &= 4n-1+2(i-1) & 1 \leq i \leq [n-1]/2 \\
 f(v_{2i-1}) &= 4n-2+2(i-1) & 1 \leq i \leq [n+1]/2 \\
 f(v_{2i}) &= 2i-1 & 1 \leq i \leq [n-1]/2 \\
 f(v'_{n+2-2i}) &= n+2(i-1) & 1 \leq i \leq [n+1]/2 \\
 f(v'_{n+1-2i}) &= 5n-1+2(i-1) & 1 \leq i \leq [n-1]/2 \\
 f(u'_{n+1-2i}) &= n-1+2i & 1 \leq i \leq [n-1]/2 \\
 f(u'_{n+2-2i}) &= 5n-2+2(i-1) & 1 \leq i \leq [n-1]/2 \\
 f(u'_1) &= 6n-2
 \end{aligned}$$

The induced edge labeling are

$$\begin{aligned}
 f^*(u_i u_{i+1}) &= 2n+i-1 & 1 \leq i \leq n-1 \\
 f^*(u_{2i-1} v_{2i}) &= 2i-1 & 1 \leq i \leq [n-1]/2 \\
 f^*(u_{2i} v_{2i+1}) &= 4n+2(i-1) & 1 \leq i \leq [n-1]/2 \\
 f^*(v_{2i-1} u_{2i}) &= 4n-1+2(i-1) & 1 \leq i \leq [n-1]/2 \\
 f^*(v_{2i} u_{2i+1}) &= 2i & 1 \leq i \leq [n-1]/2 \\
 f^*(u^l_{n+1-i} u^l_{n-i}) &= 3n+i-1 & 1 \leq i \leq n-1 \\
 f^*(v^l_{n+2-2i} u^l_{n+1-2i}) &= n+2i-1 & 1 \leq i \leq [n-1]/2 \\
 f^*(v^l_{n+1-2i} u^l_{n-2i}) &= 5n+2(i-1) & 1 \leq i \leq [n-1]/2 \\
 f^*(v^l_{n+1-2i} u^l_{n+2-2i}) &= 5n-1+2(i-1) & 1 \leq i \leq [n-1]/2 \\
 f^*(v^l_{n-2i} u^l_{n+1-2i}) &= n+2i & 1 \leq i \leq [n-1]/2 \\
 f^*(u_{(n+1)/2} u^l_{(n+1)/2}) &= 3n-1 \\
 f^*(v_{(n+1)/2} u^l_{(n+1)/2}) &= n & \text{if } n \equiv 3 \pmod{4} \\
 f^*(v_{(n+1)/2} u^l_{(n+1)/2}) &= 5n-2 & \text{if } n \equiv 1 \pmod{4} \\
 f^*(v^l_{(n+1)/2} u_{(n+1)/2}) &= 5n-2 & \text{if } n \equiv 3 \pmod{4} \\
 f^*(v^l_{(n+1)/2} u_{(n+1)/2}) &= n & \text{if } n \equiv 1 \pmod{4}
 \end{aligned}$$

Hence, distinct induced edge labels shows that $S(H_n)$ (n : odd) is a Mean graph. For example, $S(H_5)$ and $S(H_7)$ are Mean Graph as shown in the figure 3.2 and 3.3 respectively.

Figure 3.2 $S(H_5)$: $n \equiv 1 \pmod{4}$

Figure 3.3 $S(H_7) : n \equiv 3 \pmod{4}$

Theorem 3.4 : $S(H_n)$ (n : even) is Mean Graph.

Proof: Let $V[S(H_n)] = \{u_i, v_i, u_i^1, v_i^1 : 1 \leq i \leq n\}$

$$E[S(H_n)] = \{[(u_i u_{i+1}) \cup (u_i^1 u_{i+1}^1) : 1 \leq i \leq n-1] \cup (u_{n/2} u_{[n/2]+1}^1) \cup [(v_i v_{i+1}) \cup (u_i v_{i+1}) : 1 \leq i \leq n-1] \cup [(v_i^1 u_{i+1}^1) \cup (u_i^1 v_{i+1}^1) : 1 \leq i \leq n-1] \cup (v_{n/2} u_{[n/2]+1}^1) \cup (u_{n/2} v_{[n/2]+1}^1)\}$$

Let $f : V[S(H_n)] \rightarrow \{0, 1, 2, \dots, q\}$

$$\begin{aligned} f(u_{2i-1}) &= 2(i-1) & 1 \leq i \leq n/2 \\ f(u_{2i}) &= 4n-1+2(i-1) & 1 \leq i \leq n/2 \\ f(v_{2i-1}) &= 4n-2+2(i-1) & 1 \leq i \leq n/2 \\ f(v_{2i}) &= 2i-1 & 1 \leq i \leq n/2 \\ f(u_{2i-1}^1) &= n+2(i-1) & 1 \leq i \leq n/2 \\ f(u_{2i}^1) &= 5n-1+2(i-1) & 1 \leq i \leq [n/2]-1 \\ f(u_n^1) &= 6n-2 \\ f(v_{2i-1}^1) &= 5n-2+2(i-1) & 1 \leq i \leq n/2 \\ f(v_{2i}^1) &= n+1+2(i-1) & 1 \leq i \leq n/2 \end{aligned}$$

The induced edge labeling are

$$\begin{aligned} f^*(u_i u_{i+1}) &= 2n+i-1 & 1 \leq i \leq n-1 \\ f^*(u_{2i-1} v_{2i}) &= 2i-1 & 1 \leq i \leq n/2 \\ f^*(u_{2i} v_{2i+1}) &= 4n+2(i-1) & 1 \leq i \leq [n/2]-1 \end{aligned}$$

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Theorem 3.7: $S(P_n \Theta 2k_1)$ is Mean Graph.

Proof: Let $V[S(P_n \Theta 2k_1)] = \{ [(u_i, v_i) : 1 \leq i \leq n] \cup [(u_{ij}, v_{ij}) : 1 \leq i \leq n, 1 \leq j \leq 2] \}$

$E[S(P_n \Theta 2k_1)] = [(u_i, u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i, u_{ij}) \cup (v_i, u_{ij}) \cup (u_i, v_{ij}) : 1 \leq i \leq n, 1 \leq j \leq 2] \cup [(v_i, u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i, v_{i+1}) : 1 \leq i \leq n-1]$

Let $f : V[S(P_n \Theta 2k_1)] \rightarrow \{0, 1, 2, \dots, q\}$

Case: (i) when $n \equiv 0 \pmod{2}$

$f(u_{2i-1})$	$= 6i-5$	$1 \leq i \leq n/2$
$f(u_{2i})$	$= 6n+2+6(i-1)$	$1 \leq i \leq n/2$
$f(v_{2i-1,1})$	$= 6(i-1)$	$1 \leq i \leq n/2$
$f(v_{2i-1,2})$	$= 6i-4$	$1 \leq i \leq n/2$
$f(u_{2i,1})$	$= 6i-3$	$1 \leq i \leq n/2$
$f(u_{2i,2})$	$= 6i-1$	$1 \leq i \leq n/2$
$f(v_{2i})$	$= 6i-2$	$1 \leq i \leq n/2$
$f(v_{2i-1})$	$= 6(n+i)-7$	$1 \leq i \leq n/2$
$f(v_{2i,1})$	$= 6n-5+6i$	$1 \leq i \leq n/2$
$f(v_{2i,2})$	$= 6n-3+6i$	$1 \leq i \leq [n/2]-1$
$f(v_{n,2})$	$= 9n-2$	
$f(u_{2i-1,1})$	$= 6n-8+6i$	$1 \leq i \leq n/2$
$f(u_{2i-1,2})$	$= 6n-6+6i$	$1 \leq i \leq n/2$

The induced edge labeling are

$f^*(u_{2i-1}v_{2i-1,1})$	$= 6i-5$	$1 \leq i \leq n/2$
$f^*(u_{2i-1}v_{2i-1,2})$	$= 6i-4$	$1 \leq i \leq n/2$
$f^*(u_{2i}v_{2i,1})$	$= 6n-4+6i$	$1 \leq i \leq n/2$
$f^*(u_{2i}v_{2i,2})$	$= 6n-3+6i$	$1 \leq i \leq n/2$
$f^*(u_i, u_{i+1})$	$= 3n-1+3i$	$1 \leq i \leq n-1$
$f^*(u_i, u_{i,1})$	$= 3n-3+3i$	$1 \leq i \leq n$
$f^*(u_i, u_{i,2})$	$= 3n-2+3i$	$1 \leq i \leq n$
$f^*(v_{2i}, u_{2i,1})$	$= 6i-2$	$1 \leq i \leq n/2$
$f^*(v_{2i}, u_{2i,2})$	$= 6i-1$	$1 \leq i \leq n/2$
$f^*(v_{2i-1}, u_{2i-1,1})$	$= 6n-7+6i$	$1 \leq i \leq n/2$
$f^*(v_{2i-1}, u_{2i-1,2})$	$= 6(n-1)+6i$	$1 \leq i \leq n/2$
$f^*(v_{2i-1}, u_{2i})$	$= 6n-5+6i$	$1 \leq i \leq n/2$
$f^*(v_{2i}, u_{2i+1})$	$= 6i$	$1 \leq i \leq [n/2]-1$
$f^*(v_{2i}, u_{2i-1})$	$= 6i-3$	$1 \leq i \leq n/2$
$f^*(v_{2i+1}, u_{2i})$	$= 6n-2+6i$	$1 \leq i \leq [n/2]-1$

Hence, distinct induced edge labels shows that $S(P_n \Theta 2k_1)$ (n : even) is a Mean graph. For example, $S(P_4 \Theta 2k_1)$ is mean Graphs as shown in the figure 3.5.

Figure 3.8 $S(P_4 \Theta 2k_1)$; $n \equiv 0 \pmod{2}$

Case: (ii) When $n \equiv 1 \pmod{2}$

$f(u_{2i-1})$	$= 6i-5$	$1 \leq i \leq \lceil n+1 \rceil / 2$
$f(u_{2i})$	$= 6n-4+6i$	$1 \leq i \leq \lceil n-1 \rceil / 2$
$f(v_{2i-1,1})$	$= 6(i-1)$	$1 \leq i \leq \lceil n+1 \rceil / 2$
$f(v_{2i-1,2})$	$= 6i-4$	$1 \leq i \leq \lceil n+1 \rceil / 2$
$f(v_{2i,1})$	$= 6(n+i)-5$	$1 \leq i \leq \lceil n-1 \rceil / 2$
$f(v_{2i,2})$	$= 6(n+i)-3$	$1 \leq i \leq \lceil n-1 \rceil / 2$
$f(u_{2i,1})$	$= 6i-3$	$1 \leq i \leq \lceil n-1 \rceil / 2$
$f(u_{2i,2})$	$= 6i-1$	$1 \leq i \leq \lceil n-1 \rceil / 2$
$f(u_{2i-1,1})$	$= 6(n+i)-8$	$1 \leq i \leq \lceil n+1 \rceil / 2$
$f(u_{2i-1,2})$	$= 6(n+i-1)$	$1 \leq i \leq \lceil n-1 \rceil / 2$
$f(u_{n,2})$	$= 9n-2$	
$f(v_{2i})$	$= 6i-2$	$1 \leq i \leq \lceil n-1 \rceil / 2$
$f(v_{2i-1})$	$= 6(n+i)-7$	$1 \leq i \leq \lceil n+1 \rceil / 2$

The induced edge labeling are

$f^*(u_{2i-1}v_{2i-1,1})$	$= 6i-5$	$1 \leq i \leq \lceil n+1 \rceil / 2$
$f^*(u_{2i-1}v_{2i-1,2})$	$= 6i-4$	$1 \leq i \leq \lceil n+1 \rceil / 2$
$f^*(u_{2i}v_{2i,1})$	$= 6(n+i)-4$	$1 \leq i \leq \lceil n-1 \rceil / 2$
$f^*(u_{2i}v_{2i,2})$	$= 6(n+i)-3$	$1 \leq i \leq \lceil n-1 \rceil / 2$
$f^*(u_i u_{i+1})$	$= 3(n+i)-1$	$1 \leq i \leq n-1$
$f^*(u_i u_{i,1})$	$= 3(n+i-1)$	$1 \leq i \leq n$
$f^*(u_i u_{i,2})$	$= 3(n+i)-2$	$1 \leq i \leq n$
$f^*(v_{2i} u_{2i,1})$	$= 6i-2$	$1 \leq i \leq \lceil n-1 \rceil / 2$
$f^*(v_{2i} u_{2i,2})$	$= 6i-1$	$1 \leq i \leq \lceil n-1 \rceil / 2$
$f^*(v_{2i-1} u_{2i-1,1})$	$= 6(n+i)-7$	$1 \leq i \leq \lceil n+1 \rceil / 2$
$f^*(v_{2i-1} u_{2i-1,2})$	$= 6(n+i-1)$	$1 \leq i \leq \lceil n+1 \rceil / 2$
$f^*(v_{2i-1} u_{2i})$	$= 6(n+i)-5$	$1 \leq i \leq \lceil n-1 \rceil / 2$
$f^*(v_{2i} u_{2i+1})$	$= 6i$	$1 \leq i \leq \lceil n-1 \rceil / 2$
$f^*(v_{2i} u_{2i-1})$	$= 6i-3$	$1 \leq i \leq \lceil n-1 \rceil / 2$
$f^*(v_{2i+1} u_{2i})$	$= 6(n+i)-2$	$1 \leq i \leq \lceil n-1 \rceil / 2$

Hence, distinct induced edge labels shows that $S(P_n \odot 2K_1)$ ($n : \text{odd}$) is a Mean graph.

For example, $S(P_3 \odot 2K_1)$ is Mean Graphs as shown in the figure 3.6.

Figure 3.9 $S(P_3 \odot 2K_1)$; $n \equiv 1 \pmod{2}$

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