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K-Super Mean Labeling of Cycle Related Graphs

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Abstract : Let G be a (p, q) graph and $f: V(G) \rightarrow \{1, 2, 3, \dots, p + q + k - 1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called k -Super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1, 2, 3, \dots, p + q + k - 1\}$. A graph that admits a k -Super mean labeling is called k -Super mean graph.

In this paper we investigate k -super mean labeling of $\langle C_m, K_{1,n} \rangle$ and $\langle C_m * K_{1,n} \rangle$.

Keywords: k -Super mean labeling, k -Super mean graph, $\langle C_m, K_{1,n} \rangle$, $\langle C_m * K_{1,n} \rangle$ AMS Subject Classification--- 05C78

I. INTRODUCTION

All graphs in this paper are finite, simple and undirected. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G . Terms not defined here are used in the sense of Harary [7]. Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [11]. Labeled graphs serve as useful models for a broad range of applications such as X-ray, crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. The concept of mean labeling was introduced and studied by S. Somasundaram and R.Ponraj [12]. The concept of super mean labeling was introduced and studied by D. Ramya et al. [11]. Further some results on super mean graphs are discussed in [8, 9, 10, 14, 15]. B. Gayathri and M. Tamilselvi [13] extended super mean labeling to k -super mean labeling. In this paper we investigate k -Super mean labeling of $\langle C_m, K_{1,n} \rangle$ and $\langle C_m * K_{1,n} \rangle$. For brevity, we use k -SML for k -super mean labeling. Here k denoted as any positive integer greater than or equal to 1.

II. MAIN RESULTS

A. Definition 2.1

Let G be a (p, q) graph and $f: V(G) \rightarrow \{1, 2, 3, \dots, p + q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called Super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1, 2, 3, \dots, p + q\}$. A graph that admits a super mean labeling is called Super mean graph.

B. Definition 2.2

Let G be a (p, q) graph and $f: V(G) \rightarrow \{k, k + 1, k + 2, \dots, p + q + k - 1\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $f^*(e) = \frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then f is called k -Super mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k + 1, \dots, p + q + k - 1\}$. A graph that admits a k -super mean labeling is called k -Super mean graph.

C. Definition 2.3

$\langle C_m, K_{1,n} \rangle$ is the graph obtained from C_m and $K_{1,n}$ by identifying the vertex u_1 of C_m with the central vertex v of $K_{1,n}$.

D. Theorem 2.4

The graph $\langle C_m, K_{1,n} \rangle$ is a k -Super Mean Labeling for $n \leq 2$ and $m \geq 3$.

1) *Proof:* Let G denotes the graph $\langle C_m, K_{1,n} \rangle$. Let $V(G) = \{u_i; 1 \leq i \leq m\} \cup \{v_i; 1 \leq i \leq n\}$ and $E(G) = \{e_i = (u_i v_i); 1 \leq i \leq n\} \cup \{e'_i = (u_i u_{i+1}); 1 \leq i \leq m - 1\} \cup \{e''_m = (u_m u_1)\}$ be the vertices and edges of G respectively. For $m=4$, the super mean labeling of the graphs $\langle C_1, K_{1,1} \rangle$, $\langle C_2, K_{1,2} \rangle$, $\langle C_3, K_{1,3} \rangle$ and $\langle C_1, K_{1,4} \rangle$

Figure 1

Define $f : V(G) \rightarrow \{k, k+1, \dots, k+2(m+n)-1\}$

2) Case (1): m is odd

Let $m = 2l + 1$ for all $l \in \mathbb{Z}^+$

For $n=1, 2$

$$f(v_i) = k + i - 1, 1 \leq i \leq n$$

$$f(u_1) = k + 2n$$

$$f(u_j) = k + 2n + 4j - 6, 2 \leq j \leq l + 1$$

$$f(u_{l+1+j}) = k + 2n + 4l - 4j + 5, 1 \leq j \leq l$$

For $n=3, 4$

$$f(u_1) = k + 4$$

$$f(u_j) = k + 2n + 4j - 2, 2 \leq j \leq l$$

$$f(u_{l+j}) = k + 2n + 4l - 4j + 5, 1 \leq j \leq l$$

$$f(u_{2l+1}) = k + 2n + 3$$

3) Case (2): m is even

Let $m = 2l$ for all $l \in \mathbb{Z}^+$

For $n=1, 2$

$$f(v_i) = k + i, 1 \leq i \leq n$$

$$f(u_1) = k + 2n$$

$$f(u_j) = k + 2n + 4j - 5, 2 \leq j \leq l$$

$$f(u_{l+j}) = k + 2n + 4l - 3(j-1), 1 \leq j \leq 2$$

$$f(u_{l+2+j}) = k + 2n + 4l - 4j - 2, 1 \leq j \leq l-2$$

For $n=3, 4$

$$f(u_1) = k + 4$$

$$f(u_j) = k + 2n + 4j - 2, 2 \leq j \leq l-1$$

$$f(u_{l-1+j}) = k + 2n + 4l - 3(j-1) - 1, 1 \leq j \leq 2$$

$$f(u_{l+1+j}) = k + 2n + 4l - 4j - 3, 1 \leq j \leq l-2$$

$$f(u_{2l}) = 2n + 4$$

Clearly, f induces distinct edge labels and it is easy to check that f generates a k –super mean labeling and Hence $\langle C_m, K_{1,n} \rangle$ is a k –Super Mean Labeling, for all $m \geq 3$ and $n \leq 4$.

E. Example 2.5

Figure 2: 20-Super mean labeling of $\langle C_{11}, K_{1,3} \rangle$

F. Definition 2.6

$\langle C_m * K_{1,n} \rangle$ is the graph obtained from C_m and $K_{1,n}$ by identifying any one of the vertices of C_m with a pendant vertex of $K_{1,n}$ (i.e. a non-central vertex of $K_{1,n}$).

G. Theorem 2.7

The graph $\langle C_m * K_{1,n} \rangle$ is a k -Super Mean Labeling for $n \leq 6$ and $m \geq 3$.

1) Proof:

Let G denotes the graph $\langle C_m * K_{1,n} \rangle$. Let $V(G) = \{u_i; 1 \leq i \leq m\} \cup \{v_i; 1 \leq i \leq n\}$ and $E(G) = \{e_i = (u_1 v_{i+1}); 1 \leq i \leq n-1\} \cup \{e'_i = (u_i u_{i+1}); 1 \leq i \leq m-1\} \cup \{e'_m = (u_m u_1)\} \cup \{e = (v v_1)\}$ be the vertices and edges of G respectively.

For $m=4$, the super mean labeling of the graphs $\langle C_m * K_{1,n} \rangle$, $n=1, 2, 3, 4, 5, 6$ are shown in Figure 3.

Figure 3

Define $f : V(G) \rightarrow \{k, k+1, \dots, k+2(m+n)-1\}$

2) Case (1): m is odd

Let $m = 2l + 1$ for all $l \in \mathbb{Z}^+$

For $n=1,2,3$

$$f(v) = k + 2n - 2$$

$$f(u_1 = v_1) = k + 2n$$

$$f(v_{n+1-i}) = k + i - 1, 1 \leq i \leq n-1, n = 2,3$$

$$f(u_j) = k + 2n + 4j - 6, 2 \leq j \leq l+1$$

$$f(u_{l+1+j}) = k + 2n + 4l - 4j + 5, 1 \leq j \leq l$$

For $n=4,5$

$$f(v) = k + 4$$

$$f(u_1 = v_1) = k + 2n + 2$$

$$f(v_{n+1-i}) = \begin{cases} k + i - 1 & , 1 \leq i \leq 2 \\ k + 6 + 3(i - 3) & , 3 \leq i \leq n-1 \end{cases}$$

For $n=6$

$$f(v) = k + 4$$

$$f(u_1 = v_1) = k + 14$$

$$f(v_2) = k + 11$$

$$f(v_3) = k + 10$$

$$f(v_4) = k + 6$$

$$f(v_5) = k + 1$$

$$f(v_6) = k$$

$$f(u_1) = k + 2n + 2$$

$$f(u_2) = k + 2n$$

$$f(u_j) = k + 2n + 4j - 7, 3 \leq j \leq l+2$$

$$f(u_{l+2+j}) = k + 2n + 4l - 4j + 2, 1 \leq j \leq l - 1$$

m is even

Let $m = 2l$ for all $l \in \mathbb{Z}^+$

$$f(v) = k + 2n - 2$$

$$f(u_1 = v_1) = k + 2n$$

$$f(v_{n+1-i}) = k + i - 1, 1 \leq i \leq n - 1, n = 2, 3$$

$$f(u_j) = k + 2n + 4j - 6, 2 \leq j \leq l$$

$$f(u_{l+j}) = k + 2n + 4l - 3(j - 1) - 1, 1 \leq j \leq 2$$

$$f(u_{l+2+j}) = k + 2n + 4l - 4j - 3, 1 \leq j \leq l - 2$$

For $n=4, 5$

$$f(v) = k + 4$$

$$f(u_1 = v_1) = k + 2n + 2$$

$$f(v_{n+1-i}) = \begin{cases} k + i - 1, & 1 \leq i \leq 2 \\ k + 6 + 3(i - 3), & 3 \leq i \leq n - 1 \end{cases}$$

For $n=6$

$$f(v) = k + 4$$

$$f(u_1 = v_1) = k + 14$$

$$f(v_2) = k + 11$$

$$f(v_3) = k + 10$$

$$f(v_4) = k + 6$$

$$f(v_5) = k + 1 \text{ and}$$

$$f(v_6) = 1$$

$$f(u_1) = k + 2n + 2$$

$$f(u_2) = k + 2n$$

$$f(u_j) = k + 2n + 4j - 7, 3 \leq j \leq l$$

$$f(u_{l+j}) = k + 2n + 4l - 4 + 3(j - 1), 1 \leq j \leq 2$$

$$f(u_{l+2+j}) = k + 2n + 4l - 4j - 2, 1 \leq j \leq l - 2$$

Clearly, the edge labels are distinct. It can be easily verified that f generates a k -super mean labeling and Hence $\langle C_m * K_{1,n} \rangle$ is a k -Super Mean Labeling, for all $m \geq 3$ and $n \leq 6$.

H. Example 2.8

Figure 4: 60-Super mean labeling of $\langle C_{12} * K_{1,3} \rangle$

III.CONCLUSIONS

Graph Labeling has its own applications in communication network and astronomy, so enormous types of labeling have grown. Towards this, k –Super Mean Labeling is also a kind of mean labeling. In this dissertation we discussed k –Super Mean Labeling of $\langle C_m, K_{1,n} \rangle$ and $\langle C_m * K_{1,n} \rangle$ graphs.

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