iJRASET

International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 Issue: I Month of publication: January 2018

DOI: <http://doi.org/10.22214/ijraset.2018.1262>

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Fuzzy Ideals and Anti Fuzzy Ideals of Near-Ring

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Abstract: The aim of this paper is to extend the notion of a fuzzy subnear-ring, fuzzy ideals of a near ring, anti fuzzy ideals of near-ring and to give some properties of fuzzy ideals and anti fuzzy ideals of a near-ring.

Keywords: Near-ring, Near-subring, Ideals of near-ring, Fuzzy set, Fuzzy subring, Fuzzy ideals of near-ring, Anti fuzzy ideals

I. INTRODUCTION

The theory of fuzzy set was introduced by Zadeh[3], applying which Rosenfeld[6] in 1971 defined fuzzy subgroups. Salah Abou-Zaid[4] introduced the theory of a fuzzy subnear-ring and fuzzy ideals of a near-ring. Fuzzy ideals of a ring and a characterization of a regular ring studied by Lui[7]. The notion of fuzzy ideals of near rings with interval valued membership functions introduced by B. Davvaz[10] in 2001. In 2001, Kyung Ho Kim and Young BaeJun[11] in their paper entitled “Normal fuzzy R-subgroups in near-rings” introduced the concept of a normal fuzzy R-subgroup in near-rings and explored some related properties. In 2005, Syam Prasad Kuncham and Satyanarayana Bhavanari in their paper entitled “Fuzzy Prime ideal of a Gamma-near-ring” introduced fuzzy prime ideal in Γ -near-rings. The anti-fuzzy ideals of near-ring defined by F. A. Azam, A. A. Mamun and F. Nasrin. In this paper we study the concept of fuzzy ideals of a near ring and some difference properties of fuzzy ideals and anti-fuzzy ideals of a near-ring.

II. PRELIMINARIES

For the sake of continuity we recall some basic definitions.

A. Definition 2.1

A set N together with two binary operations $+$ (called *addition*) and $\cdot$ (called *multiplication*) is called a (right) *near-ring* if:

- 1) N is a group (not necessarily abelian) under addition;
- 2) multiplication is associative (so N is a semi group under multiplication); and
- 3) multiplication distributes over addition on the *right*: for any $x, y, z \in N$ it holds that $(x + y) \cdot z = (x \cdot z) + (y \cdot z)$. This near-ring will be termed as right near-ring. If $z \cdot (x + y) = z \cdot x + z \cdot y$ instead of condition (3), the set N satisfies, then we call N a left near-ring. Near-rings are generalised of a rings, addition needs not be commutative and (more important) only one distributive law is postulated.

B. Examples 2.2

(1) Z be the Set of positive and negative integers with 0. $(Z, +)$ is a group. Define ‘ $\cdot$ ’ on Z by $a \cdot b = a$ for all $a, b \in Z$. Clearly $(Z, +, \cdot)$ is a near ring. (2) Let $Z_{12} = \{0, 1, 2, 3, \dots, 11\}$. $(Z_{12}, +)$ is a group under ‘ $+$ ’ modulo 12. Define ‘ $\cdot$ ’ on Z_{12} by $a \cdot b = a$ for all $a, b \in Z$. Clearly $(Z_{12}, +, \cdot)$ is a near ring. (3) Let $M_{2 \times 2} = \{(a_{ij})/ Z : Z \text{ is treated as a near ring}\}$. $M_{2 \times 2}$ under the operation of ‘ $+$ ’ and matrix multiplication ‘ $\cdot$ ’ is defined by the following:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+b & a+b \\ c+d & c+d \end{bmatrix}$$

Because we use the multiplication in Z i.e. $a \cdot b = a$. So

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix} = \begin{bmatrix} a+b & a+b \\ c+d & c+d \end{bmatrix}.$$

It is easily verified $M_{2 \times 2}$ is a near ring. We denote xy instead of $\cdot y$. Note that $x \cdot 0 = 0$ and $x(-y) = -xy$ but in general $0x \neq 0$ for some $x \in R$. An ideal I of a near-ring R is a subset of R such that

- (1) $(I, +)$ is a normal subgroup of $(R, +)$,
- (2) $RI \subseteq I$
- (3) $(r + i)s - rs \in I$ for any $i \in I$ and any $r, s \in R$.

III. FUZZY IDEALS OF NEAR-RINGS

A. Definition 3.1

Let R be a near-ring and μ be a fuzzy subset of R . We say a fuzzy subnear-ring of R if (1) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$, (2) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in R$.

B. Definition 3.2

Let R be a near-ring and μ be a fuzzy subset of R . μ is called a fuzzy left ideal of R if μ is a fuzzy subnear-ring of R and satisfies: for all $x, y \in R$.

- 1) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$,
- 2) $\mu(y + x - y) \geq \mu(x)$,
- 3) $\mu(xy) \geq \mu(y)$ or $\mu(xy) \geq \mu(x)$

C. Definition 3.3

Let R be a near-ring and μ be a fuzzy subset of R . μ is called a fuzzy right ideal of R if μ is a fuzzy subnear-ring of R and satisfies: for all $x, y \in R$. (1) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$, (2) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$, (3) $\mu(y + x - y) \geq \mu(x)$, (4) $\mu((x + i)y - xy) \geq \mu(i)$.

D. Example 3.4

Let $R := \{a, b, c, d\}$ be a set with two binary operations as follows:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

.	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	b

The we can easily see that $(R, +)$ is a group and $(R, .)$ is an semigroup and satisfies left distributive law. Hance $(R, +, .)$ is a left near-ring. Define a fuzzy subset $\mu : R \rightarrow [0,1]$ by $\mu(c) = \mu(d) < \mu(b) < \mu(a)$. Then μ is a left fuzzy ideal of R .

E. Example 3.5

Let $R := \{a, b, c, d\}$ be a set with two binary operations as follows.

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

.	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	b	c	b

Then we can easily see that $(R, +, .)$ is a left near-ring. Define a fuzzy subset $\mu : R \rightarrow [0,1]$ by $\mu(c) = \mu(d) < \mu(b) < \mu(a)$. Then μ is a fuzzy left ideal of R , but not fuzzy right ideal of R , Since $\mu((c + d)d - cd) = \mu(d) < \mu(b)$.

F. Proposition 3.6

If a fuzzy subset μ of R satisfies the properties $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$ then

- 1) $\mu(0_R) \geq \mu(x)$
- 2) $\mu(-x) = \mu(x)$, for all $x, y \in R$

Proof.(1) We have that for any $x \in R$

$$\begin{aligned}\mu(0_R) &= \mu(x - x) \\ &\geq \min\{\mu(x), \mu(x)\} \\ &= \mu(x)\end{aligned}$$

Hence $\mu(0_R) \geq \mu(x)$.

(2) By (1), we have that

$$\begin{aligned}\mu(-x) &= \mu(0_R - x) \\ &\geq \min\{\mu(0_R), \mu(x)\} \\ &= \mu(x)\end{aligned}$$

Hence $\mu(-x) = \mu(x)$. ■

G. Proposition 3.7

Let μ be a fuzzy ideal of R . If $\mu(x - y) = \mu(0_R)$ then $\mu(x) = \mu(y)$.

Proof. Assume that $\mu(x - y) = \mu(0_R)$ for all $x, y \in R$. Then

$$\begin{aligned}\mu(x) &= \mu(x - y + y) \\ &\geq \min\{\mu(x - y), \mu(y)\} \\ &= \min\{\mu(0_R), \mu(y)\} \\ &= \mu(y)\end{aligned}$$

So, $\mu(x) \geq \mu(y)$ (1)

$$\begin{aligned}\text{Also, } \mu(y) &= \mu(y - x + x) \\ &\geq \min\{\mu(y - x), \mu(x)\} \\ &= \min\{\mu(0_R), \mu(x)\} \\ &= \mu(x)\end{aligned}$$

So, $\mu(y) \geq \mu(x)$ (2)

From equation (1) and (2), Hence $\mu(x) = \mu(y)$. □

H. Proposition 3.8

If $\mu: R \rightarrow [0, 1]$ is a fuzzy ideals of near-ring R with multiplicative identity 1_R . Then $\mu(0_R) \geq \mu(x) \geq \mu(1_R) \forall x \in R$.

Proof: We know that, $\mu(x) = \mu(-x)$

And now,

$$\mu(0_R) = \mu(x - x)$$

$$\begin{aligned}
 &= \mu(x + (-x)) \\
 &\geq \min\{\mu(x), \mu(-x)\} \\
 &= \mu(x)
 \end{aligned} \tag{1}$$

Also $\mu(x) = \mu(x, 1_R)$

$$\begin{aligned}
 &\geq \min\{\mu(x), \mu(1_R)\} \\
 &\geq \mu(1_R)
 \end{aligned} \tag{2}$$

From equation (1) and (2),

$$\mu(0_R) \geq \mu(x) \geq \mu(1_R) \quad \forall x \in R. \quad \blacksquare$$

IV. ANTI-FUZZY IDEALS OF NEAR-RING

A. Definition 4.1

Let R be a near-ring and μ be a fuzzy subset of R . μ is called an anti-fuzzy left ideal of R if μ is a fuzzy subnear-ring of R and satisfies: for all $x, y \in R$.

- 1) $\mu(x - y) \leq \max\{\mu(x), \mu(y)\}$,
- 2) $\mu(y + x - y) \leq \mu(x)$,
- 3) $\mu(xy) \leq \mu(y)$ or $\mu(xy) \leq \mu(x)$

B. Definition 4.2

Let R be a near-ring and μ be a fuzzy subset of R . μ is called a anti fuzzy right ideal of R if μ is a fuzzy subnear-ring of R and satisfies: for all $x, y \in R$,

- 1) $\mu(x - y) \leq \max\{\mu(x), \mu(y)\}$,
- 2) $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$
- 3) $\mu(y + x - y) \leq \mu(x)$
- 4) $\mu((x + i)y - xy) \leq \mu(i)$.

C. Proposition 4.3 For every anti fuzzy ideals μ of R ,

- 1) $\mu(0_R) \leq \mu(x), \forall x \in R$.
- 2) $\mu(x) = \mu(-x), \forall x \in R$.
- 3) $\mu(x - y) = \mu(0_R) \Rightarrow \mu(x) = \mu(y), \forall x, y \in R$.

Proof.(1) $\mu(0_R) = \mu(x - x)$

$$\begin{aligned}
 &\leq \max\{\mu(x), \mu(x)\} \\
 &= \mu(x) .
 \end{aligned}$$

(2) $\mu(-x) = \mu(0_R - x)$

$$\leq \max\{\mu(0_R), \mu(x)\}$$

$$= \mu(x).$$

For all $x \in R$. Since x is arbitrary, we conclude that $\mu(-x) = \mu(x)$.

(3) Assume that $\mu(x - y) = \mu(0_R)$ for all $x, y \in R$. Then

$$\begin{aligned}\mu(x) &= \mu(x - y + y) \\ &\leq \max\{\mu(x - y), \mu(y)\} \\ &= \max\{\mu(0_R), \mu(x)\} \\ &= \mu(y)\end{aligned}$$

$$\text{So, } \mu(x) \leq \mu(y) \tag{1}$$

Also,

$$\begin{aligned}\mu(y) &= \mu(y - x + x) \\ &\leq \max\{\mu(y - x), \mu(x)\} \\ &= \max\{\mu(0_R), \mu(x)\} \\ &= \mu(x)\end{aligned}$$

$$\text{So, } \mu(y) \leq \mu(x) \tag{2}$$

From equation (1) and (2)

Hence $\mu(x) = \mu(y)$. ■

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