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Border and Exterior of Soft Semi $\#g\alpha$ Closed Sets in Soft Bi-Topological Spaces

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Abstract: In this paper, we introduce a new class of closed sets via, soft semi $\#g\alpha$ -closed sets in bi-topological spaces. And also we study the concepts of border and exterior of soft semi $\#g\alpha$ -closed sets in bi-topological spaces which are denoted by $(1, 2)^*$ soft semi $\#g\alpha$ -bd (F, A) and $(1, 2)^*$ soft semi $\#g\alpha$ -ext (F, A) , where (F, A) is any soft set of (X, E) and also investigate their basic properties.

Keywords: $(1, 2)^*$ soft semi $\#g\alpha$ -closed set, $(1, 2)^*$ soft semi $\#g\alpha$ -open set, $(1, 2)^*$ soft semi $\#g\alpha$ -interior, $(1, 2)^*$ soft semi $\#g\alpha$ -closure, $(1, 2)^*$ soft semi $\#g\alpha$ -border and $(1, 2)^*$ soft semi $\#g\alpha$ -exterior.

I. INTRODUCTION

In 1963, the concepts of bi-topological spaces was originally initiated by J.C. Kelly[3]. The theory of generalized closed sets in topological spaces which was found by Levine[8] in 1970. The concepts of generalized and semi generalized closed sets was introduced and studied by Lellis[7] in classical topology. He defined a bi-topological space (X, τ_1, τ_2) to be a set X with two topologies τ_1 and τ_2 on X and initiated the systematic study of bi-topological spaces. The soft theory is rapidly processing in different field of mathematics. It was first proposed by Russian researcher Molodtsov[9] in 1999. Muhammad Shabir and Manazza Naz [10] introduced soft topological spaces in 2011. It was defined over an initial universe with a fixed set of parameters. N. Cagman and S. Karatas[2] introduced topology on a set called “soft topology” and initiated the theory of soft topological spaces in 2013. In this paper we defined and examined the basic properties of $(1, 2)^*$ soft semi $\#g\alpha$ -border and $(1, 2)^*$ soft semi $\#g\alpha$ -exterior in soft bi-topological spaces and study their properties.

II. PRELIMINARIES

In this section we have presented some of the basic definitions and results of soft set, soft topological space, bi-topological space to use in the sequel. Throughout this paper, X is an initial universe, E is the set of parameters, $P(X)$ is the power set of X , and $A \subseteq X$.

A. Definition 2.1.

Let $\tilde{\tau}$ be the collection of soft sets over X , then $\tilde{\tau}$ is called a soft topology on X if $\tilde{\tau}$ satisfies the following axioms:
 $\emptyset, \tilde{X}$ belongs to $\tilde{\tau}$.
The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
The triplet $(X, \tilde{\tau}, E)$ is called a soft topological space over X . For simplicity, we can take the soft topological space $(X, \tilde{\tau}, E)$ as X throughout the work.

B. Definition 2.2.

A set X together with two different topologies is called bi-topological space. It is denoted by (X, τ_1, τ_2) .

C. Definition 2.3.

A soft set (F, A) of a soft topological space $(X, \tilde{\tau}, E)$ is called

- 1) soft α - closed [4] if $\tilde{sc}l(\tilde{int}(\tilde{sc}l(F, A))) \subseteq (F, A)$. The complement of soft α -closed set is called soft α -open.
- 2) soft semi - closed [2] if $\tilde{sc}l(\tilde{int}(F, A)) \subseteq (F, A)$. The complement of soft semi - closed set is called soft semi-open.
- 3) soft g -closed [5] if $\tilde{sc}l(F, A) \subseteq (U, E)$, whenever $(F, A) \subseteq (U, E)$ and (U, E) is soft open in $(X, \tilde{\tau}, E)$. The complement of soft g -closed set is called soft g -open.

- 4) soft $g^\# \alpha$ -closed [6] if $\tilde{s}acl(F,A) \subseteq (U,E)$, whenever $(F,A) \subseteq (U,E)$ and (U,E) is soft g -open in $(X, \tilde{\tau}, E)$. The complement of soft $g^\# \alpha$ -closed set is called soft $g^\# \alpha$ -open. soft $\#g\alpha$ -closed [8] if $\tilde{s}acl(F,A) \subseteq (U,E)$, whenever $(F,A) \subseteq (U,E)$ and (U,E) is soft $g^\# \alpha$ -open in $(X, \tilde{\tau}, E)$. The complement of soft $\#g\alpha$ -closed set is called soft $\#g\alpha$ -open.
- 5) soft semi $\#g\alpha$ -closed [9] if $\tilde{s} scl(F,A) \subseteq (U,E)$, whenever $(F,A) \subseteq (U,E)$ and (U,E) is soft $\#g\alpha$ -open in $(X, \tilde{\tau}, E)$. The complement of soft semi $\#g\alpha$ -closed set is called soft semi $\#g\alpha$ -open
- 6) The union of all soft semi $\#g\alpha$ open sets [10] each contained in a set (F,A) of $(X, \tilde{\tau}, E)$ is called soft semi $\#g\alpha$ interior of (F,A) which is denoted by $\tilde{s} semi \#g\alpha\text{-int}(F,A)$
- 7) The intersection of all soft semi $\#g\alpha$ -closed sets [10], each containing a set (F,A) of $(X, \tilde{\tau}, E)$ is called soft semi $\#g\alpha$ -closure of (F,A) , which is denoted by $\tilde{s} semi \#g\alpha\text{-closure of } (F,A)$.

D. Definition 2.4.

Let X be a non-empty soft set on the universe X , $\tilde{\tau}_1, \tilde{\tau}_2$ are different soft topologies on $\tilde{X}$. Then $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2)$ is called a soft bi-topological space.

E. Definition 2.5.

Let $F_A \in S(U)$. Power soft set of F_A is defined by, $\tilde{P}(F_A) = \{F_{Ai} : i \in I\}$

And its cardinality is defined by $|\tilde{P}(F_A)| = 2 \sum_{x \in E} |f_A(x)|$ where $|f_A(X)|$ is cardinality of $f_A(X)$.

F. Example 2.6.

Let $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2\}$ and $F_E = X = \{(x_1, \{u_1, u_2, u_3\}), (x_2, \{u_1, u_2, u_3\})\}$. And let $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bi-topological space, where $\tilde{\tau}_1 = \{\emptyset, \tilde{\tau}_2, \tilde{\tau}_3, \tilde{\tau}_5, X\}$, $\tilde{\tau}_2 = \{\tilde{\tau}_1, \tilde{\tau}_2, \tilde{\tau}_8, \tilde{\tau}_{14}, X\}$, then $\tilde{\tau}_{1,2}$ soft open sets are $\{\tilde{\tau}_1, \tilde{\tau}_2, \tilde{\tau}_3, \tilde{\tau}_5, \tilde{\tau}_8, \tilde{\tau}_{14}, \tilde{\tau}_{17}, \tilde{\tau}_{32}, X\}$ and $\tilde{\tau}_{1,2}$ soft closed sets are $\{\tilde{\tau}_1, \tilde{\tau}_4, \tilde{\tau}_6, \tilde{\tau}_7, \tilde{\tau}_{12}, \tilde{\tau}_{31}, \tilde{\tau}_{44}, \tilde{\tau}_{46}, X\}$. Then,

$$\begin{aligned}
 \tilde{\tau}_1 &= \{ \emptyset, \tilde{\tau}_2, \tilde{\tau}_3, \tilde{\tau}_5, \tilde{\tau}_8, \tilde{\tau}_{14}, \tilde{\tau}_{17}, \tilde{\tau}_{32}, X \} \\
 \tilde{\tau}_2 &= \{ \emptyset, \tilde{\tau}_1, \tilde{\tau}_2, \tilde{\tau}_8, \tilde{\tau}_{14}, X \} \\
 \tilde{\tau}_3 &= \{ \emptyset, \tilde{\tau}_1, \tilde{\tau}_3, \tilde{\tau}_5, \tilde{\tau}_8, \tilde{\tau}_{14}, \tilde{\tau}_{17}, \tilde{\tau}_{32}, X \} \\
 \tilde{\tau}_4 &= \{ \emptyset, \tilde{\tau}_1, \tilde{\tau}_4, \tilde{\tau}_6, \tilde{\tau}_7, \tilde{\tau}_{12}, \tilde{\tau}_{31}, \tilde{\tau}_{44}, \tilde{\tau}_{46}, X \} \\
 \tilde{\tau}_5 &= \{ \emptyset, \tilde{\tau}_1, \tilde{\tau}_5, \tilde{\tau}_8, \tilde{\tau}_{14}, \tilde{\tau}_{17}, \tilde{\tau}_{32}, X \} \\
 \tilde{\tau}_6 &= \{ \emptyset, \tilde{\tau}_1, \tilde{\tau}_6, \tilde{\tau}_7, \tilde{\tau}_{12}, \tilde{\tau}_{31}, \tilde{\tau}_{44}, \tilde{\tau}_{46}, X \} \\
 \tilde{\tau}_7 &= \{ \emptyset, \tilde{\tau}_1, \tilde{\tau}_7, \tilde{\tau}_{12}, \tilde{\tau}_{31}, \tilde{\tau}_{44}, \tilde{\tau}_{46}, X \} \\
 \tilde{\tau}_8 &= \{ \emptyset, \tilde{\tau}_2, \tilde{\tau}_8, \tilde{\tau}_{14}, X \} \\
 \tilde{\tau}_9 &= \{ \emptyset, \tilde{\tau}_2, \tilde{\tau}_9, \tilde{\tau}_{14}, X \} \\
 \tilde{\tau}_{10} &= \{ \emptyset, \tilde{\tau}_2, \tilde{\tau}_{10}, \tilde{\tau}_{14}, X \} \\
 \tilde{\tau}_{11} &= \{ \emptyset, \tilde{\tau}_2, \tilde{\tau}_{11}, \tilde{\tau}_{14}, X \} \\
 \tilde{\tau}_{12} &= \{ \emptyset, \tilde{\tau}_2, \tilde{\tau}_{12}, \tilde{\tau}_{14}, X \} \\
 \tilde{\tau}_{13} &= \{ \emptyset, \tilde{\tau}_1, \tilde{\tau}_{13}, \tilde{\tau}_{14}, X \} \\
 \tilde{\tau}_{14} &= \{ \emptyset, \tilde{\tau}_1, \tilde{\tau}_{14}, \tilde{\tau}_2, \tilde{\tau}_8, \tilde{\tau}_{14}, X \} \\
 \tilde{\tau}_{15} &= \{ \emptyset, \tilde{\tau}_1, \tilde{\tau}_{15}, \tilde{\tau}_2, \tilde{\tau}_8, \tilde{\tau}_{14}, X \} \\
 \tilde{\tau}_{16} &= \{ \emptyset, \tilde{\tau}_1, \tilde{\tau}_{16}, \tilde{\tau}_2, \tilde{\tau}_8, \tilde{\tau}_{14}, X \} \\
 \tilde{\tau}_{17} &= \{ \emptyset, \tilde{\tau}_1, \tilde{\tau}_{17}, \tilde{\tau}_2, \tilde{\tau}_8, \tilde{\tau}_{14}, X \} \\
 \tilde{\tau}_{18} &= \{ \emptyset, \tilde{\tau}_1, \tilde{\tau}_{18}, \tilde{\tau}_2, \tilde{\tau}_8, \tilde{\tau}_{14}, X \} \\
 \tilde{\tau}_{19} &= \{ \emptyset, \tilde{\tau}_1, \tilde{\tau}_{19}, \tilde{\tau}_2, \tilde{\tau}_8, \tilde{\tau}_{14}, X \} \\
 \tilde{\tau}_{20} &= \{ \emptyset, \tilde{\tau}_1, \tilde{\tau}_{20}, \tilde{\tau}_2, \tilde{\tau}_8, \tilde{\tau}_{14}, X \} \\
 \tilde{\tau}_{21} &= \{ \emptyset, \tilde{\tau}_1, \tilde{\tau}_{21}, \tilde{\tau}_2, \tilde{\tau}_8, \tilde{\tau}_{14}, X \} \\
 \tilde{\tau}_{22} &= \{ \emptyset, \tilde{\tau}_1, \tilde{\tau}_{22}, \tilde{\tau}_2, \tilde{\tau}_8, \tilde{\tau}_{14}, X \}
 \end{aligned}$$

$$\begin{aligned}
 23 &= \{(1, \{3\}), (2, \{3, 1\})\} \\
 24 &= \{(1, \{1\}), (2, \{3\})\} \\
 25 &= \{(1, \{1\}), (2, \{2, 3\})\} \\
 26 &= \{(1, \{2\}), (2, \{3, 1\})\} \\
 27 &= \{(1, \{2\}), (2, \{3\})\} \\
 28 &= \{(1, \{2\}), (2, \{2, 3\})\} \\
 29 &= \{(1, \{1\}), (2, \{3, 1\})\} \\
 30 &= \{(1, \{3\}), (2, \{3\})\} \\
 31 &= \{(1, \{3\}), (2, \{2, 3\})\} \\
 32 &= \{(1, \{1, 2\}), (2, \{1\})\} \\
 33 &= \{(1, \{1, 2\}), (2, \{2\})\} \\
 34 &= \{(1, \{1, 2\}), (2, \{1, 2\})\} \\
 35 &= \{(1, \{2, 3\}), (2, \{1\})\} \\
 36 &= \{(1, \{2, 3\}), (2, \{2\})\} \\
 37 &= \{(1, \{2, 3\}), (2, \{1, 2\})\} \\
 38 &= \{(1, \{3, 1\}), (2, \{1\})\} \\
 39 &= \{(1, \{3, 1\}), (2, \{2\})\} \\
 40 &= \{(1, \{3, 1\}), (2, \{1, 2\})\} \\
 41 &= \{(1, \{1, 2\}), (2, \{3\})\} \\
 42 &= \{(1, \{1, 2\}), (2, \{2, 3\})\} \\
 43 &= \{(1, \{2, 3\}), (2, \{3\})\} \\
 44 &= \{(1, \{2, 3\}), (2, \{2, 3\})\} \\
 45 &= \{(1, \{3, 1\}), (2, \{3\})\} \\
 46 &= \{(1, \{3, 1\}), (2, \{2, 3\})\} \\
 47 &= \{(1, \{1, 2, 3\})\} \\
 48 &= \{(1, \{1, 2, 3\}), (2, \{1\})\} \\
 49 &= \{(1, \{1, 2, 3\}), (2, \{2\})\} \\
 50 &= \{(1, \{1, 2, 3\}), (2, \{1, 2\})\} \\
 51 &= \{(1, \{1, 2, 3\}), (2, \{3\})\} \\
 52 &= \{(1, \{1, 2, 3\}), (2, \{2, 3\})\} \\
 53 &= \{(1, \{1, 2, 3\}), (2, \{3, 1\})\} \\
 54 &= \{(1, \{1\}), (2, \{1, 2, 3\})\} \\
 55 &= \{(1, \{2\}), (2, \{1, 2, 3\})\} \\
 56 &= \{(1, \{1, 2\}), (2, \{1, 2, u_3\})\} \\
 57 &= \{(1, \{3\}), (2, \{1, 2, 3\})\} \\
 58 &= \{(1, \{2, 3\}), (2, \{1, 2, 3\})\} \\
 59 &= \{(1, \{1, 3\}), (2, \{1, 2, 3\})\} \\
 60 &= \{(1, \{1, 3\}), (2, \{1, 3\})\} \\
 61 &= \{(1, \{1, 2\}), (2, \{1, 3\})\} \\
 62 &= \{(1, \{2, 3\}), (2, \{1, 3\})\} \\
 63 &= \{(2, \{1, 2, 3\})\}
 \end{aligned}$$

$\sigma_4 = \{(\{I_1, \{I_1, I_2, I_3\}, \{I_2, \{I_1, I_2, I_3\}\}) = X$. Are all soft subsets of $\sim () = 2^6 = 64$

G. Definition 2.7.

A soft set (F, A) of a soft bi-topological space $(\sim, \sim_{I_1}, \sim_{I_2}, E)$ is called $(I, 2)^*$ soft semi #ga-closed if $\sim \text{scl}(F, A) \subseteq (U, E)$, whenever $(F, A) \subseteq (U, E)$ and (U, E) is $(I, 2)^*$ soft #ga-open in $(\sim, \sim_{I_1}, \sim_{I_2}, E)$. The complement of $(I, 2)^*$ soft semi #ga-closed set is called $(I, 2)^*$ soft semi #ga-open.

H. Theorem 2.8.

If (F, A) and $\sim_{I, 2}(G, B)$ are soft subset of (X, E) , then

- 1) (F, A) is $(I, 2)^*$ soft semi #ga-open iff $(I, 2)^*$ soft semi #ga-int $(F, A) \cong (F, A)$.
- 2) $(I, 2)^*$ soft semi #ga-int (F, A) is $(I, 2)^*$ soft semi #ga-open
- 3) (F, A) is $(I, 2)^*$ soft semi #ga-closed iff $(I, 2)^*$ soft semi #ga-cl $(F, A) \cong (F, A)$
- 4) $(I, 2)^*$ soft semi #ga-cl (F, A) is $(I, 2)^*$ soft semi #ga-closed.
- 5) $(I, 2)^*$ soft semi #ga-cl $((X, E) \setminus (F, A)) \cong (X, E) \setminus (I, 2)^*$ soft semi #ga-int (F, A) .
- 6) $(I, 2)^*$ soft semi #ga-int $((X, E) \setminus (F, A)) \cong (X, E) \setminus (I, 2)^*$ soft semi #ga-cl (F, A) .
- 7) If (F, A) is $(I, 2)^*$ soft semi #ga-open in $(\sim, \sim_{I_1}, \sim_{I_2}, E)$ and $\sim_{I, 2}(G, B)$ is $(I, 2)^*$ soft semi #ga-open in $(\sim, \sim_{I_1}, \sim_{I_2}, E)$, then $(F, A) \cap \sim_{I, 2}(G, B)$ is $(I, 2)^*$ soft semi #ga-open in $(\sim, \sim_{I_1}, \sim_{I_2}, E)$
- 8) A point $x \in (I, 2)^*$ soft semi #ga-cl (F, A) iff every $(I, 2)^*$ soft semi #ga-open set in (X, E) containing x intersects (F, A) .
- 9) Arbitrary intersection of $(I, 2)^*$ soft semi #ga-closed sets in $(\sim, \sim_{I_1}, \sim_{I_2}, E)$ is also $(I, 2)^*$ soft semi #ga-closed in $(\sim, \sim_{I_1}, \sim_{I_2}, E)$.

Proof We know that, $(I, 2)^*$ soft union of all $(I, 2)^*$ soft open sets contained in (F, A) is called $\sim \text{int}(F, A)$. So $(I, 2)^*$ soft semi #ga-open sets also is in (F, A) .

Since $(I, 2)^*$ soft semi #ga-int (F, A) are $(I, 2)^*$ soft open sets. Therefore $(I, 2)^*$ soft semi #ga-int (F, A) is $(I, 2)^*$ soft semi #ga-open sets.

Let the $(I, 2)^*$ soft intersection of all $(I, 2)^*$ soft closed sets containing (F, A) is called $\sim \text{cl}(F, A)$. So $(I, 2)^*$ soft semi #ga-closed sets is in (F, A) . (iv) is similar to (iii).

$(I, 2)^*$ soft complement of (X, E) and (F, A) is equal to $(I, 2)^*$ soft complement of (X, E) and $(I, 2)^*$ soft semi #ga-int (F, A) .

Similarly (vi) can be proved. (vii) and (viii) are follow from the definition of $(I, 2)^*$ soft interior. (ix) obvious from the definition of $(I, 2)^*$ soft closure.

I. Definition 2.9.

For any soft subset (F, A) of $\sim_{I, 2}(X, E)$,

The soft border of (F, A) is defined by soft bd $(F, A) \cong (F, A) \setminus (I, 2)^*$ soft int (F, A) .

The soft exterior of (F, A) is defined by soft ext $(F, A) \cong (I, 2)^*$ soft int $((X, E) \setminus (F, A))$.

III. SOFT SEMI #G-ALPHA BORDER AND EXTERIOR OF A SET IN BI-TOPOLOGICAL SPACES

In this section, we introduce and study the concepts of border and exterior of soft semi #ga-closed sets in soft bi-topological spaces.

A. Definition 3.1.

For any soft subset (F, A) of $\sim_{I, 2}(X, E)$, $(I, 2)^*$ soft semi #ga-border of (F, A) is defined by

$$(I, 2)^* \text{ soft semi #ga-bd } (F, A) \cong (F, A) \setminus (I, 2)^* \text{ soft semi #ga-int } (F, A).$$

B. Theorem 3.2.

In a soft bi-topological space $(X, \sim_{I, 2}, E)$, for any soft subset (F, A) of (X, E) , the following statements hold.

$$(I, 2)^* \text{ soft semi #ga-bd } (\square) \cong (I, 2)^* \text{ soft semi #ga-bd } (X, E) \cong \square.$$

$$(I, 2)^* \text{ soft semi #ga-bd } (F, A) \subseteq (F, A).$$

$$(F, A) \cong (I, 2)^* \text{ soft semi #ga-int } (F, A) \cup (I, 2)^* \text{ soft semi #ga-bd } (F, A).$$

$$(I, 2)^* \text{ soft semi #ga-int } (F, A) \cap (I, 2)^* \text{ soft semi #ga-bd } (F, A) \cong \square.$$

$$\text{soft semi #ga-int } (F, A) \cong (F, A) \setminus (I, 2)^* \text{ soft semi #ga-bd } (F, A).$$

Proof Let us take $(I, 2)^*$ soft semi #ga-bd $(\square)$ is in $(I, 2)^*$ soft semi #ga-bd (X, E) and is an empty set.

$(I,2)^*$ soft semi $\#g\alpha$ -bd (F,A) it should be any subset of (F,A) .

Since soft union of $(I,2)^*$ soft semi $\#g\alpha$ -bd (F,A) and $(I,2)^*$ soft semi $\#g\alpha$ -int (F,A) are for any subset of (F,A) .

The soft intersection of $(I,2)^*$ soft semi $\#g\alpha$ -bd (F,A) and $(I,2)^*$ soft semi $\#g\alpha$ -int (F,A) are empty set.

The soft complement of $(I,2)^*$ soft semi $\#g\alpha$ -bd (F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -int (F,A) .

C. Theorem 3.3.

$(I,2)^*$ soft semi $\#g\alpha$ -int $((I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A)) \cong \square$. (F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -open if and only if $(I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A) \cong \square$.

soft semi $\#g\alpha$ -bd $((I,2)^*$ soft semi $\#g\alpha$ -int $(F,A)) \cong \square$.

$(I,2)^*$ soft semi $\#g\alpha$ -bd $((I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A)) \cong (I,2)^*$ soft semi $\#g\alpha$ -bd (F,A) .

soft semi $\#g\alpha$ -bd $(F,A) \cong (F,A) \tilde{\cap} (I,2)^*$ soft semi $\#g\alpha$ -cl $((X,E) \setminus (F,A))$.

Proof: Let $x \in (I,2)^*$ soft semi $\#g\alpha$ -int $((I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A))$. Then $x \in (I,2)^*$ soft semi $\#g\alpha$ -bd (F,A) , since $(I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A) \subseteq (F,A)$, $x \in (I,2)^*$ soft semi $\#g\alpha$ -int $((I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A)) \subseteq (I,2)^*$ soft semi $\#g\alpha$ -int (F,A) . Therefore $x \in (I,2)^*$ soft semi $\#g\alpha$ -int $(F,A) \tilde{\cap} (I,2)^*$ soft semi $\#g\alpha$ -bd (F,A) which is contradiction to the above theorem (iv). Thus (i) is proved.

(F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -open iff $(I,2)^*$ soft semi $\#g\alpha$ -int $(F,A) \cong (F,A)$ [Theorem 2.8(i)]. But $(I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A) \cong (F,A) \setminus (I,2)^*$ soft semi $\#g\alpha$ -int (F,A) implies $(I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A) \cong \square$. This proves (ii) and (iii).

And when $(F,A) \cong (I,2)^*$ soft semi $\#g\alpha$ -bd (F,A) Definition 3.1 becomes $(I,2)^*$ soft semi $\#g\alpha$ -bd $((I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A)) \cong (I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A) \setminus (I,2)^*$ soft semi $\#g\alpha$ -int $((I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A))$. Using (iii), we will get (iv).

(v) $(I,2)^*$ soft semi $\#g\alpha$ -bd $(F,A) \cong (F,A) \setminus (I,2)^*$ soft semi $\#g\alpha$ -int $(F,A) \cong (F,A) \tilde{\cap} (\sim_{I,2}(X,E) \setminus (I,2)^*$ soft semi $\#g\alpha$ -int $(F,A)) \cong (F,A) \tilde{\cap} (I,2)^*$ soft semi $\#g\alpha$ -cl $((X,E) \setminus (F,A))$ [Theorem 2.8 (v)]. Hence (v) is also proved.

D. Definition.3.4.

For any soft subset (F,A) of $\sim_{I,2}(X,E)$, its $(I,2)^*$ soft semi $\#g\alpha$ -exterior is defined by, $(I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A) \cong (I,2)^*$ soft semi $\#g\alpha$ -int $((X,E) \setminus (F,A))$.

E. Theorem 3.5.

For any $\cong (I,2)^*$ soft subets (F,A) and $\sim_{I,2}(G,B)$ of $\sim_{I,2}(X,E)$, in soft bi-topological space $(X, \sim_{I,2}, E)$, the following statements hold.

$(I,2)^*$ soft semi $\#g\alpha$ -ext $(\square) \cong (I,2)^*$ soft semi $\#g\alpha$ -ext $(X,E) \cong \sim$.

If $(F,A) \subseteq \sim_{I,2}(G,B)$, then $(I,2)^*$ soft semi $\#g\alpha$ -ext $(G,B) \subseteq (I,2)^*$ soft semi $\#g\alpha$ -ext (F,A) .

$(I,2)^*$ soft semi $\#g\alpha$ -ext (F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -open.

(F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -closed if and only if $(I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A) \cong \sim_{I,2}(X,E) \setminus (F,A)$.

$(I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A) \cong (X,E) \setminus (I,2)^*$ soft semi $\#g\alpha$ -cl (F,A) .

Proof: Let us take (i) $(I,2)^*$ soft semi $\#g\alpha$ -ext $(\square)$ is in $(I,2)^*$ soft semi $\#g\alpha$ -ext (X,E) and is an empty set.

And (ii) if any soft subset of (F,A) is contained in $\sim_{I,2}(G,B)$ then, $(I,2)^*$ soft semi $\#g\alpha$ -ext (F,A) is always contained in $(I,2)^*$ soft semi $\#g\alpha$ -ext (G,B) .

Since $(I,2)^*$ soft semi $\#g\alpha$ -int (F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -open, proof of (iii) is follow from the definition 3.4.

is $(I,2)^*$ soft semi $\#g\alpha$ -cl (F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -closed.

Since $(I,2)^*$ soft semi $\#g\alpha$ -int $((X,E) \setminus (F,A)) \cong (X,E) \setminus (I,2)^*$ soft semi $\#g\alpha$ -cl (F,A) , (v) follows from definition 3.4.

F. Theorem 3.6.

$(I,2)^*$ soft semi $\#g\alpha$ -ext $((I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A)) \cong (I,2)^*$ soft semi $\#g\alpha$ -int $((I,2)^*$ soft semi $\#g\alpha$ -cl $(F,A))$.

(F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -regular, then $(I,2)^*$ soft semi $\#g\alpha$ -ext $((I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A)) \cong (F,A)$.

$(I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A) \cong (I,2)^*$ soft semi $\#g\alpha$ -ext $((X,E) \setminus ((I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A)))$.

$(I,2)^*$ soft semi $\#g\alpha$ -int $(F,A) \subseteq (I,2)^*$ soft semi $\#g\alpha$ -ext $((I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A))$.

Proof: Since $(I,2)^*$ soft semi $\#g\alpha$ -int $(X,E) \setminus (F,A) \cong (X,E) \setminus (I,2)^*$ soft semi $\#g\alpha$ -ext (F,A) , (i) follows from definition 3.4. Similarly (ii) can be proved. If (F,A) is $(I,2)^*$ soft semi $\#g\alpha$ -regular, from the above theorem (iv), we have $(I,2)^*$ soft semi $\#g\alpha$ -ext $(F,A) \cong (X,E) \setminus (F,A)$ which is also $(I,2)^*$ soft semi $\#g\alpha$ -regular.

Thus $(I,2)^* \text{soft semi } \#g\alpha\text{-ext}(\text{soft semi } \#g\alpha\text{-ext}(F,A)) \cong (F,A)$, (ii) is proved. (iii) $(I,2)^* \text{soft semi } \#g\alpha\text{-ext}((X,E) \setminus (I,2)^* \text{soft semi } \#g\alpha\text{-ext}(F,A)) \cong (I,2)^* \text{soft semi } \#g\alpha\text{-ext}((X,E) \setminus ((X,E) \setminus (I,2)^* \text{soft semi } \#g\alpha\text{-int}((X,E) \setminus (F,A)))) \cong (I,2)^* \text{soft semi } \#g\alpha\text{-int}((X,E) \setminus ((X,E) \setminus (I,2)^* \text{soft semi } \#g\alpha\text{-int}((X,E) \setminus (F,A)))) \cong (I,2)^* \text{soft semi } \#g\alpha\text{-int}((I,2)^* \text{soft semi } \#g\alpha\text{-int}(X,E) \setminus (F,A)) \cong ((I,2)^* \text{soft semi } \#g\alpha\text{-int}(X,E) \setminus (F,A)) \cong (I,2)^* \text{soft semi } \#g\alpha\text{-ext}(F,A)$. Hence (iii) is proved.

Since $(F,A) \subseteq (I,2)^* \text{soft semi } \#g\alpha\text{-cl}(F,A)$, using (i) (iv) can be proved.

G. Theorem 3.7.

$(X,E) \cong (I,2)^* \text{soft semi } \#g\alpha\text{-int}(F,A) \cup (I,2)^* \text{soft semi } \#g\alpha\text{-ext}(F,A) \cup (I,2)^* \text{soft semi } \#g\alpha\text{-fr}(B,E)$.

$(I,2)^* \text{soft semi } \#g\alpha\text{-ext}((F,A) \cup \widetilde{I,2}(G,B)) \subseteq (I,2)^* \text{soft semi } \#g\alpha\text{-ext}(F,A) \cap (I,2)^* \text{soft semi } \#g\alpha\text{-ext}(B,E)$.

$(I,2)^* \text{soft semi } \#g\alpha\text{-ext}((F,A) \cap \widetilde{I,2}(G,B)) \subseteq (I,2)^* \text{soft semi } \#g\alpha\text{-ext}(F,A) \cup (I,2)^* \text{soft semi } \#g\alpha\text{-ext}(B,E)$.

Proof If we know that complement of $(I,2)^* \text{soft semi } \#g\alpha\text{-int}(X,E)$ is in (F,A) and union of $(I,2)^* \text{soft semi } \#g\alpha\text{-int}(F,A)$ is $\widetilde{I,2}(X,E)$.

And then any subset of (F,A) of $\widetilde{I,2}(\quad)$, its exterior is complement of $\widetilde{I,2}(X,E)$, its exterior is complement of $\widetilde{I,2}(X,E)$ and (F,A) . So union of all $(I,2)^* \text{soft semi } \#g\alpha\text{-interior}$, exterior and frontier is in $\widetilde{I,2}(X,E)$. Hence (i) is proved.

Proof of (ii) union of all exterior of (F,A) and $\widetilde{I,2}(G,B)$ is contained in intersection of $(I,2)^* \text{soft semi } \#g\alpha\text{-ext}(F,A)$ and $(I,2)^* \text{soft semi } \#g\alpha\text{-ext}(B,E)$. Hence (ii) is proved.

And next, proof of (iii) intersection of all exterior of (F,A) and $\widetilde{I,2}(G,B)$ is contained in union of all $(I,2)^* \text{soft semi } \#g\alpha\text{-ext}(F,A)$ and $(I,2)^* \text{soft semi } \#g\alpha\text{-ext}(B,E)$. Hence (ii) is proved.

Example 3.8.

Let $U = \{x_1, x_2, x_3\}$, $E = \{x_1, x_2\}$ and $X = \{(x_1, \{x_1, x_2, x_3\}), (x_2, \{x_1, x_2, x_3\})\}$. And let (X, τ_1, τ_2) be a soft bi-topological space, where $\tau_1 = \{\emptyset, x_2, x_3, x_5, X\}$, $\tau_2 = \{\emptyset, x_2, x_8, x_{14}, X\}$, then $\widetilde{I,2}$ soft open sets are $\{x_1, x_2, x_3, x_5, x_8, x_{14}, x_{17}, x_{32}, X\}$ and $\widetilde{I,2}$ soft closed sets are $\{x_1, x_4, x_6, x_7, x_{12}, x_{31}, x_{44}, x_{46}, X\}$. Consider a $(I,2)^* \text{soft semi } \#g\alpha\text{-closed}$ set $\sim = \{(x_1, \{x_1, x_2, x_3\}), (x_2, \{x_1\})\}$

$(I,2)^* \text{soft semi } \#g\alpha\text{-border} :$

$\{(x_1, \{x_1, x_2, x_3\}), (x_2, \{x_1\})\}$

$(I,2)^* \text{soft semi } \#g\alpha\text{-exterior} :$

$\{(x_1, \{x_1, x_2, x_3\}), (x_2, \{x_1\})\}$

IV. CONCLUSIONS

In this paper, Border and Exterior of soft semi $g\alpha$ -closed sets in soft bi-topological spaces were introduced and studied with already existing sets in soft bi-topological spaces. The scope for further research can be focused on the applications of soft bi-topological spaces.

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