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Vague generalized b continuous mappings

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Abstract: The aim of this paper is to introduce and investigate a new class of continuous mapping in vague topological spaces namely vague generalized b continuous mapping, vague generalized b irresolute mapping and their properties are discussed.

Keywords: Vague topology, vague generalized b continuous mappings, vague generalized b irresolute mappings.

I. INTRODUCTION

In 1970, Levine [8] initiated the study of generalized closed sets. The concept of fuzzy sets was introduced by Zadeh [12] in 1965. The theory of fuzzy topology was introduced by C.L.Chang [6] in 1967. Several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. In 1986, the concept of intuitionistic fuzzy sets was introduced by Atanassov [2] as a generalization of fuzzy sets.

The theory of vague sets was first proposed by Gau and Buehre [7] as an extension of fuzzy set theory and vague sets are regarded as a special case of context dependent fuzzy sets. In this paper we introduce the concept of vague generalized b continuous mapping and vague generalized b irresolute mappings and also obtained their properties and relations with counter examples.

II. PRELIMINARIES

Definition 2.1: [3] A vague set A in the universe of discourse X is characterized by two membership functions given by:

- 1) A true membership function $t_A : X \rightarrow [0,1]$ and
- 2) A false membership function $f_A : X \rightarrow [0,1]$.

where $t_A(x)$ is lower bound on the grade of membership of x derived from the “evidence for x”, $f_A(x)$ is a lower bound on the negation of x derived from the “evidence against x” and $t_A(x) + f_A(x) \leq 1$. Thus the grade of membership of x in the vague set A is bounded by a subinterval $[t_A(x), 1 - f_A(x)]$ of $[0, 1]$. This indicates that if the actual grade of membership $\mu(x)$, then $t_A(x) \leq \mu(x) \leq 1 - f_A(x)$. The vague set A is written as, $A = \{ \langle x, [t_A(x), 1 - f_A(x)] \rangle / x \in X \}$ where the interval $[t_A(x), 1 - f_A(x)]$ is called the “vague value of x in A and is denoted by $V_A(x)$.

Definition 2.2: [3] Let A and B be vague sets of the form $A = \{ \langle x, [t_A(x), 1 - f_A(x)] \rangle / x \in X \}$ and $B = \{ \langle x, [t_B(x), 1 - f_B(x)] \rangle / x \in X \}$. Then

$A \subseteq B$ if and only if $t_A(x) \leq t_B(x)$ and $1 - f_A(x) \leq 1 - f_B(x)$.

$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

$$A^c = \{ \langle x, [f_A(x), 1 - t_A(x)] \rangle / x \in X \}$$

$$A \cap B = \{ \langle x, [(t_A(x) \wedge t_B(x)), ((1 - f_A(x)) \wedge (1 - f_B(x)))] \rangle / x \in X \}.$$

$$A \cup B = \{ \langle x, [(t_A(x) \vee t_B(x)), ((1 - f_A(x)) \vee (1 - f_B(x)))] \rangle / x \in X \}.$$

For the sake of simplicity, we shall use the notion $A = \{ \langle x, [t_A(x), 1 - f_A(x)] \rangle \}$ instead of

$$A = \{ \langle x, [t_A(x), 1 - f_A(x)] \rangle / x \in X \}$$

Definition 2.3:[9] Let (X, τ) be a topological space. A subset A of X is called:

- i) generalized closed (briefly, g-closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- ii) generalized semi closed (briefly gs-closed) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- iii) α -generalized closed (briefly α g-closed) if $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- iv) generalized pre closed (briefly gp-closed) if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- v) generalized b closed set (briefly gb-closed) if $\text{bcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

Definition 2.4:[9] A vague topology (VT in short) on X is a family τ of vague sets (VS in short) in X satisfying the following axioms.

$$0, 1 \in \tau$$

$$G_1 \cap G_2 \in \tau$$

$$\cup G_i \in \tau \text{ for any family } \{G_i / i \in J\} \subseteq \tau$$

In this case the pair (X, τ) is called vague topological space (VTS in short) and vague set in τ is known as vague open set (VOS in short) in X . The complement A^c of VOS in (X, τ) is called vague closed set (VCS in short) in X .

Definition 2.5:[9] A Vague set $A = \{ \langle x, [t_A, 1 - f_A] \rangle \}$ in a VTS is said to be a vague semi closed set (VSCS in short) if $\text{vint}(\text{vcl}(A)) \subseteq A$.

vague semi open set (VSOS in short) if $A \subseteq \text{vcl}(\text{vint}(A))$.

vague pre-closed set (VPCS in short) if $\text{vcl}(\text{vint}(A)) \subseteq A$.

vague pre-open set (VPOS in short) if $A \subseteq \text{vint}(\text{vcl}(A))$.

vague α -closed set ($V\alpha$ CS in short) if $\text{vcl}(\text{vint}(\text{vcl}(A))) \subseteq A$.

vague α -open set ($V\alpha$ OS in short) if $A \subseteq \text{vint}(\text{vcl}(\text{vint}(A)))$.

vague regular open set (VROS in short) if $A = \text{vint}(\text{vcl}(A))$.

vague regular closed set (VRCS in short) if $\text{vcl}(\text{vint}(A)) = A$.

Definition 2.6:[9] A vague set A of VTS (X, τ) is said to be

- 1) vague generalized closed set (VGCS in short) if $\text{vcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is VOS in X .
- 2) vague generalized semi closed set (VGSCS in short) if $\text{vscl}(A) \subseteq U$ whenever $A \subseteq U$ and U is VOS in X .
- 3) vague alpha generalized closed set ($V\alpha$ GCS in short) if $\text{v}\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is VOS in X .
- 4) vague generalized pre-closed set (VGPCS in short) if $\text{vpcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is VOS in X .

Definition 2.10:[11] Let (X, τ) be an VTS and $A = \{ \langle x, [t_A, 1 - f_A] \rangle \}$ be a vague set in X . Then the vague b closure of A ($\text{vbcl}(A)$ in short) and vague b interior of A ($\text{vbint}(A)$ in short) are defined as

$\text{vbint}(A) = \cup \{ G / G \text{ is an VbOS in } X \text{ and } G \subseteq A \}$, $\text{vbcl}(A) = \cap \{ K / K \text{ is VbCS in } X \text{ and } A \subseteq K \}$

Definition 2.11:[11] A vague set A in $\text{VTS}(X, \tau)$ is said to be vague generalized b closed set (VGbCS short) if $\text{vbcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is VOS in (X, τ) . The family of all VGbCS of a $\text{VTS}(X, \tau)$ is denoted by $\text{VGbC}(X)$.

Definition 2.12:[10] Let (X, τ) and (Y, σ) be any two vague topological spaces. A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be

- 1) vague continuous (V continuous in short) if $f^{-1}(V)$ is vague closed set in (X, τ) for every vague closed set V of (Y, σ) .
- 2) vague semi-continuous (VS continuous in short) if $f^{-1}(V)$ is vague semi-closed set in (X, τ) for every vague closed set V of (Y, σ) .
- 3) vague pre-continuous (VP continuous in short) if $f^{-1}(V)$ is vague pre-closed set in (X, τ) for every vague closed set V of (Y, σ) .
- 4) vague α -continuous ($V\alpha$ -continuous in short) if $f^{-1}(V)$ is vague α -closed set in (X, τ) for every vague closed set V of (Y, σ) .
- 5) vague regular continuous (VR continuous in short) if $f^{-1}(V)$ is vague regular closed set in (X, τ) for every vague closed set V of (Y, σ) .
- 6) vague generalized continuous (VG continuous in short) if $f^{-1}(V)$ is vague generalized closed set in (X, τ) for every vague closed set V of (Y, σ) .
- 7) vague generalized semi-continuous (VGS continuous in short) if $f^{-1}(V)$ is vague generalized semi-closed set in (X, τ) for every vague closed set V of (Y, σ) .
- 8) vague α -generalized continuous ($V\alpha G$ continuous in short) if $f^{-1}(V)$ is vague α generalized closed set in (X, τ) for every vague closed set V of (Y, σ) .
- 9) vague generalized pre-continuous (VGP continuous in short) mapping if $f^{-1}(V)$ is VGPCS in (X, τ) for every vague closed set V of (Y, σ) .

Definition 2.13:[11] A $\text{VTS}(X, \tau)$ is called

- 1) vague $bT_{1/2}$ space ($V_bT_{1/2}$ space in short) if every VbCS in X is VCS in X .
- 2) vague $gbT_{1/2}$ space ($V_{gb}T_{1/2}$ space in short) if every VGbCS in X is VCS in X .
- 3) vague gbT_b space ($V_{gb}T_b$ space in short) if every VGbCS in X is VbCS in X .

III. VAGUE GENERALIZED b CONTINUOUS MAPPINGS

Definition 3.1: A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be vague b continuous (Vb continuous in short) if $f^{-1}(V)$ is vague b closed set in (X, τ) for every vague closed set V of (Y, σ) .

Definition 3.2: A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be vague generalized b continuous (VGb continuous in short) mapping if $f^{-1}(V)$ is VGbCS in (X, τ) for every vague closed set V of (Y, σ) .

Theorem 3.3: Let (X, τ) and (Y, σ) be any two vague topological spaces. For any vague continuous function $f: (X, \tau) \rightarrow (Y, \sigma)$ we have the following:

- 1) Every V continuous mapping is Vb continuous mapping.
- 2) Every V continuous mapping is VGb continuous mapping.
- 3) Every $V\alpha$ continuous mapping is Vb continuous mapping.
- 4) Every $V\alpha$ continuous mapping is VGb continuous mapping.
- 5) Every $V\alpha G$ continuous mapping is VGb continuous mapping.
- 6) Every Vb continuous mapping is VGb continuous mapping.
- 7) Every VP continuous mapping is VGb continuous mapping.
- 8) Every VR continuous mapping is VGb continuous mapping.
- 9) Every VS continuous mapping is VGb continuous mapping.
- 10) Every VGP continuous mapping is VGb continuous mapping.

- 11) Every VP continuous mapping is Vb continuous mapping.
- 12) Every VR continuous mapping is Vb continuous mapping.
- 13) Every VS continuous mapping is Vb continuous mapping.
- 14) Every VWG continuous mapping is VGb continuous mapping.

Proof: It is obvious.

Remark 3.4: The converse of the above theorem need not be true as shown by the following examples.

Example 3.5: Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \{\langle x, [0.3, 0.8], [0.5, 0.7] \rangle\}$ and $G_2 = \{\langle y, [0.2, 0.3], [0.6, 0.8] \rangle\}$ then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a Vb continuous mapping but not V continuous, since $G_2^c = \{\langle y, [0.7, 0.8], [0.2, 0.4] \rangle\}$ is VbCS in Y but $f^{-1}(G_2^c)$ is not VCS in X.

Example 3.6: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{\langle x, [0.5, 0.7], [0.5, 0.8] \rangle\}$ and $G_2 = \{\langle y, [0.1, 0.2], [0.8, 0.9] \rangle\}$ Then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a VGb continuous mapping but not V continuous, since $G_2^c = \{\langle y, [0.8, 0.9], [0.1, 0.2] \rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VCS in X.

Example 3.7: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{\langle x, [0.5, 0.7], [0.5, 0.8] \rangle\}$, $G_2 = \{\langle y, [0.4, 0.7], [0.5, 0.7] \rangle\}$ then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a Vb continuous mapping but not V α continuous, since $G_2^c = \{\langle y, [0.3, 0.6], [0.3, 0.5] \rangle\}$ is VbCS in Y but $f^{-1}(G_2^c)$ is not V α CS in X.

Example 3.8: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{\langle x, [0.2, 0.4], [0.6, 0.8] \rangle\}$, $G_2 = \{\langle y, [0.7, 0.9], [0.3, 0.5] \rangle\}$ then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a VGb continuous mapping but not V α continuous, since $G_2^c = \{\langle y, [0.1, 0.3], [0.5, 0.7] \rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not V α CS in X.

Example 3.9: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{\langle x, [0.3, 0.5], [0.4, 0.6] \rangle\}$, $G_2 = \{\langle y, [0.2, 0.5], [0.3, 0.6] \rangle\}$ then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a VGb continuous mapping but not V α G continuous, since $G_2^c = \{\langle y, [0.5, 0.8], [0.4, 0.7] \rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not V α GCS in X.

Example 3.10: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{\langle x, [0.5, 0.7], [0.6, 0.8] \rangle\}$, $G_2 = \{\langle y, [0.6, 0.7], [0.5, 0.7] \rangle\}$ then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a VGb continuous mapping but not Vb continuous, since $G_2^c = \{\langle y, [0.3, 0.4], [0.3, 0.5] \rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VbCS in X.

Example 3.11: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{\langle x, [0.2, 0.3], [0.1, 0.5] \rangle\}$, $G_2 = \{\langle y, [0.3, 0.8], [0.2, 0.6] \rangle\}$ then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and

$f(b) = v$. Then f is a VGb continuous mapping but not VP continuous, since $G_2^c = \{\langle y, [0.2, 0.7], [0.4, 0.8] \rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VPCS in X .

Example 3.12: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{\langle x, [0.3, 0.5], [0.4, 0.8] \rangle\}$ $G_2 = \{\langle y, [0.2, 0.5], [0.3, 0.6] \rangle\}$ then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a VGb continuous mapping but not VR continuous, since $G_2^c = \{\langle y, [0.5, 0.8], [0.4, 0.7] \rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VRCS in X .

Example 3.13: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{\langle x, [0.7, 0.9], [0.1, 0.2] \rangle\}$ $G_2 = \{\langle y, [0.3, 0.4], [0.6, 0.7] \rangle\}$ then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a VGb continuous mapping but not VS continuous, since $G_2^c = \{\langle y, [0.6, 0.7], [0.3, 0.4] \rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VSCS in X .

Example 3.14: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{\langle x, [0.2, 0.3], [0.1, 0.4] \rangle\}$ $G_2 = \{\langle y, [0.3, 0.6], [0.4, 0.5] \rangle\}$ then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a VGb continuous mapping but not VGP continuous, since $G_2^c = \{\langle x, [0.4, 0.7], [0.5, 0.6] \rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VGPCS in X .

Example 3.15: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{\langle x, [0.2, 0.3], [0.1, 0.5] \rangle\}$ $G_2 = \{\langle y, [0.3, 0.8], [0.2, 0.6] \rangle\}$ then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a Vb continuous mapping but not VP continuous, since $G_2^c = \{\langle y, [0.2, 0.7], [0.4, 0.8] \rangle\}$ is VbCS in Y but $f^{-1}(G_2^c)$ is not VPCS in X .

Example 3.16: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{\langle x, [0.3, 0.5], [0.4, 0.8] \rangle\}$ $G_2 = \{\langle y, [0.2, 0.5], [0.3, 0.6] \rangle\}$ then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a Vb continuous mapping but not VR continuous, since $G_2^c = \{\langle y, [0.5, 0.8], [0.4, 0.7] \rangle\}$ is VbCS in Y but $f^{-1}(G_2^c)$ is not VRCS in X .

Example 3.17: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{\langle x, [0.7, 0.9], [0.1, 0.2] \rangle\}$ $G_2 = \{\langle y, [0.3, 0.4], [0.6, 0.7] \rangle\}$ then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a Vb continuous mapping but not VS continuous, since $G_2^c = \{\langle y, [0.6, 0.7], [0.3, 0.4] \rangle\}$ is a VbCS in Y but $f^{-1}(G_2^c)$ is not VSCS in X .

Example 3.18: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{\langle x, [0.3, 0.4], [0.6, 0.7] \rangle\}$ $G_2 = \{\langle y, [0.5, 0.7], [0.8, 0.9] \rangle\}$ then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is a VWG continuous mapping but not VGb continuous, since $G_2^c = \{\langle y, [0.3, 0.5], [0.1, 0.2] \rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VWGCS in X .

Remark 3.19: From the above theorem and examples we have the following diagrammatic representation

Theorem 3.20: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is VGb continuous mapping if and only if the inverse image of each VOS in Y is VGbOS in X .

Proof: Necessity Let A be VOS in Y . This implies A^c is VCS in Y . Since f is VGb continuous mapping, $f^{-1}(A^c)$ is VGbCS in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is VGbOS in X .

Sufficiency: It is obvious.

Theorem 3.21: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be mapping and let $f^{-1}(A)$ is VRCS in X for every VCS A in Y . Then f is VGb continuous but not conversely.

Proof: Let A be VCS in Y . Then $f^{-1}(A)$ is VRCS in X . Since every VRCS is VGbCS, $f^{-1}(A)$ is VGbCS in X . Hence f is VGb continuous mapping.

Theorem 3.22: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be VGb continuous mapping, then f is vague continuous mapping if X is $V_{gb}T_{1/2}$ space.

Proof: Let A be VCS in Y . Then $f^{-1}(A)$ is VGbCS in X , by hypothesis. Since X is $V_{gb}T_{1/2}$ space, $f^{-1}(A)$ is VCS in X . Hence f is vague continuous mapping.

Theorem 3.23: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be VGb continuous mapping, then f is Vb continuous mapping if X is $V_{gb}T_b$ space.

Proof: Let A be VCS in Y . Then $f^{-1}(A)$ is VGbCS in X , by hypothesis. Since X is $V_{gb}T_b$ space, $f^{-1}(A)$ is VbCS in X . Hence f is Vb continuous mapping.

Theorem 3.24: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from VTS X into VTS Y . Then the following conditions are equivalent if X is $V_{gb}T_b$ space.

- i) f is VGb continuous mapping.
- ii) $f^{-1}(B)$ is VGbCS in X for every VCS B in Y .
- iii) $vcl(vint(f^{-1}(A))) \cap vint(vcl(f^{-1}(A))) \subseteq f^{-1}(vcl(A))$ for every vague set A in Y .

Proof: i) $\Rightarrow$ ii): It is obvious.

ii) $\Rightarrow$ iii): Let A be vague set in Y . Then $vcl(A)$ is VCS in Y . By hypothesis, $f^{-1}(vcl(A))$ is VGbCS in X . Since X is $V_{gb}T_b$ space, $f^{-1}(vcl(A))$ is VbCS. Therefore $vcl(vint(f^{-1}(vcl(A)))) \cap vint(vcl(f^{-1}(vcl(A)))) \subseteq f^{-1}(vcl(A))$. Hence $vcl(vint(f^{-1}(A))) \cap vint(vcl(f^{-1}(A))) \subseteq f^{-1}(vcl(A))$.

iii) $\Rightarrow$ i): Let A be VCS in Y . By hypothesis $vcl(vint(f^{-1}(A))) \cap vint(vcl(f^{-1}(A))) \subseteq f^{-1}(vcl(A)) = f^{-1}(A)$. This implies $f^{-1}(A)$ is VbCS in X and hence it is VGbCS. Thus f is VGb continuous mapping.

Theorem 3.25: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from VTS X into VTS Y . Then the following conditions are equivalent if X is $V_{gb}T_b$ space

- i) f is VGb continuous mapping.
- ii) $f^{-1}(A)$ is VGbOS in X for every VOS A in Y .
- iii) $f^{-1}(vint(A)) \subseteq vcl(vint(f^{-1}(A))) \cup vint(vcl(f^{-1}(A)))$ for every VS A in Y .

Proof: i) $\Rightarrow$ ii): It is obvious.

ii) $\Rightarrow$ iii): Let A be vague set in Y . Then $vint(A)$ is VOS in Y . By hypothesis, $f^{-1}(vint(A))$ is VGbOS in X . Since X is $V_{gb}T_b$ space, $f^{-1}(vint(A))$ is VbOS in X . Therefore $f^{-1}(vint(A)) \subseteq vcl(vint(f^{-1}(vint(A)))) \cup vint(vcl(f^{-1}(vint(A))))$. Hence $f^{-1}(vint(A)) \subseteq vcl(vint(f^{-1}(vint(A)))) \cup vint(vcl(f^{-1}(vint(A))))$.

iii) $\Rightarrow$ i): Let A be VCS in Y . Then its complement, say A^c is VOS in Y , then $vint(A^c) = A^c$. Now by hypothesis $f^{-1}(vint(A^c)) \subseteq vcl(vint(f^{-1}(vint(A^c)))) \cup vint(vcl(f^{-1}(vint(A^c))))$. This implies $f^{-1}(A^c) \subseteq vcl(vint(f^{-1}(A^c))) \cup vint(vcl(f^{-1}(A^c)))$. Hence $f^{-1}(A^c)$ is VGbOS in X . Thus $f^{-1}(A)$ is VGbCS in X . Hence f is VGb continuous mapping.

Theorem 3.26: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be VGb continuous mapping and $g: (Y, \sigma) \rightarrow (Z, \mu)$ is vague continuous mapping, then $g \circ f: (X, \tau) \rightarrow (Z, \mu)$ is VGb continuous mapping.

Proof: Let A be VCS in Z . Then $g^{-1}(A)$ is VCS in Y , by hypothesis. Since f is VGb continuous mapping, $f^{-1}(g^{-1}(A))$ is VGbCS in X . Hence $g \circ f$ is VGb continuous mapping.

Remark 3.27: The composition of two VGb continuous mapping need not be VGb continuous mapping.

Example 3.28: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $Z = \{p, q\}$ vague sets G_1, G_2 and G_3 defined as follows: $G_1 = \{\langle x, [0.3, 0.8], [0.5, 0.7] \rangle\}$, $G_2 = \{\langle y, [0.2, 0.3], [0.6, 0.8] \rangle\}$ and $G_3 = \{\langle z, [0.1, 0.2], [0.8, 0.9] \rangle\}$ then $\tau = \{0, G_1, 1\}$, σ

$= \{0, G_2, 1\}$ and $\mu = \{0, G_3, 1\}$ be vague topologies on X, Y and Z respectively. Let the mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$, $g: (Y, \sigma) \rightarrow (Z, \mu)$ by $g(x) = p$ and $g(y) = q$. Then the f and g are VGb continuous mapping but the mapping $g \circ f: (X, \tau) \rightarrow (Z, \mu)$ is not VGb continuous mapping.

Definition 3.29: Let (X, τ) be VTS. The vague generalized b closure ($vgbcl(A)$ in short) for any vague set A is defined as follows, $vgbcl(A) = \bigcap \{K/K \text{ is a VGbCS in } X \text{ and } A \subseteq K\}$. If A is VGbCS, then $vgbcl(A) = A$.

Theorem 3.30: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be VGb continuous mapping. Then the following conditions hold.

- i) $f(vgbcl(A)) \subseteq vcl(f(A))$ for every vague set A in X .
- ii) $vgbcl(f^{-1}(B)) \subseteq f^{-1}(vcl(B))$ for every vague set B in Y .

Proof: i) Since $vcl(f(A))$ is VCS in Y and f is VGb continuous mapping, then $f^{-1}(vcl(f(A)))$ is VGbCS in X . That is $vgbcl(A) \subseteq f^{-1}(vcl(f(A)))$. Therefore $f(vgbcl(A)) \subseteq vcl(f(A))$ for every vague set A in X .

ii) Replacing A by $f^{-1}(B)$ in (i), we get $f(vgbcl(f^{-1}(B))) \subseteq vcl(f(f^{-1}(B))) \subseteq vcl(B)$. Hence $vgbcl(f^{-1}(B)) \subseteq f^{-1}(vcl(B))$ for every vague set B in Y .

IV. VAGUE GENERALIZED b IRRESOLUTE MAPPINGS

Definition 4.1: A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be vague generalized b irresolute (VGb irresolute in short) mapping if $f^{-1}(A)$ is VGbCS in (X, τ) for every VGbCS A in (Y, σ) .

Theorem 4.2: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be VGb irresolute mapping, then f is VGb continuous mapping but not conversely.

Proof: Let f be VGb irresolute mapping. Let A be any VCS in Y . Since every VCS is VGbCS, A is VGbCS in Y . Since f is VGb irresolute mapping, by definition $f^{-1}(A)$ is VGbCS in X . Hence f is VGb continuous mapping.

Example 4.3: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{x, [0.5, 0.7], [0.6, 0.8]\}$, $G_2 = \{y, [0.6, 0.7], [0.5, 0.7]\}$ then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and

$f(b) = v$. Then f is VGb continuous mapping but not Vb continuous, since $G_2^c = \{y, [0.3, 0.4], [0.3, 0.5]\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VbCS in X .

Theorem 4.4: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is VGb irresolute mapping if and only if the inverse image of each VGbOS in Y is VGbOS in X .

Proof: Necessity Let A be VGbOS in Y . This implies A^c is VGbCS in Y . Since f is VGb irresolute mapping $f^{-1}(A^c)$ is VGbCS in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is VGbOS in X .

Sufficiency: It is obvious.

Theorem 4.5: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be VGb irresolute mapping, then f is vague irresolute mapping if X is $V_b T_{1/2}$ space.

Proof: Let A be VCS in Y . Then A is VGbCS in Y . Since f is VGb irresolute mapping, $f^{-1}(A)$ is VGbCS in X , by hypothesis. Since X is $V_b T_{1/2}$ space, $f^{-1}(A)$ is VCS in X . Hence f is vague irresolute mapping.

Theorem 4.6: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be VGb irresolute mapping, then f is Vb irresolute mapping if X is $V_{gb} T_b$ space.

Proof: Let A be VbCS in Y . Then A is VGbCS in Y . Since f is VGb irresolute mapping, $f^{-1}(A)$ is VGbCS in X , by hypothesis. Since X is $V_{gb}T_b$ space, $f^{-1}(A)$ is VbCS in X . Hence f is Vb irresolute mapping.

Theorem 4.7: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \mu)$ be VGb irresolute mapping, where X, Y and Z are VTS, then $g \circ f$ is VGb irresolute mapping.

Proof: Let A be VGbCS in Z . Since g is VGb irresolute mapping, $g^{-1}(A)$ is VGbCS in Y . Since f is VGb irresolute mapping, $f^{-1}(g^{-1}(A))$ is VGbCS in X . Hence $(g \circ f)^{-1}$ is VGbCS in X . Therefore $g \circ f$ is VGb irresolute mapping.

Theorem 4.8: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be VGb irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \mu)$ be VGb continuous mapping, where X, Y and Z are VTS, then $g \circ f$ is VGb continuous mapping.

Proof: Let A be VCS in Z . Since g is VGb continuous mapping, $g^{-1}(A)$ is VGbCS in Y . Since f is VGb irresolute mapping, $f^{-1}(g^{-1}(A))$ is VGbCS in X . Hence $(g \circ f)^{-1}$ is VGbCS in X . Therefore $g \circ f$ is VGb continuous mapping.

Theorem 4.9: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from VTS X into VTS Y . Then the following conditions are equivalent if X and Y are $V_{gb}T_b$ space.

- i) f is VGb irresolute mapping.
- ii) $f^{-1}(B)$ is VGbOS in X for each VGbOS B in Y .
- iii) $f^{-1}(vbint(B)) \subseteq vbint(f^{-1}(B))$ for each VS B of Y .
- iv) $vbcl(f^{-1}(B)) \subseteq f^{-1}(vbcl(B))$ for each VS B of Y .

Proof: i) $\Rightarrow$ ii) It is obvious.

ii) $\Rightarrow$ iii) Let B be VS in Y and $vbint(B) \subseteq B$. Also $f^{-1}(vbint(B)) \subseteq f^{-1}(B)$. Since $vbint(B)$ is VbOS in Y , it is VGbOS in Y . Therefore $f^{-1}(vbint(B))$ is VGbOS in X , by hypothesis. Since X is $V_{gb}T_b$ space $f^{-1}(vbint(B))$ is VbOS in X .

Hence $f^{-1}(vbint(B)) = vbint(f^{-1}(vbint(B))) \subseteq vbint(f^{-1}(B))$

iii) $\Rightarrow$ iv) It is obvious by taking complement in (iii).

iv) $\Rightarrow$ i) Let B be VGbCS in Y . Since Y is $V_{gb}T_b$ space, B is VbCS in Y and $vbcl(B) = B$. Hence $f^{-1}(B) = f^{-1}(vbcl(B)) \subseteq vbcl(f^{-1}(B))$. Therefore $vbcl(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is VbCS and hence it is VGbCS in X . Thus f is VGb irresolute mapping.

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