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Special Dio3-Tuples for Pronic Number –I

S.Vidhya¹, G.Janaki²

^{1, 2} Department of Mathematics, Cauvery College for Women, Trichy-18, Tamilnadu, India.

Abstract: We search for three distinct polynomials with integer coefficients such that the product of any two members of the set added with their sum and increased by a non-zero integer (or polynomial with integer coefficients) is a perfect square.

Keywords: NDio 3-tuples, Pronic numbers, Polynomials.

Notation: Pro_n = Pronic number of rank n .

I. INTRODUCTION

Many mathematicians considered the problem of the existence of Diophantine quadruples with the property $D(n)$ for any arbitrary integer n [1] and also for any linear polynomials n . Further, various authors considered the connections of the problem of Diaphanous, Davenport and Fibonacci numbers in [2-14].

In this communication, we present a few special dio 3-tuples for Pronic numbers of different ranks with their corresponding properties.

II. BASIC DEFINITION

A set of three distinct polynomials with integer coefficients (a_1, a_2, a_3) is said to be a special dio 3-tuple with property $D(n)$ if $a_i * a_j + (a_i + a_j) + n$ is a perfect square for all $1 \leq i < j \leq 3$, where n may be non-zero integer or polynomial with integer coefficients.

A. Method of Analysis

1) Case 1: Construction of Dio 3-tuples for Pronic number of rank $n-1$ and n .

Let $a = \text{Pro}_{n-1}$, $b = \text{Pro}_n$ be Pronic number of rank $n-1$ and n respectively such that $ab + (a+b) + n^2 + 1$ is a perfect square say α^2 .

Let c be any non-zero integer such that

$$ac + (a+c) + n^2 + 1 = \beta^2 \quad (1)$$

$$bc + (b+c) + n^2 + 1 = \gamma^2 \quad (2)$$

On solving equations (1) and (2), we get

$$(a-b) + (n^2+1)(b-a) = (b+1)\beta^2 - (a+1)\gamma^2 \quad (3)$$

Assume $\beta = x + (a+1)y$ and $\gamma = x + (b+1)y$ and it reduces to

$$x^2 = (a+1)(b+1)y^2 + n^2 \quad (4)$$

The initial solution of the equation (4) is given by

$$x_0 = n^2 + 1, \quad y_0 = 1$$

Therefore, $\beta = 2n^2 - n + 2$

On substituting the values of a and β in equation (1), we get

$$c = 4n^2 + 3 = \text{Pro}_{2n-2} + 6n + 1$$

Hence, The triple $(\text{Pro}_{n-1}, \text{Pro}_n, \text{Pro}_{2n-2} + 6n + 1)$ is a Dio 3-tuple with property $D(n^2 + 1)$.

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.

Table 1

n	(a, b, c)	$D(n)$
1	(0,2,7)	2
2	(2,6,19)	5
3	(6,12,39)	10
4	(12,20,67)	17
5	(20,30,103)	26

We present below the Dio 3-tuple for Pronic number of the rank mentioned above with suitable properties.

Table 2

a	b	c	$D(n)$
Pro_{n-1}	Pro_n	$\text{Pro}_{2n-2} + 6n + 3$	$D(3n^2 + 4)$
Pro_{n-1}	Pro_n	$\text{Pro}_{2n-2} + 6n + 5$	$D(5n^2 + 9)$
Pro_{n-1}	Pro_n	$\text{Pro}_{2n-2} + 6n + 7$	$D(7n^2 + 16)$
Pro_{n-1}	Pro_n	$\text{Pro}_{2n-2} + 6n + 9$	$D(9n^2 + 25)$
Pro_{n-1}	Pro_n	$\text{Pro}_{2n-2} + 6n + 11$	$D(11n^2 + 36)$

In general, it is noted that the triple $(\text{Pro}_{n-1}, \text{Pro}_n, \text{Pro}_{2n-2} + 6n + 2k + 1)$ is a Dio 3-tuple with the property $D((2k + 1)n^2 + t^2)$, where $k = 2, 3, 4, \dots$ and $t = 1, 2, \dots$

2) Case 2: Construction of Dio 3-tuples for Pronic number of rank $n - 2$ and n .

Let $a = \text{Pro}_{n-2}$, $b = \text{Pro}_n$ be Pronic number of rank $n - 2$ and n respectively such that $ab + (a + b) + 2n^2 - 2n - 1$ is a perfect square say α^2 .

Let c be any non-zero integer such that

$$ac + (a + c) + 2n^2 - 2n - 1 = \beta^2 \quad (5)$$

$$bc + (b + c) + 2n^2 - 2n - 1 = \gamma^2 \quad (6)$$

On solving equations (5) and (6), we get

$$(a - b) + (2n^2 - 2n - 1)(b - a) = (b + 1)\beta^2 - (a + 1)\gamma^2 \quad (7)$$

Assume $\beta = x + (a + 1)y$ and $\gamma = x + (b + 1)y$ and it reduces to

$$x^2 = (a + 1)(b + 1)y^2 + 2n^2 - 2n - 2 \quad (8)$$

The initial solution of the equation (8) is given by

$$x_0 = n^2 - n + 1, \quad y_0 = 1$$

Therefore, $\beta = 2n^2 - 4n + 4$

On substituting the values of a and β in equation (5), we get

$$c = 4n^2 - 4n + 5 = \text{Pro}_{2n-2} + 2n + 3$$

Hence, The triple $(\text{Pro}_{n-2}, \text{Pro}_n, \text{Pro}_{2n-2} + 2n + 3)$ is a Dio 3-tuple with property $D(2n^2 - 2n - 1)$.

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.

Table 3

n	(a, b, c)	$D(n)$
1	(0,2,5)	-1
2	(0,6,13)	3
3	(2,12,29)	11
4	(6,20,53)	23
5	(12,30,85)	39

We present below, a few Dio 3-tuple for Pronic number of rank mentioned above with suitable properties.

Table 4

a	b	c	$D(n)$
Pro_{n-2}	Pro_n	$\text{Pro}_{2n-2} + 2n + 5$	$D(4n^2 - 4n + 2)$
Pro_{n-2}	Pro_n	$\text{Pro}_{2n-2} + 2n + 7$	$D(6n^2 - 6n + 7)$
Pro_{n-2}	Pro_n	$\text{Pro}_{2n-2} + 2n + 9$	$D(8n^2 - 8n + 14)$
Pro_{n-2}	Pro_n	$\text{Pro}_{2n-2} + 2n + 11$	$D(10n^2 - 10n + 23)$
Pro_{n-2}	Pro_n	$\text{Pro}_{2n-2} + 2n + 13$	$D(12n^2 - 12n + 34)$

3) Case 3: Construction of Dio 3-tuples for Pronic number of rank $n - 2$ and $n - 1$.

Let $a = \text{Pro}_{n-2}$, $b = \text{Pro}_{n-1}$ be Pronic number of rank $n - 2$ and $n - 1$ respectively such that

$ab + (a + b) + (-n^2 + 2n - 1)$ is a perfect square say α^2 .

Let c be any non-zero integer such that

$$ac + (a + c) + (-n^2 + 2n - 1) = \beta^2 \quad (9)$$

$$bc + (b + c) + (-n^2 + 2n - 1) = \gamma^2 \quad (10)$$

On solving equations (9) and (10), we get

$$(a - b) + (-n^2 + 2n - 1)(b - a) = (b + 1)\beta^2 - (a + 1)\gamma^2 \quad (11)$$

Assume $\beta = x + (a + 1)y$ and $\gamma = x + (b + 1)y$ and it reduces to

$$x^2 = (a + 1)(b + 1)y^2 + (-n^2 + 2n - 2) \quad (12)$$

The initial solution of the equation (12) is given by

$$x_0 = n^2 - 2n + 1, \quad y_0 = 1$$

Therefore, $\beta = 2n^2 - 5n + 4$

On substituting the values of a and β in equation (9), we get

$$c = 4n^2 - 8n + 5 = \text{Pro}_{2n-2} - 2n + 3$$

Hence, The triple $(\text{Pro}_{n-2}, \text{Pro}_{n-1}, \text{Pro}_{2n-2} - 2n + 3)$ is a Dio 3-tuple with property $D(-n^2 + 2n - 1)$.

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.

Table 5

n	(a,b,c)	$D(n)$
1	(0,0,1)	0
2	(0,2,5)	-1
3	(2,6,17)	-4
4	(6,12,37)	-9
5	(12,20,65)	-16

We present below, a few Dio 3-tuple for Pronic number of rank mentioned above with suitable properties.

Table 6

a	b	c	$D(n)$
Pro_{n-2}	Pro_{n-1}	$\text{Pro}_{2n-2} - 2n + 5$	$D(n^2 - 2n + 2)$
Pro_{n-2}	Pro_{n-1}	$\text{Pro}_{2n-2} - 2n + 7$	$D(3n^2 - 6n + 7)$
Pro_{n-2}	Pro_{n-1}	$\text{Pro}_{2n-2} - 2n + 9$	$D(5n^2 - 10n + 14)$
Pro_{n-2}	Pro_{n-1}	$\text{Pro}_{2n-2} - 2n + 11$	$D(7n^2 - 14n + 23)$
Pro_{n-2}	Pro_{n-1}	$\text{Pro}_{2n-2} - 2n + 13$	$D(9n^2 - 18n + 34)$

III. CONCLUSION

In this paper we have presented a few examples of constructing a special Dio 3-tuples for Pronic number of different ranks with suitable properties. To conclude one may search for Dio 3-tuples for higher order Pronic number with their corresponding suitable properties.

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