Suspension Model of Fluid Particle for Blood Flow Through Balloon Catheterized Elliptic Shaped Stenosed Artery

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ABSTRACT

This current work is concerned with the realization of fluid flow through arterial catheterization (inserting of balloon inside the arteries to fracture the stenosis). The blood is considered as two-phase fluid, in which first one is the particle phase which consists the red blood cells (erythrocyte) and the second is fluid phase, consists the plasma. It is found that blood, which is mixture of red cells in plasma, and behaves as non-Newtonian at a low shear rate. Solutions for fluid velocity (u_f), shear stress at wall (τ_w) and the shearing stress on the wall at throat of the stenosis (τ_s) are obtained analytically. The insertion of balloon with catheter into diseased i.e. Stenosed artery leads to adequate change in the magnitudes of fluid flow characteristics. It is noticed that the WSS (τ_w) decreases with increasing values of the displacement in axial direction of the balloon, while the WSS (τ_w) increases with the increasing volume fraction density of the particles, the radius of the inner tube, which keeps the balloon in position k and the maximum height attained by the balloon.

KEY WORDS: STENOSIS, HEMATOCRIT, SUSPENSION MODEL, SHEAR STRESS, CATHETERIZED ARTERY.

INTRODUCTION

The circulatory system of blood in mammals due to its complicated nature is always the subject of research that leads to the diagnosis of a number of cardiovascular diseases like angina myocardial infarction. Various theoretical and experimental efforts are carried out just to find out the blood flow behavior when it flows in artery and veins of living beings. Stenosis is the unusual and non-natural growth in the arterial wall thickness in cardiovascular system under unhealthy condition. In Biology, the meaning of stenosis is narrowing of any body passage, tube or orifice or in simple form it can say that stenosis is the accumulation of cholesterol and other substances on the endothelium and by the proliferation of connective tissues in an arterial wall. The stenosis blocks the supply of blood to the brain and heart and causes, heart attack and various cardiovascular diseases. Researchers Fry, Dintenfass explained that the fluid

dynamics properties play a significant role in circulation of fluid .

To understand the role of stenosis in blood flow through an artery, several works have been carried out by Young Shukla et al., Most of the work is based on the assumption that blood is considered as Newtonian fluid. But in practical nature, blood shows non-Newtonian behaviour under some conditions. It is practically found that blood, being a suspension of red cells (erythrocyte) in plasma, blood behaves like non-Newtonian fluid at a low shear rate (Han and Barnett, Whitmore). The studies relating variation of the resistance to flow and the shear stress at wall with the axial distance and the depth of the stenosis has explained by many authors. Fung analyzed that an accurate description of the blood in small vessels requires the consideration of red cells as discrete particles.

The length of artery is taken large enough as compared to radius of artery so that the onset point, end point and special wall effects can be neglected. Young observed that blood can be assumed as Newtonian fluid when it passes through narrow arteries at low shear rate particularly in diseased state, while blood being suspension of corpuscles in plasma acts as a non-Newtonian fluid, when it flows inside larger arteries at high shear rate [8]. An effort has been made to analyze the influence of arterial catheterization (widening of arteries by a balloon in the existence of mild stenosis) on the flow characteristics

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of fluid flowing in arteries explained by a two-fluid model of particle-fluid suspension. The arterial wall has been considered hard and elastic. The wall of the artery is assumed relatively rigid, when stenoses develop in circulatory system. In the present time, the use of catheters has become an important tool for diagnosis and treatment in medical Science.

It is a polyester based thermoplastic polyurethane, medical grade polyvinyl chloride. Catheter is moved to the location of stenosis sincerely in the arteries and balloon is then inflated to break the fatty deposit substances and widen the narrowed portion. An inserted catheter modifies the pressure distribution and changes the flow of fluid with the assumption of slip velocity or zero slip at the vessel wall as explained by Haynes and Cokelet et al. . Blood flow model for flow of blood through catheterized stenosed artery has been prepared by Mekheimer and Kothari. During the blood flow red blood cells act as discrete particles i.e. blood is considered as two-phase fluid in which the particle phase consist the red blood cells (erythrocyte) and the fluid phase consists the plasma. The equations in the case of mild stenosis and the boundary conditions has solved analytically for the axial velocity. By considering the blood vessels to be cylindrical in shape, Prakash has studied the flow of blood in stenosed artery for blood as the Casson fluid. Maiti studied the role of arterial stenosis on blood as Herschel-Bulkley fluid under slip velocity. Wu et al. prepared a fluid suspension model for blood flow. Therefore, it is perhaps more appropriate to treat the blood like a double -phase fluid, that is, a suspension of red blood cells and plasma.

In this research paper, generalized equations for fluid in motion and particle phases has been solved and expressions for blood flow characteristics basically, the shear stress at wall (τ_w), and shearing stress at the maximum height (τ_s) of the stenosis have formulated and has discussed qualitatively.

Mathematical formulation

Consider the axisymmetric flow of blood, through the gap between an artery of circular cross section with elliptic shaped stenosis as depicted in figure -1 and the inner tube have a balloon. Mathematically, the geometry of artery with elliptic shaped mild stenosis and balloon model is defined by functions $R(z)$ and $R_b(z)$ respectively

$$\frac{R(z)}{R_0} = \begin{cases} 1 - \frac{\delta}{R_0} \sin\left[\frac{\pi(z-d)}{l_0}\right] & d \leq z \leq d + l_0, \\ 1 & \text{otherwise,} \end{cases} \quad (1)$$

$$\frac{R_b}{R_0} = \begin{cases} k + \delta_1 \exp\left[-\pi^2 \left\{z - z_d - \left(\frac{l_0}{2}\right)^2\right\}\right] & d \leq z \leq d + l_0, \\ k & \text{otherwise.} \end{cases} \quad (2)$$

where

$R(z)$ Radius of stenotic portion of the artery .

R_0 Radius of non- stenotic artery.

l length of the arterial segment..

l_0 length of the stenotic portion of the artery.

d : stenosis location

Blood is assumed as suspension of all erythrocytes i.e.

red cell and plasma mixture. Equations representing two layered suspension blood flow in case of mild stenosis $\frac{\delta}{R_0} \ll 1$, are given as.

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_f}{\partial r} \right) = - \frac{1}{(1-c)\mu_s} \frac{dp}{dz}, \quad (3)$$

$$\frac{\partial p}{\partial r} = 0, \quad (4)$$

$$C \frac{dp}{dz} = CS(u_f - u_p). \quad (5)$$

Where

u_f = axial velocity component of fluid,

u_p = axial velocity component of particle,

C = Volume fraction density of the particle which is constant also called haematocrit,

S = drag coefficient of interaction.

ρ_f = Fluid density,

ρ_p = Particle density,

μ_s = suspension viscosity of fluid,

kR_0 = Inner radius of artery which keep balloon in position $k \ll 1$,

Z_d = Displacement in axial direction of the balloon.

The boundary condition are

$$\begin{cases} u_f = 0, & u_p = 0 & \text{at } r = R(z) \\ u_f = 0, & u_p = 0 & \text{at } r = R_b(z) \end{cases} \quad (6)$$

$$\mu_s(c) = \frac{\mu_0}{1 - mc}, \quad (7)$$

$$m = 0.070 \exp[2.49C + (1107/T)\exp(-1.69C)], \quad (8)$$

$$S = \frac{9}{2} \frac{\mu_0}{b_0^2} \frac{4+3[8c-3c^2]^{1/2} + 3c}{(2-3c)^2}. \quad (9)$$

μ_0 = fluid (suspending medium) viscosity,

b_0 = radius of erythrocytes,

T = temperature.

Solution of problem

Solving equations (3), (4) and (5) with boundary conditions equation (6) yields the following expression for the fluid velocity u_f and particle velocity u_p as given below

$$u_f = - \frac{R_0^2}{4(1-c)\mu_s} \left\{ \left(\frac{R}{R_0} \right)^2 - \left(\frac{r}{R_0} \right)^2 - \frac{\left(\left(\frac{R}{R_0} \right)^2 - \left(\frac{R_b}{R_0} \right)^2 \right) \log\left(\frac{r}{R}\right)}{\log\left(\frac{R_b}{R}\right)} \right\}, \quad (10)$$

$$u_p = \left[- \frac{R_0^2}{4(1-c)\mu_s} \left\{ \left(\frac{R}{R_0} \right)^2 - \left(\frac{r}{R_0} \right)^2 - \frac{\left(\left(\frac{R}{R_0} \right)^2 - \left(\frac{R_b}{R_0} \right)^2 \right) \log\left(\frac{r}{R}\right)}{\log\left(\frac{R_b}{R}\right)} + \frac{4(1-c)\mu_s}{8R_0^2} \right\} \right]. \quad (11)$$

The volumetric flow rate given as

$$Q = 2(1-c) \int_{R_b}^R r u_f dr + 2C \int_{R_b}^R r u_p dr,$$

Using equations (10) and (11) and then solving, it yields :

$$Q = - \frac{\left(\frac{dp}{dz} \right) R_0^4}{8(1-c)\mu_s} \left\{ \left(\frac{R}{R_0} \right)^2 - \beta^2 \right\} \left\{ \left(\frac{R}{R_0} \right)^2 + \beta^2 - \frac{\left(\left(\frac{R}{R_0} \right)^2 - \beta^2 \right)}{\log\left[\frac{R_b}{R}\right]} + \alpha \right\}. \quad (12)$$

Where $\alpha = \frac{8C(1-c)\mu_s}{8R_0^2}$ is suspension parameter.

$$\beta = \frac{R_0}{R_c},$$

$$Q = -\frac{(dp/dz)}{8(1-c)\mu_0} \left[\frac{1}{\Phi(z)} \right],$$

Where $\Phi(z) = \frac{1}{\left(\left(\frac{R_0}{R_c} \right)^2 - \beta^2 \right) \left(\frac{R_0}{R_c} \right)^2 + \beta^2 - \frac{\left(\left(\frac{R_0}{R_c} \right)^2 - \beta^2 \right)}{\log \left(\frac{R_0}{R_c} \right)} + \alpha}$ (13)

$$\Rightarrow \frac{dp}{dz} = -8(1-c)\mu_0 Q \Phi(z). \quad (14)$$

Shear stress at wall (WSS) is given by

$$\bar{\tau}_R = -\frac{R}{2} \frac{dp}{dz},$$

$$\bar{\tau}_R = \frac{4(1-c)\left(\frac{R_0}{R_c}\right)Q\mu_0\Phi(z)}{\pi R^2},$$

$$\tau_0 = -\frac{R}{2} \left[\frac{dp}{dz} \right]_{z=0} = \frac{4\mu_0 Q}{\pi R^2},$$

$$\tau_R = \frac{\bar{\tau}_R}{c_0},$$

$$\tau_R = \frac{(1-c)\mu_0 \left(\frac{R_0}{R_c} \right)}{\left(\left(\frac{R_0}{R_c} \right)^2 - \beta^2 \right) \left(\frac{R_0}{R_c} \right)^2 + \beta^2 - \frac{\left(\left(\frac{R_0}{R_c} \right)^2 - \beta^2 \right)}{\log \left(\frac{R_0}{R_c} \right)} + \alpha} \quad (15)$$

Shear stress at the stenosis throat (τ_s) is given as

$$\tau_s = \left[\tau_R \right]_{\frac{R_0}{R_c} = \left(1 - \frac{\delta_1}{R_0} \right)} \quad (16)$$

RESULTS AND DISCUSSION

To evaluate analytic results, obtained for axial velocity profile of fluid (u_f) the shear stress at wall (τ_w) and shearing stress at stenosis computer codes are developed. In order to estimate the quantitative effects of the various parameters at the temperature of 25°C.

Figure 1: Geometry of the problem.

Figure 2: u_f with r for distinct $\delta_1 = 0.1, 0.15, 0.2$.

The values of parameters using following data; δ/R_0 (stenosis height) = 0, 0.05, 0.10, 0.15, 0.2 ; δ_1/R_0 (balloon

height) = 0, 0.1, 0.2, 0.3; $z_d = 0.75, 0.95, 1.15$; $d = 0.75$; $k = 0.01, 0.05, 0.10$ and presented graphically.

Figures 2-4 indicate, how the axial velocity (u_f) of fluid is influenced by the radius of catheter by fixing the other parameters. The nature of the axial velocity (u_f) is seen parabolic in the annular space between the artery wall and the balloon wall. It is observed from figures that the axial velocity (u_f) attains its maximum value at $r = 0.5$ then decreases gradually. Graphs 2 and 3 demonstrate that axial velocity (u_f) decreases with increasing values of δ_1 , k while graph 4 reveals that axial velocity increases with increasing values of z_d .

Figure 3: u_f with r for distinct $k = 0.01, 0.05, 0.10$.

Figure 4: u_f with r for distinct $z_d = 0.75, 0.95, 1.15$.

Shear stress at wall is very important flow characteristic to understand the development of stenosed arterial disease. The variations of wall shear stress (WSS) (τ_w) and shear stress at stenosis throat (τ_s) with different parameters have been shown in figures 5 –11. The magnitude of shear stress at wall (τ_w) in the stenosis constriction region ($0.75 \leq z \leq 1.75$) starts increasing at the

beginning of the stenosis at ($z = 0.75$) upto its maximum constriction ($z = 1.25$) at the throat of the stenosis then the curve is decreasing gradually to reach end point ($z = 1.75$) of the constriction.

Figure 5: τ_w with z for distinct $C = 0, 0.2, 0.4$

Figure 6: τ_w with z for distinct $\delta_1 = 0.10, 0.15, 0.20$

Figure 7: τ_w with z for distinct $z_d = 0.75, 0.95, 1.15$

The nature of variations in wall shear stress distribution, (τ_w) and its value at the stenosis throat, with respect to the parameters C , δ_1 , k and z_d are shown in figures 5-7. Figure 5 describes the distribution of shear stress at wall, (τ_w) in the stenosis constriction region for different values of the volume fraction density C . It increases with

increasing value of C for fixed value of δ/R_0 , δ_1 and k . The wall shear stress distribution, (τ_w) increases with increasing value of δ_1 for some fixed values of δ/R_0 , C and k in the stenotic region (Figure 6), while it decreases with increasing value of z_d in the same region for fixed δ/R_0 , C , δ_1 and k (Figure 7). Also (τ_w) increases with increasing value of k in the stenotic region for fixed δ/R_0 , C and δ_1 (Figure 8).

Figure 8: τ_w with z for distinct $k = 0.01, 0.05, 0.10$.

Figure 9: τ_s with δ/R_0 distinct values of (k, C) .

The variations of shearing stress at the stenosis throat, (τ_s) with the stenosis size, δ/R_0 are shown in figures 9-11. Figure-9 describes, (τ_w) increases with increasing values of k and C $\{(k, C) = (0.01, 0.2), (0.05, 0.2), (0.1, 0.2), (0.03, 0.1), (0.03, 0.2), (0.03, 0.3)\}$ for fixed z , δ_1 , z_d . Figure 10 shows (τ_s) decreases with increasing of z_d and fixed value $\delta_1(0.1)$ i.e. $\{(z_d, \delta_1) = (0.75, 0.1), (0.95, 0.1), (1.15, 0.1)\}$ and increases with increasing values of δ_1 i.e $\{(z_d, \delta_1) = (0.75, 0.15), (0.75, 0.20), (0.75, 0.25)\}$ for fixed values of z , C and k .

In figure 11, the variations of shearing stress at stenosis throat (τ_s) with the volume fraction density C have been demonstrate for different values of δ/R_0 and δ_1 . It is seen that shearing stress at the stenosis throat, τ_s increases with $\{(\delta/R_0, \delta_1) = (0, 0.1), (0, 0.2), (0, 0.2), (0.1, 0), (0.2, 0), (0.3, 0)\}$ increasing values of stenosis size, δ/R_0 , δ_1 . Also

(τ_s) increases with increasing values of C , because of the apparent viscosity of fluid increases with increasing erythrocytes concentration in blood.

Figure 10: τ_s with δ/R_0 distinct (z_d, δ_1).

CONCLUSION

A mathematical model has been developed, for the flow of blood through axi-symmetric coaxial tubes such that the outer tube consist an axially symmetric mild elliptic shaped stenosis and the inner tube consist a balloon which is also axi-symmetric in nature. Graphical results are depicted for the fluid velocity profile in axial direction, distributions of WSS, shear stress at stenosed throat. The main finding in brief are given as follows:

- Axial velocity decreases corresponding with raising values of δ_1 , k while it shows reverse nature for balloon displacement i.e. z_d .
- It is seen that the flow characteristics WSS consider higher magnitude in particle fluid suspension than that in particle free flow (i.e. $C = 0$). WSS attains its highest value at maximum heights of stenosis (throat) and lowest at the ending of the stenosis and at the onset of the normal region of the artery.
- Shearing stress at throat of stenosis (τ_s) with the volume fraction density C have been studied for different values of δ/R_0 and δ_1 . It is seen that, (τ_s) increases corresponding with, raising values of stenosis size, δ/R_0 , C and δ_1 . Also it shows same nature in variation of k and C .

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Figure 11: τ_s with C for distinct ($\delta/R_0, \delta_1$).

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