Root Cube Mean Cordial Labeling of Some Standard Graphs

N. G. Gidheeba¹ and A. Vijayan²

¹Ph.D. Research Scholar (Part time), Register Number : 18233112092024, Nesamony

Memorial Christian College, Marthandam, Kanyakumari District, Tamil Nadu, India

²Research Department of Mathematics, Nesamony Memorial Christian College,

Marthandam, Kanyakumari District, Tamil Nadu, India

ABSTRACT

Let $G=(V,E)$ be a graph and f be a mapping from $V(G) \rightarrow \{0,1,2\}$. For each edge uv of G assign the label $\left\lfloor \sqrt{\frac{f(u)^3 + f(v)^3}{2}} \right\rfloor$. f is called a root cube mean cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with x , $x \in \{0,1,2\}$ respectively. A graph with a root cube mean cordial labeling is called root cube mean cordial graph. In this paper, root cube mean cordiality of some standard graphs such as cycle, path, star are discussed.

KEY WORDS: CYCLE, PATH, ROOT CUBE MEAN CORDIAL LABELING, ROOT CUBE MEAN CORDIAL GRAPHS.

INTRODUCTION

The graphs considered here are finite, undirected and simple. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. Labeled graphs are useful models for a broad range of applications such as coding theory, X-ray crystallography, astronomy, circuit design etc. The concept of cordial labeling was introduced by Cahit in the year 1987. Let $f: V(G) \rightarrow \{0,1\}$ be a function. For each edge uv assign the label $|f(u) - f(v)|$. f is called a cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, $i, j \in \{0,1\}$ where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges respectively labeled with x ($x = 0,1$). A graph which admits a cordial labeling is called cordial graph.

2. Root Cube Mean Cordial Labeling

Definition 2.1

Let $G = (V,E)$ be a graph and f be a mapping from $V(G) \rightarrow \{0,1,2\}$. For each edge uv of G assign the label

$$\left\lfloor \sqrt{\frac{f(u)^3 + f(v)^3}{2}} \right\rfloor$$

f is called a root cube mean cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with x , $x \in \{0,1,2\}$ respectively. A graph with a root cube mean cordial labeling is called root cube mean cordial graph. We illustrate the definition using the following example.

Example 2.2 A graph that admits a root cube mean cordial labeling is given below.

Here $v_f(0) = 2$, $v_f(1) = 3$, $v_f(2) = 3$ and $e_f(0) = 3$, $e_f(1) = 2$, $e_f(2) = 3$ and $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0,1,2\}$.

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Theorem 2.3. The graph P_n is a root cube mean cordial graph.

Proofs

Let P_n be the path $u_1 u_2 \dots u_n$ where $|V(P_n)| = n$

and $|E(P_n)| = n - 1$.

Define $f: V(P_n) \rightarrow \{0, 1, 2\}$ as follows.

Case (i) $n \equiv 0 \pmod{3}$

Let $n = 3t$, Define $f(u_i) = 0, 1 \leq i \leq t$

$$f(u_{t+i}) = 1, 1 \leq i \leq t$$

$$f(u_{2t+i}) = 2, 1 \leq i \leq t$$

Then $v_f(0) = t, v_f(1) = t, v_f(2) = t$

$$e_f(0) = t, e_f(1) = t - 1, e_f(2) = t.$$

Therefore, $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)|$

for all $i, j \in \{0, 1, 2\}$.

Hence P_n is a root cube mean cordial graph.

Case (ii) $n \equiv 1 \pmod{3}$

Let $n = 3t + 1$. Define $f(u_i) = 0, 1 \leq i \leq t$

$$f(u_{t+i}) = 1, 1 \leq i \leq t + 1$$

$$f(u_{2t+i}) = 2, 1 \leq i \leq t$$

Then $v_f(0) = t, v_f(1) = t + 1, v_f(2) = t$

$$e_f(0) = t, e_f(1) = t, e_f(2) = t.$$

Therefore, $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)|$

for all $i, j \in \{0, 1, 2\}$.

Hence P_n is a root cube mean Cordial graph.

Case (iii) $n \equiv 2 \pmod{3}$

Let $n = 3t + 2$. Define $f(u_i) = 0, 1 \leq i \leq t + 1$

$$f(u_{t+1+i}) = 1, 1 \leq i \leq t + 1$$

$$f(u_{2t+2+i}) = 2, 1 \leq i \leq t$$

Then $v_f(0) = t + 1, v_f(1) = t + 1, v_f(2) = t$

$$e_f(0) = t + 1, e_f(1) = t, e_f(2) = t.$$

Therefore, $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$

for all $i, j \in \{0, 1, 2\}$.

Hence P_n is a root cube mean Cordial graph.

Example 2.4: Root cube mean cordial labeling of the path P_9, P_{10}, P_{11} is given below.

Theorem: 2.5

The cycle C_n is a root cube mean cordial graph if $n \equiv 1 \pmod{3}$ and $n \equiv 2 \pmod{3}$

Proof:

Let C_n be the cycle $u_1 u_2 \dots u_n u_1$ where $|V(C_n)| = n$ and $|E(C_n)| = n$.

Define $f: V(C_n) \rightarrow \{0, 1, 2\}$ as follows.

Case (i) $n \equiv 1 \pmod{3}$

Let $n = 3t + 1$. Define $f(u_i) = 0, 1 \leq i \leq t$

$$f(u_{t+i}) = 1, 1 \leq i \leq t + 1$$

$$f(u_{2t+i}) = 2, 1 \leq i \leq t$$

Then $v_f(0) = t, v_f(1) = t + 1, v_f(2) = t$

$$e_f(0) = t + 1, e_f(1) = t, e_f(2) = t.$$

Therefore, $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$

for all $i, j \in \{0, 1, 2\}$.

Hence C_n is a root cube mean cordial graph.

Case (ii) $n \equiv 2 \pmod{3}$

Let $n = 3t + 2$. Define $f(u_i) = 0, 1 \leq i \leq t$

$$f(u_{t+i}) = 1, 1 \leq i \leq t + 1$$

$$f(u_{2t+1+i}) = 2, 1 \leq i \leq t$$

$$f(u_n) = 0.$$

$$\text{Then } v_f(0) = t, v_f(1) = t+1, v_f(2) = t+1$$

$$e_f(0) = t+1, e_f(1) = t, e_f(2) = t+1.$$

$$\text{Therefore, } |v_f(i) - v_f(j)| \leq 1$$

$$\text{for all } i, j \in \{0, 1, 2\} \text{ and } |e_f(i) - e_f(j)| \leq 1$$

$$\text{for all } i, j \in \{0, 1, 2\}.$$

Hence C_n is a root cube mean cordial graph.

Remarks: 2.6

The cycle C_n is not a root cube mean graph if

$$n \equiv 0 \pmod{3}$$

Proof:

Let C_n be the cycle $u_1 u_2 \dots u_n u_1$ where $|V(C_n)| = n$ and $|E(C_n)| = n$.

$$\text{Define } f: V(C_n) \rightarrow \{0, 1, 2\}$$

.Consider $n \equiv 0 \pmod{3}$.

Suppose the cycle C_n admits a root cube mean cordial graph if $n \equiv 0 \pmod{3}$.

$$\text{Let } n = 3t.$$

Then $v_f(0) = t, v_f(1) = t, v_f(2) = t$ and $e_f(0) = t, e_f(1) = t-1, e_f(2) = t+1$. Therefore, $|v_f(i) - v_f(j)| \leq 1$ for all $i, j \in \{0, 1, 2\}$ but $|e_f(i) - e_f(j)| \leq 1$ for all $i, j \in \{0, 1, 2\}$ not satisfied, Since $|e_f(0) - e_f(1)| > 1$, which is a contradiction.

∴ Our assumption is wrong. Hence C_n is not root cube mean cordial graph.

Example 2.7: Root cube mean cordial labeling of the cycle C_4 and C_5 is given below:

Theorem 2.8 The star $K_{1,n}$ is a root cube mean cordial graph.

Proof :

Let $K_{1,n}$ be the star with $n + 1$ vertices and n edges. Let $V(K_{1,n}) = \{u, u_i: 1 \leq i \leq n\}$ and $E(K_{1,n}) = \{uu_i: 1 \leq i \leq n\}$. Let u be the centre of $K_{1,n}$. Define $f: V(K_{1,n}) \rightarrow \{0, 1, 2\}$ as follows.

$$\text{Let } f(u) = 1.$$

Case (i) $n \equiv 0 \pmod{3}$

$$\text{let } n = 3t. \text{ Define } f(u_i) = 0, 1 \leq i \leq t$$

$$f(u_{t+i}) = 1, 1 \leq i \leq t$$

$$f(u_{2t+i}) = 2, 1 \leq i \leq t$$

$$\text{Then } v_f(0) = t, v_f(1) = t+1, v_f(2) = t \text{ and}$$

$$e_f(0) = t, e_f(1) = t, e_f(2) = t.$$

$$\text{Therefore, } |v_f(i) - v_f(j)| \leq 1 \text{ and}$$

$$|e_f(i) - e_f(j)| \leq 1 \text{ for all } i, j \in \{0, 1, 2\}.$$

Hence $K_{1,n}$ is a root cube mean cordial graph

Case (ii) $n \equiv 1 \pmod{3}$

$$\text{Let } n = 3t+1. \text{ Define } f(u_i) = 0, 1 \leq i \leq t+1$$

$$f(u_{t+1+i}) = 1, 1 \leq i \leq t$$

$$f(u_{2t+1+i}) = 2, 1 \leq i \leq t$$

$$\text{Then } v_f(0) = t+1, v_f(1) = t+1, v_f(2) = t \text{ and}$$

$$e_f(0) = t+1, e_f(1) = t, e_f(2) = t.$$

$$\text{Therefore, } |v_f(i) - v_f(j)| \leq 1 \text{ and } |e_f(i) - e_f(j)| \leq$$

$$1 \text{ for all } i, j \in \{0, 1, 2\}.$$

Hence $K_{1,n}$ is a root cube mean cordial graph

Case (iii) $n \equiv 2 \pmod{3}$

$$\text{Let } n = 3t+2. \text{ Define } f(u_i) = 0, 1 \leq i \leq t+1$$

$$f(u_{t+1+i}) = 1, 1 \leq i \leq t$$

$$f(u_{2t+1+i}) = 2, 1 \leq i \leq t+1$$

$$\text{Then } v_f(0) = t+1, v_f(1) = t+1, v_f(2) = t+1$$

$$e_f(0) = t+1, e_f(1) = t, e_f(2) = t+1.$$

$$\text{Therefore, } |v_f(i) - v_f(j)| \leq 1 \text{ and } |e_f(i) - e_f(j)| \leq$$

$$1 \text{ for all } i, j \in \{0, 1, 2\}.$$

Hence $K_{1,n}$ is a root cube mean cordial graph

Example 2.9. Root cube mean cordial labeling of the star $K_{1,t}$ is given below

Theorem 2. 10

Bistar graph $B_{n,n}$ is a root cube mean cordial graph

Proof :

Let $B_{n,n}$ be the Bistar graph with vertex s
 $V(B_{n,n}) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$ and edges $E(B_{n,n})$
 $\{uv, uu_i, vv_i : 1 \leq i \leq n\}$.

Define $f : V(B_{n,n}) \rightarrow \{0, 1, 2\}$ as follows.

Let $f(u) = 1, f(v) = 1$.

Case (i)

$n \equiv 2 \pmod{3}$. Let $n = 3t - 1$

Define $f(u_i) = 0, 1 \leq i \leq 2t$

$$f(u_i) = 1, 2t + 1 \leq i \leq n$$

$$f(v_i) = 2, 1 \leq i \leq 2t$$

$$f(v_i) = 1, 2t + 1 \leq i \leq n$$

Then $v_f(0) = v_f(1) = v_f(2) = 2t$

$$e_f(0) = 2t, e_f(1) = 2t - 1, e_f(2) = 2t.$$

Therefore, $|v_f(i) - v_f(j)| \leq 1 \forall i, j \in \{0, 1, 2\}$ and

$$|e_f(i) - e_f(j)| \leq 1 \forall i, j \in \{0, 1, 2\}$$

Case (ii) $n \equiv 0 \pmod{3}$, let $n = 3t$

Define $f(u_i) = 0, 1 \leq i \leq t + 1$

$$f(u_i) = 1, t + 2 \leq i \leq n$$

$$f(v_i) = 2, 1 \leq i \leq 2t$$

$$f(v_i) = 0, 2t + 1 \leq i \leq n$$

Then $v_f(0) = v_f(1) = 2t + 1, v_f(2) = 2t$

$$e_f(0) = 2t + 1, e_f(1) = 2t, e_f(2) = 2t$$

Therefore $|v_f(i) - v_f(j)| \leq 1 \forall i, j \in \{0, 1, 2\}$

$$|e_f(i) - e_f(j)| \leq 1 \forall i, j \in \{0, 1, 2\}$$

Case (iii)

$n \equiv 1 \pmod{3}$, let $n = 3t + 1$

Define $f(u_i) = 0, 1 \leq i \leq 2t + 1$

$$f(u_i) = 1, 2t + 2 \leq i \leq n$$

$$f(v_i) = 2, 1 \leq i \leq 2t + 1$$

$$f(v_i) = 1, 2t + 2 \leq i \leq n$$

Then $v_f(0) = 2t + 1, v_f(1) = 2t + 2, v_f(2) = 2t + 1$

$$e_f(0) = e_f(1) = e_f(2) = 2t + 1$$

Therefore $|v_f(i) - v_f(j)| \leq 1 \forall i, j \in \{0, 1, 2\}$

$$|e_f(i) - e_f(j)| \leq 1 \forall i, j \in \{0, 1, 2\}$$

From all the above cases, we have $B_{n,n}$ is a root cube mean cordial graph.

Theorem 2. 11.

The complete graph K_n is not a root cube mean cordial graph for any $n > 3$.

Proof:

Let G be the complete graph K_n with n vertices and $n(n-1)/2$ edges. Clearly K_1, K_2, K_3 are root cube mean cordial graphs. Let us check for the complete graphs $K_n, n > 3$

Define $f : V(K_n) \rightarrow \{0, 1, 2\}$ as follows :

Case (i) $n \equiv 0 \pmod{3}$

Let $n = 3t$

Then $v_f(0) = v_f(1) = v_f(2) = t$

$$e_f(0) = \binom{t}{2} + t^2$$

$$e_f(1) = \binom{t}{2},$$

$$e_f(2) = \binom{t}{2} t^2 + t^2, \text{ a contradiction,}$$

since $|e_i(i) - e_i(j)| > 1$

Case (ii) $n \equiv 1 \pmod{3}$

Let $n = 3t + 1$

Subcase (i) If $v_K(0) = t + 1, v_K(1) = v_K(2) = t$

$$\text{then } e_K(0) = \binom{t+1}{2} + t(t+1)$$

$$e_K(1) = \binom{t+1}{2} - t$$

$$e_K(2) = \binom{t}{2} + t(t+1) + t^2, \text{ a contradiction,}$$

since $|e_i(i) - e_i(j)| > 1$

Subcase (ii) If $v_K(0) = v_K(2) = t, v_K(1) = t + 1$

$$\text{then } e_K(0) = \binom{t+1}{2} + t^2$$

$$e_K(1) = \binom{t+1}{2}$$

$$e_K(2) = \binom{t}{2} + t(t+1) + t^2, \text{ a contradiction,}$$

since $|e_i(i) - e_i(j)| > 1$

Subcase (iii) If $v_K(0) = v_K(1) = t, v_K(2) = t + 1$

$$\text{then } e_K(0) = \binom{t}{2} + t^2$$

$$e_K(1) = \binom{t+1}{2} \square t$$

$$e_K(2) = \binom{t+1}{2} + t(t+1) + t(t+1),$$

a contradiction

Case (iii) $n \equiv 2 \pmod{3}$

Let $n = 3t + 2$

Subcase (i) If $v_K(0) = t, v_K(1) = v_K(2) = t + 1$

$$\text{then } e_K(0) = \binom{t+1}{2} + t^2$$

$$e_K(1) = \binom{t+1}{2}$$

$$e_K(2) = \binom{t+2}{2} + t(t+1) + t(t+1),$$

a contradiction, since $|e_i(i) - e_i(j)| > 1$

Subcase (ii) If $v_K(1) = t, v_K(0) = v_K(2) = t + 1$

$$\text{then } e_K(0) = \binom{t+1}{2} + t(t+1)$$

$$e_K(1) = \binom{t+1}{2} - t$$

$$e_K(2) = \binom{t+2}{2} + t(t+1) + t(t+1),$$

a contradiction, since $|e_i(i) - e_i(j)| > 1$

Subcase (iii) If $v_K(2) = t, v_K(0) = v_K(1) = t + 1$

$$\text{then } e_K(0) = \binom{t+1}{2} + (t+1)^2$$

$$e_K(1) = \binom{t+1}{2}$$

$$e_K(2) = \binom{t+1}{2} + t(t+1) + t^2, \text{ a contradiction,}$$

since $|e_i(i) - e_i(j)| > 1$

From all the above cases, K_n is not a root cube mean cordial graph.

Theorem 2. 12

The complete bipartite graph $K_{2,n}$ is not root cube mean cordial for any $n > 2$.

Proof:

Let $V(K_{2,n}) = A \cup B$, where $A = \{u, v\}$

and $B = \{u_1, u_2, \dots, u_n\}$. Then $E(K_{2,n}) = \{uu_i, vu_i : 1 \leq i \leq n\}$. Clearly $K_{2,1}$ and $K_{2,2}$ are root cube mean cordial graph. Assume that $n > 2$. Here $K_{2,n}$ has $2 + n = m$ (say) vertices and $2n = 2(m-2)$ edges.

Case (i) $m \equiv 0 \pmod{3}$. Let $m = 3t$

Without violating the vertex label differences assign labels to the vertices $v_f(0) = v_f(1) = v_f(2) = t$, we have the following possibilities.

Subcase (a)

Let $f(u)=0, f(v)=0$. Then $e_f(0) = 4t-4, e_f(1) = 0, e_f(2) = 2t$.

Subcase (b)

Let $f(u) = 0, f(v) = 1$. Then $e_f(0) = 3(t-1), e_f(1) = t-1, e_f(2) = 2t$

Subcase (c)

Let $f(u) = 1, f(v) = 0$. Then $e_f(0) = 3(t-1), e_f(1) = t-1, e_f(2) = 2t$

Subcase (d)

Let $f(u) = 0, f(v) = 2$. Then $e_f(0) = 2t-1, e_f(1) = 0, e_f(2) = 4t-3$

Subcase (e)

Let $f(u) = 2, f(v) = 0$. Then $e_f(0) = 2t-1, e_f(1) = 0, e_f(2) = 4t-3$

Subcase (f)

Let $f(u) = 1, f(v) = 1$. Then $e_f(0) = 2t, e_f(1) = 2t-4, e_f(2) = 2t$

Subcase (g)

Let $f(u) = 1, f(v) = 2$. Then $e_f(0) = t, e_f(1) = t-1, e_f(2) = 4t-3$

Subcase (h)

Let $f(u) = 2, f(v) = 1$. Then $e_f(0) = t, e_f(1) = t-1, e_f(2) = 4t-3$

Subcase (i)

Let $f(u) = 2, f(v) = 2$.

Then $e_f(0) = 0, e_f(1) = 0, e_f(2) = 6t-4$

All the above subcases, the condition that $|e_f(i) - e_f(j)| \leq 1 \forall i, j$ is not satisfied. Hence we can see that $K_{2,n}$ is not root cube mean cordial.

Thus in the following cases, we consider only the subcases in which the vertices u and v are having labels 1 and 2 respectively.

Case (ii)

$m \equiv 1 \pmod{3}$. Let $m = 3t + 1$. Suppose $f(u) = 1$ and $f(v) = 2$

Subcase (a)

If $v_f(0) = t+1, v_f(1) = v_f(2) = t$. Then $e_f(0) = t+1, e_f(1) = t-1, e_f(2) = 4t-2$

Subcase (b)

If $v_f(1) = t+1, v_f(0) = v_f(2) = t$. Then $e_f(0) = t, e_f(1) = t, e_f(2) = 4t-2$

Subcase (c)

If $v_f(2) = t+1, v_f(0) = v_f(1) = t$. Then $e_f(0) = t, e_f(1) = t-1, e_f(2) = 4t-1$

Case (iii)

$m \equiv 2(\text{mod } 3)$, Let $m = 3t + 2$. Suppose $f(u) = 1$ and $f(v) = 2$

Subcase (a)

If $v_f(0) = v_f(1) = t+1$, $v_f(2) = t$. Then $e_f(0) = t+1$, $e_f(1) = t$, $e_f(2) = 4t-1$

Subcase (b)

If $v_f(0) = v_f(2) = t+1$, $v_f(1) = t$. Then $e_f(0) = t+1$, $e_f(1) = t-1$, $e_f(2) = 4t$.

Subcase (c)

If $v_f(0) = t$, $v_f(1) = v_f(2) = t+1$. Then $e_f(0) = t$, $e_f(1) = t$, $e_f(2) = 4t$. f does not satisfies the conditions $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, for $i, j \in \{0, 1, 2\}$. Hence $K_{2,m}$ is not root cube mean cordial graph.

Figure 1: A root cube mean cordial graph

Figure 1.1

Figure 1.2

Figure 1.3

Figure: 4

Figure: 5

Figure: 6

CONCLUSION

In this paper root cube mean cordiality of some standard graphs such as cycle, path, star, complete graph, complete bipartite graph and bistar are studied.

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