An Analysis of The Actual Application of Middle School Textbooks in Respect to The Competencies in Mathematics

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Abstract

In this paper, the researcher analyzes the middle-school-mathematics textbooks that were developed in accordance with the 2015 revision-curriculum, in Korea. Its aim is to provide implications for mathematics education by analyzing the actual application of the competencies in the mathematics textbooks. In order to accomplish this, the analysis of the textbooks was performed on: number & operations; variables & expressions; functions; geometry and probability & statistics. The analytical criteria for judging whether or not such competencies are applied in the content of textbooks consists of the six mathematics competencies, presented by Ministry of Education[1]. Moreover, as the results of this study demonstrate problem-solving competency, communication competency and attitude & practice competency were applied throughout the math textbooks well. However, the application rate of: the reasoning competency, creativity & convergence competency and information processing competency were low.

Keywords : Analysis, Competency, Mathematics, Middle School, Textbook

1. Introduction

In the 2015 revision curriculum, the emphasis is placed on the development of creative and convergent talents with core competencies required by the knowledge and information society. In particular, the mathematics curriculum emphasizes the six core competencies such as problem solving, reasoning, communication, creativity & convergence, information processing and attitude & practice. Three competencies like problem solving, reasoning and communication were emphasized in the 2009 revision mathematics curriculum[2], which are mathematical processes already mentioned in the Principle and Standards for School Mathematics[3]. It is meaningful to look at how many the competencies are applied in mathematics textbooks according to the revised curriculum because many Korean teachers design and write lesson plans based on textbooks. The purpose of this study is to provide implications for teaching

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methods in mathematics by analyzing the actual cases of the competencies shown in the revised middle-school-mathematics-textbooks.

2. Study Method

2.1 Subject

In this study, five textbooks[4-8] of the seventh-grade mathematics were analyzed. The textbooks were developed in accordance with the 2015 revision curriculum. Five textbooks were widely used in educational field and on-line materials were well developed also. The analysis of the textbooks was conducted on: number & operations, variables & expressions, functions, geometry and probability & statistics. The below [Table 1] summarizes the common elements extracted from the textbook[4-8].

[Table 1] The Analyzed Units of Textbooks

Part	Section	Subsection
Number & operations	Fractional Decomposition	Fractional Decomposition
	Integral and Rational Numbers	Properties of Number Systems
		Addition and Subtraction
		Multiplication and Division
Variables & expression	Variables and Expression	Using Variables
		Calculating Linear Expression
	Linear Equation	Definition of Equation
		Linear Equation
Functions	Coordinate Planes and Graphs	Ordered Pair and Coordinate
		Graphs
	Direct Proportion and Reciprocal Proportion	Direct Proportion
		Reciprocal Proportion
Geometry	Basic Figure	Point, Line, Plane, Angle
		Position Relation
		Properties of Parallel Line
	Construction and Congruent	Constructing Triangle
		Congruence Condition of Triangle
	Plane Figure	Polygon
		Circle and Sector

	Solid figure	Polyhedron
		Solid of Revolution
		Surface Area and Volume(Prism and Cylinder)
		Surface Area and Volume(Pyramid and Cone)
		Surface Area and Volume(Sphere)
Probability & Statistic	Data analysis	Stems and Leaves
		Frequency Distribution Table and Histogram
		Frequency Distribution Polygon
		Relative Frequency

2.2 Procedure

The researcher presented the analytical criteria of the math textbooks using the 2015 revision mathematics curriculum. The following [Table 2] shows that the analytical criteria for judging whether or not such competencies are reflected in the content of textbooks, focusing on the six mathematics competencies and their sub-factors[1].

[Table 2] An Analyzing Criteria of Mathematics Textbook

Competencies Subsection	P	R		C			Cm	I			A
		c	j	cr	i	o		d	e	t	
Fractional Decomposition											
...											
Relative Frequency											

P: Problem solving, R: Reasoning, C: Creativity & Convergence, Cm: Communication, I: Information processing, A: Attitude & Practice, c: conjecture, j: justification, cr: creativity, i: inner connection, o: outer connection, d: data analysis, e: engineering, t: tools for math education

Using the above analytic criteria, the researchers requested three mathematics teachers with more than 10 years of experience to analyze each of the five mathematics textbooks[4-8], and they were marked as '1' if they showed the competence and 'blank' if they did not. The researchers confirmed the agreement of the analyzed data, and tried to secure the validity of the inconsistent parts through a specialist council composed of one professor of mathematics

education and three mathematics teachers. The below [Table 3] is a part of the analysis of textbook[5].

[Table 3] An Examples of Analysis

Competencies Subsection	P	R		C			Cm	I			A
		c	j	cr	i	o		d	e	t	
Fractional Decomposition	1	1	1			1	1	1		1	1
Properties of Number Systems	1	1					1				1
...											
Relative Frequency	1					1	1	1			1

3. Result

3.1 The Application Rate of The Competencies in Math Textbooks

Problem-solving competency was reflected well throughout the math textbooks such as number & operations, variables & expressions, functions, geometry and probability & statistics. The application rate of communication competency and attitude & practice competency were generally high. However, the application rate of reasoning competency, creativity & convergence competency(except outer connection) and information processing competency were low. In the textbook B, the rate of 'justification(7.1%)' which is the sub-factor of reasoning competency was quite low. In the textbook[6], the rate of 'conjecture(10.7%)' which is the sub-factor of reasoning competency was very low. In the creativity & convergence competency, the rate of 'creativity and inner connection' was low. Looking closely, the application rate of 'creativity' in the textbook[4], textbook[5] and textbook[8] were each 25%. The application rate of 'inner connection' in the textbook B was 21.4%. Textbook[5] and textbook[6] were each 25%. The application rate of the sub-factors included in the information processing competency was low. Looking detail, the application rate of 'data analysis' in the textbook[4] and textbook[5] were each 21.4%, 17.9%. The application rate of 'engineering' in the textbook[4] was 25%. The rate of 'tools for math education' in the textbook[4] and textbook[8] are each 14.3%, 21.4%.

As a result of analyzing the seventh grade mathematics textbooks according to the revised curriculum, the actual application rate(%) of the competencies of the five mathematics textbooks is as shown in the following table.

[Table 4] The Result of Analysis

unit(%)

Competencies Textbooks	P	R		C			Cm	I			A
		c	j	cr	i	o		d	e	t	
[4]	100	35.7	7.1	25.0	21.4	64.3	92.9	21.4	35.7	14.3	92.9
[5]	100	60.7	50.0	25.0	25.0	82.1	92.9	17.9	25.0	28.6	92.9
[6]	100	10.7	28.6	39.3	25.0	92.9	92.9	28.6	32.1	28.6	92.9
[7]	100	35.7	57.1	46.4	35.7	71.4	92.9	28.6	32.1	42.9	92.9
[8]	100	32.1	32.1	25.0	32.1	78.6	85.7	28.6	39.3	21.4	89.3
Average	100	35	35	32.1	27.8	77.9	91.5	25	32.8	27.2	92.2

3.2 The Actual Application Cases of The Competencies in Math Textbooks

The qualitative analysis of the actual application of competencies in the revised math textbooks can provide an objective basis for the quantitative analysis. In addition, providing examples of how the competencies is reflected in math textbook contents can provide guidance for mathematics teachers to design their lessons that take the competencies into account.

The following [Fig. 1] is shown that the example of problem solving competency, reasoning competency(conjecture, justification), communication competency, information processing competency(data analysis) and attitude & practice competency.

The below [Fig. 2] shows the process of solving the 'surface area of the prism' of a solid figure unit in the middle school geometry with algebraic methods using real life materials. It is shown that the example of problem solving competency, reasoning competency(conjecture), communication competency, creativity & convergence(inner connection, outer connection) and attitude & practice competency.

수학 역량 기르기

문제를 해결하고 설명할 때는

- 문제의 조건과 정보를 파악하고 풀이 전략을 생각한다.
- 자신의 생각을 명확하게 전달한다.

문제 해결 | 의사소통

다음은 어느 모임의 구성원의 나이를 글로 나타낸 것이다. 구성원의 나이를 두 가지 이상의 방법으로 구하고 자신의 풀이 방법을 설명하시오.

• 준호와 기훈이는 쌍둥이이다.

• 소희는 성준이보다 2살이 많고, 성준이는 쌍둥이보다 2살이 많다.

• 구성원의 나이를 모두 더하면 54살이다.

준호: _____ 기훈: _____ 소희: _____ 성준: _____

When describing and solving a problem.

- ✓ Identify the conditions and information of the problem and think about the solution strategy.
- ✓ Communicate Self-thought clearly

The following is the age of members of a group. Find the age of the member in more than one way and describe your work.

- Junho and Kihun are twins.
- Sohee is two years older than Sung Jun. Sung Jun is two years older than twins.
- The sum of the members age is 54 years old.

[Fig. 1] The Example of Problem Solving Competency, Reasoning Competency(Conjecture, Justification), Communication Competency, Information Processing Competency(Data Analysis) and Attitude & Practice Competency([7], pp.87)

생각 열기

선물 상자 만들기

미선은 오른쪽 그림과 같은 직육면체 모양의 선물 상자를 만들려고 합니다. 상자의 전개도를 이용하여 선물 상자를 만들기 위해 필요한 종이의 넓이를 어떻게 구할지 생각해 봅시다.

→

활동 1 전개도의 빈칸에 알맞은 수를 써넣고, 한 밑면의 넓이와 옆면 전체의 넓이를 각각 구해 보자.

활동 2 선물 상자를 만드는 데 필요한 종이의 넓이를 구하는 방법을 말해 보자.

Let's make a gift box

Misun will create a rectangular box shaped gift box like the one on the right. Let's think about how to find the width of the paper needed to make a gift box using the spreadsheet of the box.

Activity(1) Let's write the appropriate number in the empty space of the development chart and find the width of the bottom surface and the total width of the side surface.

Activity(2) Let's say how to get the width of paper needed to make a gift box.

[Fig. 2] The Example of Problem Solving Competency, Reasoning Competency(Conjecture), Communication Competency, Creativity & Convergence(Inner Connection, Outer Connection) and Attitude & Practice Competency([5], pp.223)

The following [Fig. 3] shows the process of representing graph with the 'relation of direct proportion and reciprocal proportion' of a function part. It is shown that example of problem solving competency, information processing competency(engineering) and attitude & practice competency.

컴퓨터 기하 프로그램을 이용하여 그래프를 그려 보자.

1 정비례 관계와 반비례 관계의 그래프 그리기

입력창에 ' $y=2x$ ', ' $y=-2/x$ '를 입력하고 을 누르면 오른쪽 그림과 같이 정비례 관계 $y=2x$ 와 반비례 관계 $y=-\frac{2}{x}$ 의 그래프를 각각 그릴 수 있다.

Draw a graph using a computer gauging program

(1) Draw a graph of direct relationship and inverse relationship

If you type ' $y=2x$ ',' $y=-2/x$ ' in the input window and press "enter", you can draw graphs of direct relationship $y=2x$ and inverse relationship $y=-2/x$ as shown in the right figure.

[Fig. 3] The Example of Problem Solving Competency, Information Processing Competency(Engineering) and Attitude & Practice Competency([4], pp.137)

The below [Fig. 4] is shown that the example of problem solving competency, reasoning competency(conjecture, justification), communication competency, information processing competency(tools for math education) and attitude & practice competency.

설명하기

색종이를 다음 두 가지 방법으로 잘라 말아 보고 원뿔의 옆면이 부채꼴인 이유를 말해 보자.

[방법 1]

[방법 2]

Explanation

Cut the colored paper in two ways and tell why the sides of the cone are fan-shaped.

[Fig. 4] The Example of Problem Solving Competency, Reasoning Competency(Conjecture, Justification), Communication Competency, Information Processing Competency(Tools for Math Education) and Attitude & Practice Competency([6], pp.236)

The following [Fig. 5] shows the process of representing an expression 'using variables' of a variables & expression unit in the middle school algebra using real context. It is shown that the example of problem solving competency, reasoning competency(justification), communication competency, creativity & convergence(creativity, outer connection) and attitude & practice competency.

생각
넓히기

의사소통

문자로 나타낼 수 있는 다양한 상황을 실생활에서 찾아 다음 보기와 같이 써 보고 이를 식으로 나타내어 친구들에게 설명하여 보자.

보기

[상황] 선형이가 자전거를 타고 동네 공원을 시속 20 km로 a 시간 동안 달렸을 때 이동한 거리

[식] $20a$ km

[상황]

[식] _____

Find out the various situations that can be expressed as problems in real life, and write them down as follows to explain them to your friends.

-Example

[Situation] When Sun-hyung ran a bicycle in a neighborhood park for a hours, Distance during this time is [Formula] $20a$ km

[Situation] / [Formula] _____

[Fig. 5] The Example of Problem Solving Competency, Reasoning Competency(Justification), Communication Competency, Creativity & Convergence (Creativity, Outer Connection) and Attitude & Practice Competency([8], pp.78)

4. Conclusion and Suggestions

In the mathematics textbooks according to the 2015 revision curriculum, problem-solving competency, communication competency and attitude & practice competency were applied to all areas such as number & operations, variables & expressions, functions, geometry, and probability & statistics. On the other hand, the application rate of reasoning competency and information processing competency were relatively low. In the case of creativity & convergence competency, the application rate of 'outer connection' factor was high, but the rate of 'inner connection' factor and 'creativity' factor was low.

Therefore, the curriculum or textbook developers should fully consider the factors for expanding reasoning competency, creativity & convergence competency and information

processing competency in middle school mathematics curriculum.

As the results of the analysis, only one of the five textbooks was well reflected in the 'conjecture' factor, and two of the five textbooks were well reflected in the 'justification' factor. Three of the five textbooks were less than 25% in the 'creativity' factor, and only one of the five textbooks was more than 35% in the 'inner connection' factor. Within the information processing competency, the application rate of 'data analysis' and 'tools for math education' factors were low.

Teachers need to be aware of the following when designing a class that takes core competencies into account in mathematics. First, it is necessary to utilize 'conjecturing' and 'justifying' in the classroom so that students can expand their reasoning competency. Second, creativity & convergence competency can be enhanced by presenting 'real life problems' or 'open-ended problems' to students. Third, in order to enhance the information processing competency of students, it is needed to collect data from 'social phenomena' or 'real life materials', to organize and interpret them, and to use various 'educational tools' when 'justifying'.

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