Effect of Tooth Surface Modification on Dynamic Transmission Efficiency of Double Helical Gear Trains

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To analyze the influence mechanism of vibration loads on dynamic transmission efficiency (DTEF) of double helical gear trains (DHGT), a calculation method of DTEF which takes into consideration the real-time dynamic loads is proposed from the perspective of improving both efficiency and precision. The average DTEF is taken as the dynamic objective function of optimization for three-dimensional modification technology of tooth surface. An engineering example is given in this form of a numerical simulation, which proves that the average DTEF with the consideration of dynamic loads is obviously lower than the static transmission efficiency (STEF), especially for the resonance speed region. The results also show that the DTEF after 3-D modification is significantly improved, owing to the improvement of the meshing state of the tooth surface. The comparison of DTEF with different loads and speeds via simulation and experiment further demonstrates the credibility of the theoretical perspectives. This study gives the explanation on the importance of DTEF in the determination of driveline parameters and estimation of transmission quality of DHGT. The developed method for DTEF analysis is practical in engineering and establishes a reliable foundation for further research on design of high-quality double helical gear trains.

1. INTRODUCTION

Double helical gear trains are characterized by a large torque transmission capacity, superior transmission smoothness, and low axial force, which accounts for their large scale applications in ships and other heavy machinery.^{1–4} With the development of Pratt & Whitney's geared turbofan engine, double helical gear trains have also become the key transmission parts of an aircraft engine, which work under variable speed processes.^{5–9}

Owing to the relatively high mesh ratio and high sensitivity to error, the transfer efficiency of a powertrain is a crucial factor that influences the quality of double helical gear trains, especially in the high speed and heavy-load driveline system of an aircraft engine. Power loss of double helical gear trains is caused by various factors, such as mechanical friction of meshing teeth, supporting bearings, shaft parts, and resistance while stirring the lubricating oil, etc. One of the most important steps during the design of a double helical gear trains is estimation of transmission efficiency, therefore the research on the mechanism of power loss in double helical gear trains is critical for the process of design, optimization, and operation. Extensive research on the static loss mechanism of gear transmission has been carried out by scholars worldwide. Kolivand¹⁰ proposed a mathematical efficiency model incorporating hybrid elasto-

hydrodynamic lubrication (EHL) and an efficiency model for calculating the friction power loss based on the tooth surface traction model, the efficiency models were also applied with different misalignments to examine the influences of critical operating conditions and design parameters on the meshing power loss of mechanical gears. Furthermore, Li and Kahraman^{11,12} offered a transient contact model of cylindrical gear tooth surface, which took into consideration a series of time-varying incentive factors, including the curvature radius, external torque and slip-roll ratio of contact surface. Recently some simulations based on computational fluid dynamics have been proposed.^{13–15} A considerable time-saving numerical methods based on some specific techniques for mesh processing are also widely employed to improve the calculation efficiency of computational fluid dynamics simulations.¹⁶

With the increase in operating speed of modern transmission machinery, the influence of dynamic loads on transmission efficiency cannot be ignored. The transmission efficiency with dynamic load is defined as DTEF; and some previous work on DTEF has been introduced in literature. Chen and Liang¹⁷ put forward the analytical expressions of a 2-DOF transmission efficiency based on a methodology of virtual power, which was associated with the varying loads. In order to make an in-depth analysis of DTEF of a vehicle powertrain, a calculation

method for power losses during the process of gear transmission, were deduced from the EHL theory, and the validity of the DTEF model was proved by comparisons with experimental data.^{18–20} However, most of these works assume a constant vibration load (i.e., dynamic load factor and average dynamic load), the dynamic relationship between the complex variable load and transmission efficiency is overlooked. Thusly the inclusion of real-time dynamic loads is essential to improve the accuracy of the DTEF calculation.

In order to meet the requirements of high transmission efficiency, a variety of approaches and technologies have been used to improve the efficiency of gear transmission. Petrescu²¹ analyzed the influence of a few parameters on gear efficiency (including the teeth number of the active gear, the teeth number of the passive gear, pressure angle of divided circle, the helix angle), and these parameters were optimized to improve efficiency. Tüma²² pointed out that the smallest possible peak-to-peak value of the transmission error could reduce the internal excitation source of the vibration in gear train and improve the contact state of the tooth surface, which eventually brought about higher meshing smoothness and transmission efficiency of the gear train. Alberto^{23,24} focused specifically on the influences of tooth profile modification on load distribution. The results showed that the modification of gear tooth tip brought about a reduction in length of the mesh line and an increase in transmission efficiency. In previous study of Concli,²⁵ the load-dependent loss was reduced by approximately 35% and the total efficiency at nominal speed and torque was increased by 1%, by reducing the module and changing the planet carrier. The modification of gear surface, especially the 3-D modifications, is the principal method to improve meshing performance and transmission efficiency.^{26–28}

With the increasing demand for low vibration and high transmission efficiency of double helical gear trains, it is worth considering comprehensive effects so that time-varying mesh stiffness and dynamic meshing torque can be observed simultaneously. The goal of this study is to improve the DTEF of double helical gear trains via 3-D modification of the tooth profile. Additionally, the problem of enhancing DTEF is converted into finding the optimal coefficients of the 3-D modification tooth by a genetic algorithm. In particular, the calculation method of DTEF considering time-varying mesh stiffness and friction torque excitations will fill the research gap compared with previous studies. The main contribution of the proposed dynamic transmission efficiency enhancement method lie in the following three aspects:

1. For double helical gear trains, the time-varying mesh stiffness and friction torque excitations are taken into consideration comprehensively to calculate more accurately the dynamic meshing load.
2. Based on the analysis of the influence of the dynamic meshing load, a novel methodology is proposed for double helical gear trains to obtain the DTEF, which is closer to the practicality.
3. To elevate the DTEF, the methodology for 3-D tooth surface modification is accordingly designed based on these analyses, then the experiments are carried out, including

the manufacture of gear sets, closed-power-flow rig tests and data acquisition; the feasibility, effectiveness and advances in practical application are verified.

This paper is organized as follows: The first part aims to review the published papers that dealt with the power loss mechanism of gear transmission system and the corresponding method for optimization. With the purpose of calculating accurately and quickly the DTEF of double helical gear trains under various working conditions introduced in Section 3, the power losses of double helical gear trains, including the friction loss between meshing teeth surfaces and rolling bearing contact surfaces, as well as windage resistance loss, are illustrated in Section 2, the comparison of DTEF with different torques and rotational speeds is also presented in Section 2. In Section 4, to improve the DTEF of double helical gear trains, teeth modification and optimization are carried out. Subsequently, in Section 5, based upon a double helical gear test bench with mechanical closed power flow, the DTEF are carefully measured and used to validate the model. The last part summarizes the principal findings of this research.

2. TRANSMISSION POWER LOSS MECHANISM OF DHGT

Gear transmission is the most efficient solution for continuous drive, the efficiency of cylindrical gear transmission is higher than bevel gear, arc tooth gear and other gear systems.^{29,30} The double helical gear trains (DHGT) feature a larger contact surface area compared with spur/helical gear, which makes it difficult to generate the lubricating oil film on gear teeth, therefore, the study of transmission power loss mechanism is of great significance. It is generally believed that the power loss of the enclosed DHGT mainly includes the friction loss of contacting teeth surfaces, friction loss of rolling bearing and windage loss. In this section, the power loss of double helical gear trains considering real-time dynamic loads will be calculated in detail. The primary innovations of transmission power loss mechanism model built in this research can be summarized as follows:

(1) The meshing friction loss of the tooth pair is calculated based on loaded tooth contact analysis (LTCA), in which the geometric meshing characteristics of gear tooth surface is taken into consideration. To obtain a more accurate surface load distribution, the accuracy of meshing friction loss calculation can be elevated accordingly. (2) The dynamic load, which is closer to the practicality and intended to describe the actual real-time meshing force, is introduced for the analysis of meshing friction loss of tooth pair, therefore the calculation accuracy of dynamic meshing power loss can be further improved.

2.1. Tooth Pair Meshing Friction Loss of DHGT

The calculation process of LTCA using the double-helical gear pair model is illustrated in Fig. 1. where *I*, *II*, *III*, *IV* represent the contacting gear teeth at a given instant during the meshing period. The profile of gear teeth shown in Fig. 1 is along the normal direction of the contacting tooth surface. And

jq ($q = I, II, III, IV$) denote the meshing point, iq denote the central contacting position along the principal direction of contacting surfaces. The gap vector between all contact pairs in the initial state of contact is represented by $[w]_k$.

Under the action of external loads P_L and P_R , the driving gear rotates and makes an angular displacement Z , meanwhile, the teeth deformation causes the redistribution of contacting load between contacting teeth. The force equilibrium with plastic deformation is described in Eq. (1).

$$[p]_k [\delta]_k + [w]_k = [d]_k + [Z], q = I, II, III, IV; \quad (1)$$

$$\begin{cases} \sum_{j=1}^n p_{iI} + \sum_{j=1}^n p_{iIII} = P_L \\ \sum_{i=1}^n p_{iIII} + \sum_{j=1}^n p_{iIV} = P_R \\ P_L + P_R = P \end{cases}; \quad (2)$$

$$\begin{cases} p_{iq} > 0 & \text{when } d_{iq} = 0 \\ d_{iq} > 0 & \text{when } p_{iq} = 0 \end{cases}; \quad (3)$$

where $[p]_k$ denotes the contact loads matrix of teeth surfaces; $[d]_k$ denotes the teeth clearance matrix after deformation; $[\delta]_k$ denotes the flexibility matrix of teeth pair.

Due to the multi-surface contact and heavy-load operation, the mesh power loss in teeth pairs accounts for a major proportion of transmission power loss. The mesh power loss is primarily associated with the transverse load distribution factor of the gear pair, the relative sliding speed and relative rolling velocity between contacting teeth surfaces.³¹⁻³⁴

First, mesh power loss along the entire mesh period of every contacting teeth pair should be calculated. There is no absolute smooth gear tooth surface under the conditions of practical application, nor an adequate lubrication, therefore the teeth pairs usually operate under the EHL condition. As illustrated in Fig. 2, the oil film of EHL (point C) and contacting peak of boundary lubrication (point D) move simultaneously on meshing tooth surfaces.

In the situation of mixed EHL, the synthetic friction coefficient of contacting surfaces of the meshing gear pair is calculated by Eq. (4).

$$\mu_M = \frac{1 - f_\epsilon}{\mu_B} + f_\epsilon \mu_E; \quad (4)$$

where μ_M is the equivalent friction coefficient with mixed elastohydrodynamic lubrication; μ_E is the equivalent friction coefficient with elastohydrodynamic lubrication; μ_B is the equivalent friction coefficient with boundary lubrication; f_ϵ represents the percentage that the peak-to-peak contacts take in all teeth surface contacts, and the corresponding calculation is illustrated in Eq. (5).

$$\begin{cases} f_\epsilon = \frac{1.21\epsilon^{0.64}}{1+0.37\epsilon^{1.26}} \\ \epsilon = \frac{H_{min}}{\sigma} \\ \sigma = \sqrt{\sigma_p^2 + \sigma_g^2} \end{cases}; \quad (5)$$

where ϵ indicates the film thickness ratio, which is within [1,3] in the condition of mixed elastohydrodynamic lubrication; σ indicates the relative roughness of contacting teeth surfaces, and σ_p , σ_g denote the roughness of active gear and passive gear; H_{min} indicates the minimum allowable film thickness in the condition of EHL, the corresponding calculation are elaborated in literature.^{35,36}

In the condition of boundary lubrication, the tooth surface after the running-in process is considered to be in the state of elastic micro peak contact, and the boundary film are not broken. It is also assumed that the friction coefficients of each asperity are the same, and the boundary lubrication friction coefficient μ_B is calculated by Eq. (6).

$$\mu_B = \frac{\sum_{i=1}^N \oint_{A_{ci}} \tau_{ci} dA_{ci}}{P_B}; \quad (6)$$

where N represents the asperity number of contacting teeth surface; A_{ci} represents the contacting area of asperity i ; τ_i denotes the shear stress of asperity i ; P_B denotes the asperity contact pressure. The value of μ_B varies within a narrow range of 0.07–0.15, which is identified by the experiment.³⁷

The equivalent friction coefficient μ_{EL} under the condition of full-film lubrication can be deduced from the EHL model, as shown in Eq. (7)–(9).

$$\begin{cases} \mu_E = e^{fk} \cdot (P_h(t))^{b_2} \cdot |SR(t)|^{b_3} \cdot (v_e(t))^{b_6} \\ \eta_M \cdot (\rho(t))^{b_8} \\ fk = b_9 \cdot e^S + b_4 \cdot \gamma + b_4 \cdot e^{-\gamma} + b_1 \\ \gamma = \log_{10}(\eta_M) \cdot |SR(t)| \cdot P_h(t) \end{cases}; \quad (7)$$

$$\begin{cases} v_e(t) = |\nu_p(t) + \nu_g(t)| \\ \nu_r(t) = |\nu_p(t) - \nu_g(t)| \end{cases}; \quad (8)$$

$$\begin{cases} SR(t) = \frac{2\nu_r(t)}{\nu_e(t)} \\ \rho(t) = \frac{\rho_p(t) \cdot \rho_g(t)}{\rho_p(t) + \rho_g(t)} \end{cases}; \quad (9)$$

where $v_e(t)$, $v_r(t)$ are rolling speed and relative sliding speed of contact point, respectively; $\nu_p(t)$, $\nu_g(t)$ are linear velocity of meshing position of driven and driving gear pair; $SR(t)$ denotes the sliding-rolling ratio of the tooth contacting point; $\rho(t)$ denotes the equivalent curvature radius at the teeth contacting point; $\rho_p(t)$, $\rho_g(t)$ denote the curvature radius of driven gear and driving gear at the contacting point; $P_h(t)$ is the Hertzian stress at the tooth contact point; η_M is the dynamic viscosity of lubricating oil. The coefficients $b_1 - b_9$ are given in Tab. 1, which is introduced in Ref.³⁸⁻⁴²

The method for the friction coefficient calculation in the condition of a single given point between meshing teeth surfaces is presented above. Whereas for the gear pairs with high contact ratio, as in the case of double helical gears, there are usually more than one pair of teeth mesh at the same time, the engagement mark of a meshing double helical gear pair obtained with LTCA method is illustrated in Fig. 3.

It is observed that there are 16 contact lines in the duration from start to end of entire engagement, and a meshing cycle is formed every five intervals in the LTCA analysis program. Tab. 2 lists the instantaneous contacting teeth pairs in a meshing cycle, Δt denotes the meshing period of a mesh cycle.

Consequently, the transverse loading distribution factor of gear pair, the relative rolling and sliding velocity between contacting teeth surfaces are acquired with LTCA method,⁴³ and the mesh friction loss of double helical gear trains in the case

Figure 1. Double helical gear mesh model for LTCA.

Figure 2. Mixed EHL of meshing gear pair.

Figure 3. Engagement trajectory of helical gear pair.

of a given static load is deduced from Eq. (1)~Eq. (10).

$$P_{fk} = \sum_{i=1}^{\text{ceil}(\frac{n+1-k}{6})} \left\{ \sum_{j=1}^{m_{k+6(i-1)}} (|V_{k+6(i-1),j}| \cdot F_n \lambda_{k+6(i-1),j} \cdot \mu_{k+6(i-1),j}) \right\} \dots \dots (k = 1, 2, 3, 4, 5, 6); \quad (10)$$

where P_{fk} is the mesh friction power loss of teeth surface under a given load F_n ; n is the order of contact lines of a single gear tooth engagement; k is the rotating sequence number; ceil is the ceiling function; $m_{k+6(i-1)}$ is the number of scattered mesh points in $k + 6(i - 1)$ order contact line; $\mu_{k+6(i-1),j}$, $\lambda_{k+6(i-1),j}$, $V_{k+6(i-1),j}$ are the comprehensive equivalent friction coefficient, transverse loading distribution factor of gear pair, and the sliding speed at j th mesh point of $k+6(i-1)$ order contact line.

Additionally, there are two conventional methods to calculate the dynamic power loss of meshing teeth pairs in the case of time-varying meshing force. An easy way is just to replace the static torque with the vibration torque at each instance, and the transverse loading distribution factor of gear pair, the relative rolling and sliding velocities between contacting teeth sur-

Figure 4. A novel mesh power loss calculation method considering dynamic loads.

faces are kept constant. Another accurate but time-consuming approach is to execute repeatedly the LTCA program and solve Eq. (4)–(10) for every transient vibration load, Fig. 4 illustrates a novel method that integrates the advances of the methods mentioned above to calculate the mesh power loss.

According to the analysis above, to calculate the vibration loads ($F_{dL}(t)$ and $F_{dR}(t)$) of double helical gear trains, a dynamic double-helical gear model is initially put forward with the consideration of corner mesh impact force excitation, time-varying mesh stiffness excitation and dynamic friction force excitation of tooth surface. The dynamic double-helical gear model denotes a kinetic system in which the translational and rotational dynamic equations are both employed to describe the motion of system, as illustrated in Fig. 5. It is worth noting that the contacting force of the tooth surface can be decomposed

Table 1. Value of the parameters $b_1 - b_9$

Variable	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
Value	-8.91	1.03	1.04	-0.35	2.81	-0.101	0.75	-0.39	0.62

Table 2. Instantaneous contact teeth pairs in a meshing cycle.

Meshing period	0	$1/6 \times \Delta t$	$2/ \times \Delta t$	$3/6 \times \Delta t$	$4/6 \times \Delta t$	$5/6 \times \Delta t$
Number of mesh lines	17;13	2;8;14	3;9;15	4;10;16	5;11	6;12

Figure 5. Vibration model of double helical gear pairs.

into two forces in different directions: the direction of meshing line and the direction of gear shaft. The direction of y-axis in this vibration model is defined as the direction of meshing line, and the direction of z-axis is defined as the direction of the gear shaft, therefore, the spring that supports the bearings in the x-axis direction in Fig. 5 is ignored in this practical application.

In accordance with the mutual movement relationship in Fig. 5, the vibration equations of double-helical gear pair are finally given in Eq. (11)–(16).

$$\begin{cases} m_{p1}\ddot{y}_{p1} + k_{p1y}y_{p1} + c_{p1y}\dot{y}_{p1} = -F_{y1} + m_{p1}g \\ m_{g1}\ddot{y}_{g1} + k_{g1y}y_{g1} + c_{g1y}\dot{y}_{g1} = F_{y1} + m_{g1}g \\ m_{p2}\ddot{y}_{p2} + k_{p2y}y_{p2} + c_{p2y}\dot{y}_{p2} = -F_{y2} + m_{p2}g \\ m_{g2}\ddot{y}_{g2} + k_{g2y}y_{g2} + c_{g2y}\dot{y}_{g2} = F_{y2} + m_{g2}g \end{cases}; \quad (11)$$

$$\begin{cases} m_{p1}\ddot{z}_{p1} + k_{p12z}(z_{p1} - z_{p2}) + c_{p12z}(\dot{z}_{p1} - \dot{z}_{p2}) = -F_{z1} \\ m_{g1}\ddot{z}_{g1} + k_{g1z}z_{g1} + c_{g1z}\dot{z}_{g1} + k_{g12z}(z_{g1} - z_{g2}) + \\ c_{g12z}(\dot{z}_{g1} - \dot{z}_{g2}) = F_{z1} \\ m_{p2}\ddot{z}_{p2} - k_{p12z}(z_{p1} - z_{p2}) - c_{p12z}(\dot{z}_{p1} - \dot{z}_{p2}) = -F_{z2} \\ m_{g2}\ddot{z}_{g2} + k_{g2z}z_{g2} + c_{g2z}\dot{z}_{g2} - k_{g12z}(z_{g1} - z_{g2}) - \\ c_{g12z}(\dot{z}_{g1} - \dot{z}_{g2}) = F_{z2} \end{cases} \quad (12)$$

$$\left\{ \begin{array}{l} I_{p1} \ddot{\theta}_{p1} = -F_{y1} \cdot R_p + F_{dL} \cdot \chi_{p1} + T_{p1} - F_{s1} \cdot R_p \\ I_{g1} \ddot{\theta}_{g1} = F_{y1} \cdot R_g - F_{dL} \cdot \chi_{g1} - T_{g1} + F_{s1} \cdot R_g \\ I_{p2} \ddot{\theta}_{p2} = -F_{y2} \cdot R_p + F_{dR} \cdot \chi_{p2} + T_{p2} - F_{s2} \cdot R_p \\ I_{g2} \ddot{\theta}_{g2} = F_{y2} \cdot R_g - F_{dR} \cdot \chi_{g2} - T_{g2} + F_{s2} \cdot R_g \end{array} \right. ; \quad (13)$$

$$\begin{cases} F_{dL} = c_{m1} \left[\cos \beta_1 \left(R_p \dot{\theta}_{p1} - R_g \dot{\theta}_{g1} + \dot{y}_{p1} - \dot{y}_{g1} \right) + \right. \\ \left. \sin \beta_1 \left(\dot{z}_{p1} - \dot{z}_{g1} \right) \right] + \\ k_{m1} \left[\cos \beta_1 \left(R_p \theta_{p1} - R_g \theta_{g1} + y_{p1} - y_{g1} \right) + \right. \\ \left. \sin \beta_1 \left(z_{p1} - z_{g1} \right) \right] \\ F_{dR} = c_{m2} \left[\cos \beta_2 \left(R_p \dot{\theta}_{p2} - R_g \dot{\theta}_{g2} + \dot{y}_{p2} - \dot{y}_{g2} \right) + \right. \\ \left. \sin \beta_2 \left(\dot{z}_{p2} - \dot{z}_{g2} \right) \right] + \\ k_{m2} \left[\cos \beta_2 \left(R_p \theta_{p2} - R_g \theta_{g2} + y_{p2} - y_{g2} \right) + \right. \\ \left. \sin \beta_2 \left(z_{p2} - z_{g2} \right) \right] \end{cases} ; \quad (14)$$

$$\begin{cases} F_{y1} = \cos \beta_1 F_{dL} \\ F_{z1} = \sin \beta_1 F_{dL} \end{cases} ; \quad (15)$$

$$\begin{cases} F_{y2} = \cos \beta_2 F_{dR} \\ F_{z2} = \sin \beta_2 F_{dR} \end{cases} \quad ; \quad (16)$$

where I_{ij} , m_{ij} ($j = 1, 2; i = p, g$) indicate the moment of inertia and mass of active and passive gears; R_p , R_g denote the base radii of active and passive gears; θ_{ij} , and z_{ij} , y_{ij} ($j = 1, 2; i = p, g$) denote the dynamic rotational angles and the vibration displacements of every gear pairs; F_{dL} , F_{dR} denote the vibration mesh forces of right helical mesh teeth pair and left helical mesh teeth pair along the normal direction of meshing tooth surface; c_{ijz} , c_{ijy} and k_{ijz} , k_{ijy} ($j = 1, 2; i = p, g$) denote the axial and radial supporting damp and stiffness of rolling bearings; k_{p12z} , k_{g12z} , and c_{p12z} , c_{g12z} indicate the axial supporting stiffness and damping of knife retracting groove; F_{z1} , F_{z2} , and F_{y1} , F_{y2} denote the vibration mesh forces in axial and mesh line directions; χ_{p1} , χ_{p2} , χ_{g1} , χ_{g2} indicate the equivalent friction coefficients of driving torque and driven torque; c_{m1} , c_{m2} denote the mesh damping of right helical gear teeth pair and left helical gear teeth pair; F_{s1} , F_{s2} indicate the corner mesh impact forces in right helical gear teeth pair and left helical gear teeth pair; k_{m1} , k_{m2} denote the time-varying mesh stiffness of right helical gear teeth pair and left helical gear teeth pair, which can be obtained by LTCA program, and the calculation methods of meshing stiffness and equivalent meshing damping coefficients are elaborated in;⁴⁴ β_1 , β_2 denote the helix angles of right helical gear teeth pair and left helical gear teeth pair, where $-\beta_2 = \beta_1$.

2.2. Rolling Bearing Friction Loss of DHGT

Calculation methods of rolling bearing power loss are complex and diverse, according to reference, ?? the power loss of rolling bearing primarily includes the loss correlated with the load and the loss correlated with viscosity, and the rolling bearing friction loss of double helical gear trains can be obtained by Eq. (17)–(18).

$$P_B = \sum_{i=1}^{n_b} \omega_b \left(0.0009 \left(\frac{F_{ST}}{C_S} \right)^{0.55} \cdot F_\beta D_m + M_V \right); \quad (17)$$

$$M_V = \begin{cases} 1.42 \times 10^{-5} f_0 (v \omega_b)^{2/3} D_m^3 C_1 & \text{when } v \cdot \omega_b > 2000 \\ 3.492 \times 10^{-3} f_0 D_m^3 C_2 & \text{when } v \cdot \omega_b \leq 2000 \end{cases}; \quad (18)$$

where n_b is the number of rolling bearing; F_{ST} is the equivalent static load of bearing; C_S is the rating static load of bearing; F_β is the equivalent normal contacting load considering the bearing cone angle; D_m is the bearing diameter; M_V is the viscous power loss of rolling bearing; f_0 is the lubrication coefficient of rolling bearing; ω_b is the relative speed between the inner and outer rings of bearing; v is the kinetic viscosity coefficient of lubricating oil; C_1, C_2 are constants of proportionality, $C_1 = C_2 = 6894$.

To ensure the accuracy of calculation in the case of vibration loads, according to Fig. 5, the equivalent static loads are replaced by the dynamic loads, as shown in Eq. (19).

$$\begin{cases} F_{ij}(t) = k_{ij} y_{ij} \cdots (i = p, g, j = 1, 2) \\ F_a(t) = k_{g1z} z_{g1} + k_{g2z} z_{g2} \end{cases}. \quad (19)$$

2.3. Windage Loss of DHGT

The windage loss appears in gearbox when mixed oil and gas are involved, and increases with the increase of rotating speed.^{17,46} Based upon the method for calculating the working resistance of turbine rotor, the windage loss of double helical gear trains is given in Eq. (20).

$$\begin{cases} P_{Wp} = 2.82 \times 10^{-7} \left(1 + 2.3 \frac{b}{R_p}\right) (n_p)^{2.8} \cdot R_p^{4.6} (0.028v + 0.019)^{0.2} \\ P_{Wg} = 2.82 \times 10^{-7} \left(1 + 2.3 \frac{b}{R_g}\right) \left(\frac{n_p}{m_g}\right)^{2.8} \cdot R_g^{4.6} (0.028v + 0.019)^{0.2} \end{cases}; \quad (20)$$

where P_{Wp}, P_{Wg} indicate the windage loss in left and right helical gear pairs; b denotes the total tooth width including the width of cutter groove; n_p represents the rotational speed of active gear; m_g represents the total mass of passive gear.

2.4. Dynamic Transmission Efficiency of DHGT

DTEF of double helical gear trains is subsequently established after a comprehensive analysis of friction power loss between contacting teeth pairs, friction loss of rolling bearings and windage loss.

$$\eta_M(t) = 1 - \frac{P_f \{F_d(t)\} + P_B(t) + P_{Wp} + P_{Wg}}{P_0}; \quad (21)$$

where η_M is the time-varying DTEF of double helical gear trains; P_0 is the input power of double helical gear trains.

The basic parameters of double helical gear trains are listed in Tab. 3.

According to the analysis of DTEF, an example is given in Fig. 6 to demonstrate the difference between DTEF and the static transmission efficiency (STEF).

It can be seen that the STEF varies with the tooth meshing position, and the average STEF is 98.01%; however, the

Table 3. Key parameters of double helical gear trains

Parameter name	Driving gear	Driven gear
Module [mm]	5	5
Pressure angle [°]	20	20
Helix angle [°]	26.1	-26.1
Face contact ratios	1.67	1.67
Profile contact ratios	1.21	1.21
External torque [N·m]	–	828
Density [g·cm ³]	7.80	7.80
Number of teeth	17	44
Tooth width [mm]	55	55
Input speed [r/min]	2 000	–
Bearing designation	NSK-NU211EW	NSK-HR32914J
Width of groove [mm]	50	50
Center distance error [μm]	5	5
Axis angle error [°]	0.05	0.05

Figure 6. Comparison of STEF and DTEF.

Figure 7. Comparison of TEF under different loads (2000 r/min).

DTEF in the case of dynamic load features an obvious fluctuation, and the average DTEF is 97.74%. It also indicates that the DTEF with vibration force is lower than STEF, this is consistent with the conclusion that the transmission quality is reduced, and total power loss is increased due to the tooth surface error, stiffness fluctuation, meshing impact and other excitations. Therefore, it is necessary to evaluate the system efficiency by DTEF in practical application.

Further analysis of the differences between DTEF and STEF from the perspective of loads and rotating speeds are shown in Fig. 7 and Fig. 8.

As shown in Fig. 7, under the condition of constant input speed, both of the two efficiencies increase slightly with the increase of external loads at first, when the external load exceeds a certain value (1800 N·m), the efficiencies tend to be

Figure 8. Comparison of TEF under different speeds (828 N·m).

stable. As shown in Fig. 8, the STEF and DTEF increase with the increase of input speeds, since the lubrication film is easier to form at higher input speed; however, the DTEF decreases rapidly when the input speed reaches the first-order resonance speed (4700 r/min), since the resonant dynamic load is obviously larger than the static load. The result shows that the DTEF is of great significance for the design of high-speed gear.

2.5. Teeth Modification Optimization of DTEF

The 3-D modification of double helical gear trains is an efficient technology in which the profile and lead modifications are employed to elevate the comprehensive contact quality of the tooth surface, especially the reduction of vibration load and friction coefficient. The 3-D modification surface of a double helical gear is shown in Fig. 9.

To obtain the optimal design parameters of teeth shape for 3-D modification, if the average DTEF is set as the objective of optimization, then the optimization function of the tooth surface modification for a double helical gear pair is established, as shown in Eq. (22)–(24).

$$f_C(y_{ij}) = \min \left(\frac{1}{\eta_{AVG}} \right); \quad (22)$$

$$\begin{cases} |y_{1L} - y_{3L}| \leq Q_{y0}, & |y_{2L} - y_{4L}| \leq l_{y0} \\ Q_{ymin} \leq y_{1R}, y_{3R} \leq Q_{ymax}, & l_{ymin} \leq y_{2R}, y_{4R} \leq l_{ymax}; \\ |y_{5L} - y_{7L}| \leq Q_{z0}, & |y_{6L} - y_{8L}| \leq l_{z0} \\ Q_{zmin} \leq y_{5R}, y_{7R} \leq Q_{zmax}, & l_{zmin} \leq y_{6R}, y_{8R} \leq l_{zmax} \end{cases} \quad (23)$$

$$\begin{cases} |y_{1R} - y_{3R}| \leq Q_{y0}, & |y_{2R} - y_{4R}| \leq l_{y0} \\ Q_{ymin} \leq y_{1L}, y_{3L} \leq Q_{ymax}, & l_{ymin} \leq y_{2L}, y_{4L} \leq l_{ymax}; \\ |y_{5R} - y_{7R}| \leq Q_{z0}, & |y_{6R} - y_{8R}| \leq l_{z0} \\ Q_{zmin} \leq y_{5L}, y_{7L} \leq Q_{zmax}, & l_{zmin} \leq y_{6L}, y_{8L} \leq l_{zmax} \end{cases} \quad (24)$$

where f_C denotes the multi-objective optimization function under a given working condition; y_{ij} indicates the design parameters of left-helical and right-helical gear pairs; η_{AVG} is the average transmission efficiency during one mesh cycle; P_0 is the input power of double helical gear trains; l_{ymin} , l_{ymax} , l_{zmin} , l_{zmax} , Q_{ymin} , Q_{ymax} , Q_{zmin} , Q_{zmax} denote the boundary values of each variable; l_{y0} , l_{z0} , Q_{y0} , Q_{z0} represent the constant parameters for teeth profile modification. All the variables to be optimized are illustrated in Fig. 10.

The process of DTEF optimization includes the execution of LTAC program and calculation of DTEF equations, while altering the micro-modification parameters of the meshing tooth surface. The variables to be optimized are associated with the parameters for teeth modification, and the objective function of optimization is associated with the DTEF of double helical gear trains, there are no explicit formulas to describe these relationships. The optimization process based on a modified genetic algorithm and the theory of neural network is elaborated in Fig. 11.^{48,49}

The optimization results of modification parameters are finally obtained with the proposed method and are listed in Tab. 4. To demonstrate the effectiveness of parameter optimization and the advance of the optimal parameter, two other groups of parameters for teeth modifications have been configured, and corresponding calculations have been carried out as comparison, the additional parameters are listed in Tab. 5.

The comparison of DTEF before and after the teeth modification is given in Fig. 12. It is apparent that the fluctuation of DTEF is significantly reduced after the teeth modification; additionally, for the given condition (2000 r/min and 828 N·m), the highest average DTEF (98.39%) is realized by the gears modified with optimal parameters.

As shown in Fig. 13 and Fig. 14, for a more comprehensive analysis of the effects of shape modification on DTEF, the DTEF before and after the teeth modification with different external loads and speeds is calculated and compared.

As shown in Fig. 13, due to a lower actual contact ratio of double-helical gear pair when the external torque is below 630 N·m, the average DTEF of double helical gear trains after 3-D modification is lower than the standard tooth. However, the actual contact ratio of the gear pair with modified tooth increases when the external load increases, which accounts for the advantage of double helical gear in terms of transmission smoothness compared with the standard tooth. Therefore the performance of the double helical gear trains with 3-D modification is higher at this stage. In other words, the practical application will significantly benefit from the optimization process mentioned above and the 3-D modification for the gear teeth, in the case of a fixed external load. Similarly, Fig. 14 shows that the DTEF after the 3-D modification is significantly improved at different input speeds, owing to the decreased dynamic loads.

3. EXPERIMENT AND THEORETICAL VERIFICATION

The findings of this research cannot be practically implemented until the model is validated by experiments on the double helical gear-shaft-bearing system. The closed power flow experiment facilities have been successfully developed in recent years to overcome the drawback of the conventional test bench, such as high-power consumption and low energy efficiency. The closed power flow experiment facilities are distinguished by a closed-loop topology of energy flow. In other words, the energy consumed are partially collected and sent back in order to drive the test system, and in this way the energy conservation and the stability under overload conditions

Figure 9. 3-D modification teeth shape of double helical gear.

Figure 10. The schematic diagram of all variables to be optimized.

Figure 11. The optimization process of DTEF.

are strengthened. The configuration of closed power flow test bench employed in this research is illustrated in Fig. 15.

Torque encoder is used to measure the torques in input and output axes, and the Heidenhain optical encoders are used to measure the rotational speed accurately. The installation positions of torque encoder and the Heidenhain optical encoders are shown in Fig. 16.^{50–56} The modification of the tooth surface is achieved via grinding method, and the gear pairs to be tested

Figure 12. The comparison of DTEF before and after the teeth modification.

are machined by a CNC gear-grinding machine produced by Qinchuan Machine Tool Group Co., Ltd. We took some necessary measures for the gear pair and the rolling bearing system in experiments to ensure that the conditions for the experiments are as close as possible to the simulation environment. The techniques are applied primarily from the following three perspectives: (1) the tooth surface is manufactured with 4th precision grade to reduce the difference between ideal tooth surface and actual tooth surface; (2) a tolerable installation error is ensured during the assembly of test bench; and, (3) the quantity and temperature of lubricating oil is strictly selected for the gear pair and the rolling bearing.

Table 4. The results of parameter optimization.

Parameters/unit	$y_{1L}/\mu\text{m}$	y_{2L}/mm	$y_{3L}/\mu\text{m}$	y_{4L}/mm	$y_{5L}/\mu\text{m}$	y_{6L}/mm	$y_{7L}/\mu\text{m}$	y_{8L}/mm
Value	16	1.6	18	3.2	14	11.6	14	11.2
Parameters/unit	$y_{1R}/\mu\text{m}$	y_{2R}/mm	$y_{3R}/\mu\text{m}$	y_{4R}/mm	$y_{5R}/\mu\text{m}$	y_{6R}/mm	$y_{7R}/\mu\text{m}$	y_{8R}/mm
Value	15	1.8	19	2.5	15	10.6	14	10.2

Table 5. Modification parameters of comparison groups

Parameters/unit	$y_{1L}/\mu\text{m}$	y_{2L}/mm	$y_{3L}/\mu\text{m}$	y_{4L}/mm	$y_{5L}/\mu\text{m}$	y_{6L}/mm	$y_{7L}/\mu\text{m}$	y_{8L}/mm
Modification tooth A	14	1.4	15	3.0	13	10.6	13	10.6
Modification tooth B	17	1.8	19	3.5	16	12.1	16	11.9
Parameters/unit	$y_{1R}/\mu\text{m}$	y_{2R}/mm	$y_{3R}/\mu\text{m}$	y_{4R}/mm	$y_{5R}/\mu\text{m}$	y_{6R}/mm	$y_{7R}/\mu\text{m}$	y_{8R}/mm
Modification tooth A	13	1.6	17	2.3	13	9.6	12	9.2
Modification tooth B	16	1.9	21	2.8	17	11.2	16	11.2

Figure 13. Comparison of DTEF with different loads (2000 r/min). **Figure 14.** Comparison of DTEF with different speeds (828N·m).

The experiment results of DTEF are calculated by Eq. (25).

$$\eta_T(t) = \frac{T_{out}(t) \cdot n_{out}(t)}{T_{in}(t) \cdot n_{in}(t)}; \quad (25)$$

where η_T denotes the experiment results of DTEF; $T_{in}(t)$, $T_{out}(t)$ represent the input torque and output torque of double helical gear trains; n_{in} , n_{out} indicate the input speed and the output speed of the double helical gear trains.

As shown in Fig. 18, the fluctuation tendency of time-varying DTEF measured in the experiment is consistent with the results of theoretical calculation. The average time-varying DTEF before and after teeth modification are 97.48% and 98.06%, respectively. Therefore, the modification of the tooth surface proves efficient in improving DTEF under the given operating condition. In addition, due to the inevitable factors that influence the experiment accuracy, e.g., the manufacture and installation errors, the experimental results are slightly

Figure 15. Test rig for double helical gear trains(1.DC motor; 2.the master gearbox; 3.coupling; 4.low-speed torsion shaft; 5.Simple loader; 6.test box; 7.Heidenhain optical encoders; 8.high-speed torsion shaft; 9.rotating speed and torque sensor) **Figure 16.** The installation positions of torque encoder and the Heidenhain optical encoders. **Figure 17.** The meshing double helical gear pair.

lower than the ideal value.

The experimental data are measured 5 times to show the data range of experiments in Tab. 6 and Tab. 7, and the average values are used as the final results in Fig. 19 and Fig. 20.

The comparisons of DTEF from simulation and experiment with different loads and speeds are shown in Fig. 19 and Fig. 20, the differences between the results of theoretical calculation and experiment for this two conditions are basically the same. The engineering practices indicate the feasibility of this method for calculation of the DTEF.

Table 6. The mean value of each measurement result of DTEF (2000 r/min).

	Number of measurements	Load torque /N·m								
		428	628	828	1028	1228	1428	1628	1828	2028
Transmission efficiency <i>I</i> %(Standard tooth)	1st	97.16	97.25	97.53	97.57	97.77	97.87	98.00	98.03	98.06
	2ed	97.21	97.31	97.58	97.56	97.79	97.93	98.01	98.03	98.08
	3rd	97.15	97.26	97.53	97.54	97.79	97.88	98.03	98.06	98.03
	4th	97.21	97.32	97.52	97.61	97.80	97.90	97.99	98.08	98.10
	5th	96.98	97.07	97.29	97.41	97.56	97.63	97.91	97.85	97.89
	Mean value	97.14	97.24	97.48	97.54	97.74	97.84	97.99	98.01	98.03
Transmission efficiency <i>I</i> %(Modification tooth)	1st	97.00	97.17	98.09	98.35	98.46	98.60	98.68	98.77	98.78
	2ed	97.01	97.21	98.11	98.36	98.47	98.64	98.69	98.80	98.78
	3rd	97.03	97.17	98.11	98.33	98.47	98.56	98.68	98.76	98.79
	4th	97.01	97.14	98.12	98.39	98.50	98.60	98.71	98.86	98.79
	5th	96.95	96.96	97.86	98.12	98.39	98.40	98.49	98.60	98.71
	Mean value	97.00	97.13	98.06	98.31	98.46	98.56	98.65	98.76	98.77

Table 7. The mean value of each measurement result of DTEF (828 N·m).

	Number of measurements	Wheel speed /r/min											
		500	1000	1500	2000	2500	3000	3500	4000	4500	5000	5500	6000
Transmission efficiency / % (Standard tooth)	1st	97.08	97.17	97.43	97.52	97.67	97.82	97.82	97.95	97.46	97.85	97.87	97.75
	2ed	97.14	97.16	97.47	97.50	97.69	97.82	97.84	98.01	97.53	97.83	97.92	97.81
	3rd	97.07	97.15	97.43	97.50	97.65	97.85	97.80	97.91	97.46	97.83	97.88	97.74
	4th	97.13	97.18	97.42	97.50	97.69	97.87	97.85	98.00	97.48	97.90	97.90	97.80
	5th	96.78	97.04	97.20	97.44	97.50	97.70	97.54	97.68	97.23	97.63	97.63	97.59
	Mean value	97.04	97.14	97.39	97.48	97.64	97.81	97.77	97.91	97.43	97.81	97.84	97.74
Transmission efficiency / % (Modification tooth)	1st	97.41	97.69	97.91	98.10	98.31	98.53	98.62	98.52	98.34	98.65	98.62	98.51
	2ed	97.45	97.71	97.92	98.08	98.37	98.53	98.63	98.56	98.35	98.70	98.65	98.47
	3rd	97.42	97.70	97.95	98.09	98.33	98.52	98.61	98.54	98.32	98.62	98.61	98.47
	4th	97.48	97.71	97.96	98.12	98.31	98.53	98.67	98.56	98.36	98.63	98.61	98.47
	5th	97.13	97.45	97.81	97.91	98.22	98.44	98.52	98.36	98.18	98.45	98.46	98.38
	Mean value	97.38	97.65	97.91	98.06	98.31	98.51	98.61	98.51	98.31	98.61	98.59	98.46

Figure 18. Experimental measurement for time-varying DTEF

4. CONCLUSION

A series of feasible methods for DTEF calculation and influence analysis for tooth surface modifications have been developed in this research. Based on the dynamics model and elasto-hydrodynamic lubrication model, a genetic algorithm was applied to the direct 3-D tooth surface modification for double helical gears; the proposed DTEF calculation method incorporating the real-time dynamic loads features high efficiency and accuracy. According to the simulation and experiments, the following conclusions concerning the DTEF can be drawn from this research:

(1) Compared with the conventional methods for calculation of static and quasi-dynamic transmission efficiencies, the new method for calculation of DTEF given due consideration of real-time dynamic loads performed better in terms of calculation speed and precision. The simulation showed that the fluctuation of DTEF was more obvious when the vibration force

Figure 19. Comparison of theoretical and experimental results of DTEF with different torques (2000 r/min).

of tooth surface was considered, and the average DTEF was lower than STEF. It is worth noting that the DTEF at the resonant speed is significantly lower than STEF due to the rapid increase of dynamic load.

(2) The optimization of DTEF via 3-D modification technology was systematically elaborated in this paper, and according to the analysis of optimal solution data, the proposed modification model proved feasible and effective. Due to the decrease of dynamic loads, the DTEF after the 3-D modification was significantly higher than standard tooth at different input speeds. However, the effect of tooth surface modification was conditional, for instance, when the external torque was below 630 N·m, the modified tooth was inferior to the standard tooth in terms of DTEF.

(3) Experimental data of DTEF were obtained by the torque

Figure 20. Comparison of theoretical and experimental results of DTEF with different speeds (828 N·m).

encoder (for torque measurements) and the Heidenhain optical encoders (for rotational speed measurements). The key characteristics of the DTEF in theory and experiment were highly consistent. The engineering practices support the feasibility of this method for the calculation of the DTEF.

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