Structural Damage Identification Based on Principal Curvatures of Mode Shape

Rahim Gorgin

Faculty of Civil Engineering and Mechanics, Jiangsu University, Zhenjiang 212013, Jiangsu, China

Ziping Wang

Faculty of Civil Engineering and Mechanics, Jiangsu University, Zhenjiang 212013, Jiangsu, China

(Received 16 May 2020; accepted 1 November 2020)

This paper presents a procedure for damage identification and characterization on plates, based on the principal curvatures of their first mode shape. Each mode shape represents the displacement of the structure at its corresponding natural frequency. Since, variations in the geometry due to cracks or material property degradation, make changes in the mode shapes of the structure, such changes can be used for damage identification methods. The presented procedure only requires the first mode shape of the intact and damaged structure. It is shown that the principal curvatures of the surface defined by the first mode shape of the structure, are sensitive to damage and the maximum principal curvature can be used to highlight damages on the structure. The performance of the developed method is firstly evaluated using finite element analysis. To this aim, the procedure is applied to highlight both single and multi-damages in different locations of the plate with different boundary conditions. It is shown that the location of the maximum curvature variation coincides well with the location of damages and the amount of the maximum curvature change can be used as a parameter to describe damage severity. The accuracy of the proposed method is also experimentally verified by test on an aluminum plate and it is demonstrated that the proposed method remains effective even in experimental condition when only a limited number of measurements are available.

1. INTRODUCTION

Structural health monitoring techniques have found important roles throughout the aerospace, mechanical, and civil engineering communities. Among various techniques for SHM systems, the vibration-based damage detection method has been widely used as an effective method.^{1–6} This method relies on the fact that changes in structural properties, such as damage, affect the overall dynamic properties of the structure.⁷

Various modal parameters, including damping ratio, natural frequencies, mode-shapes and their derivatives as well as frequency response functions (FRFs) have been used by researchers to develop vibration based damage detection techniques. Since damping ratio has a complicated nature, it was rarely used in damage detection techniques.⁸ Natural frequencies can be easily determined. Therefore, they are used widely to assess the overall structural integrity.^{9–12} However, natural frequencies are less sensitive to concentrated damages.^{13–17} In comparison, using mode shapes and their derivatives for damage detection has many advantages. First, mode shapes contain local information, which makes them more sensitive to local damages and enables them to be used to detect multiple damages.^{18–21} Second, the mode shapes are less sensitive to environmental effects.^{22–24}

Pandey et al.²⁵ showed that the curvature of the mode shapes is changed considerably due to existing damage in structures. Ratcliffe²⁶ used finite difference approximation of Laplacian operator on mode shape data to develop a vibration based damage detection technique. Abdel Wahab and De Roeck²⁷ examined mode shapes of damaged beams and showed that some mode shapes did not demonstrate the location of dam-

ages. Hence, they introduced a curvature damage factor based on a combination of curvature variation for different mode shapes. Ismail et al.²⁸ introduced a damage index based on fourth order derivative of mode shapes. In their method, they used a curve-fitting technique employing Chebyshev series rationals to eliminate random errors from experimentally measured mode shapes. Roy and Ray-Chaudhuri²⁹ have used first-order perturbation approach to derive a closed form solution for change in mode shape due to damage. A damage indicator is introduced by Yazdanpanah et al.,³⁰ based on the mode shape, mode shape slope and mode shape curvature of a beam before and after damage. Hansen et al.³¹ developed a method based on the sensitivity of mass-normalized experimental determined mode shapes. Talaei et al.³² developed a method based on structural mode-shapes and natural frequencies and by means of Twin Gaussian Process.

Previous analytical methods based on the mode shape of structure, were only able to localize the location of damages in the structure. However, in structural health monitoring systems, predicting the size and severity of damages is also a matter of importance.^{33,34}

In this study, a procedure is developed to detect the size and location of structural damages based on the principal curvatures of the first mode shape. A cubic spline surface is fitted to the first mode shape of the intact and damaged structures. The curvatures of the spline surface, which are defined in an orthogonal coordinate system, are then used to obtain the principal curvatures. It is shown that variation in the first mode shape and subsequently the principal curvatures of the spline surface is very sensitive to damage and the maximum principal curvature can be used to predict the size and location of dam-

Figure 1. The first mode shape of intact and damaged cantilever beam.

ages. The efficiency of the proposed method is firstly assessed using finite element analysis. To achieve this, the procedure is used to highlight single and multi-damages located in different locations of plate with different boundary conditions. The damages are introduced by reducing elastic modulus in specific regions. It is shown that the stiffness reduction causes a sharp deviation in the maximum curvature of the surface. Hence, the location of the maximum curvature variation, coincides well with the location of damages and the amount of the maximum curvature change can be used as a parameter to describe damage severity. Finally, the efficiency of the presented method is experimentally evaluated by test on an aluminum plate.

2. DAMAGE DETECTION TECHNIQUE

Damages referred to the local changes in the structural geometry and/or reduction of material properties in some regions of the structure. Damages cause a local deviation from smooth deformed shape of the structure.³⁵ When the structure is subjected to small amplitude vibration, the time dependent deformed geometry of the structure can be described by linear combination of mode shapes. It was shown that the first mode shape, is a dominate mode for most linear vibration of structures, because such mode shape required lowest energy to be activated. Fig. 1 schematically illustrates the first mode shape of the intact and damaged cantilever beam.

As is shown in Fig. 1, damage presence causes local deviation in the first mode shape of the structure, at damaged region. Moreover, there is an abrupt change in the curvature of mode shape in the exact location of damage. Hence, the difference in the first mode shape curvature of the intact and damaged structure can be used as an appropriate index to localize damages.

This procedure was used in this study to highlight damages in plates. It should be noted that for such geometries, the curvature of mode shapes is dependent on the direction at a specific point. Hence, in order to evaluate the curvature changes, the principal curvatures were used in this study. Moreover, the amount of stiffness reduction has an effective role in the curvature variations and the absolute value of the principal curvature changes is used as an index to determine the severity of damages.

3. PRINCIPAL CURVATURES OF MODE SHAPES

Consider a flat plate parallel to the $x - y$ plane. The deformed geometry of its middle surface can be described by mode shape. A curve is defined by a hypothetical plane normal to initial geometry, which intersecting the deformed geometry of middle surface. The intersection of the hypothetical plane with $x - y$ plane is a line which is described by vector $\mathbf{t}$. The

curvature of the curve for small deformation of the middle surface is expressed by:

$$\frac{1}{\rho_t} = -w_{,tt}; \quad (1)$$

where w is the transverse displacement component along z -axis, ρ_t is the radius of curvature and $w_{,tt}$ is the second order derivative along vector $\mathbf{t}$. For a given point, the curvatures in planes parallel to $x - z$ plane ($1/\rho_x$) and $y - z$ plane ($1/\rho_y$) are respectively $-w_{,xx}$ and $-w_{,yy}$. The twist of middle surface is given by:

$$\frac{1}{\rho_{xy}} = -w_{,xy}. \quad (2)$$

The curvature along vector $\mathbf{t}$ can be determined using $1/\rho_x$, $1/\rho_y$ and $1/\rho_{xy}$ as follows:

$$\frac{1}{\rho_t} = \frac{1}{\rho_x} \cos^2 \alpha - \frac{1}{\rho_{xy}} \sin 2\alpha + \frac{1}{\rho_y} \sin^2 \alpha; \quad (3)$$

where α is the angle between vector $\mathbf{t}$ and x -axis. The surface curvature is continuously varying with angle α . The angles in which the curvature has the maximum or minimum values, are the principal directions and the maximum and minimum curvatures are called the principal curvatures. The principal directions are determined by taking derivative of Eq. (3) with respect to α and equating it to zero, namely:

$$\tan 2\alpha = -\frac{2}{\frac{1}{\rho_x} - \frac{1}{\rho_y}} \frac{\rho_{xy}}{\rho_{xy}}. \quad (4)$$

As is expressed by Eq. (4), the maximum and minimum curvature directions differ by 90° . Substituting the angles determined by Eq. (4) in Eq. (3) yields the principal curvatures at a given point of plate whose deformation is described by a mode shape.

4. COMPUTATIONAL METHOD

The first mode shapes of the intact and damaged structures were first determined using finite element analysis. A cubic spline surface was then fitted to the nodal displacements corresponding to the first mode shapes of the intact and damaged structures. Since the second order derivatives were required to calculate the surface curvatures, the second order continuity was considered in the spline surface definition. As is described in the previous section, the principal directions and curvatures at a given point were independent on the second order derivatives of $w_{,xx}$, $w_{,yy}$ and $w_{,xy}$. Such derivatives are defined based on a global coordinate system assign to the initial geometry of the structure.

The derivative of the spline equation was analytically taken and its values were obtained using nodal displacement. Using Eq. (4), two orthogonal principal directions were obtained for each point of structure. The maximum and minimum curvatures were determined using Eq. (3). Then, the principal directions and second order derivatives $w_{,xx}$, $w_{,yy}$ and $w_{,xy}$ were determined. Finally, the absolute value of the difference between maximum curvatures of the intact and damaged structures was computed for each point. The points with the maximum curvature difference highlight the most probable location of damages.

Figure 2. The geometry of the simulated intact plate Φ .

5. NUMERICAL STUDY

In order to evaluate the performance of the presented damage identification procedure, finite element analysis was firstly used. ABAQUS CAE software was utilized to model and analyze the dynamic response of plates. Fig. 2 illustrates the geometry of the simulated intact plate with 1 mm thickness and length of $2a = 2b = 100$ mm.

Steel material was assigned to the plate, which the density, elastic modulus and Poisson's ratio were respectively, 7800 kg/m^3 , $207 \times 10^9 \text{ Pa}$ and 0.33. The dynamic response of the plate was analyzed using ABAQUS/Linear perturbation/Frequency analysis. Four-node shell elements were used to mesh the plate. The sides of the plate were divided into 100 segments, so that 10,000 elements with 10,201 nodes were considered to mesh the plate. Each node has six degrees of freedom including three displacement components (u , v and w along x , y and z axes, respectively), two rotation angles (ϕ_1 and $-\phi_2$ are rotation angles about x and y axis, respectively), and one twist angle (ϕ_3 about z -axis). The boundaries of the intact plate were clamped, namely,

$$u = v = w = \phi_1 = \phi_2 = \phi_3 = 0;$$

For points located on boundaries. (5)

In order to simulate a single crack in the plate, a rectangular region was considered with low density and elastic modulus equal to $1 \times 10^{-7} \text{ kg/m}^3$ and 10 Pa, respectively. As is shown in Fig. 3, the size of the damaged rectangular region is 20 mm $\times$ 1 mm.

Using ABAQUS/Linear perturbation/Frequency analysis, the first mode shape of the intact and damaged plates, were determined. Figs. 4a and 4b show the orthonormal first mode shape of the intact and damaged plates.

As can be seen in Figs. 4a and 4b, although there were some variations in the geometry of the first mode shape of the damaged plate, in comparison with to the first mode shape of the intact plate, the damaged region was hard to detect based on the values of mode shapes.

Figure 3. A plate with a rectangular damage (dimensions in mm).

Based on the procedure described in the previous section, the maximum curvature of the first mode shape of the intact and damaged plates were determined and are shown in Figs. 4c and 4d, respectively. As can be seen in Fig. 4d, there was an abrupt change in the maximum curvature value, exactly at the location of the simulated damage. In order to highlight the damaged region more clearly, the absolute value of the difference between the maximum curvatures of the intact and damaged plates at each node was calculated and is shown in Fig. 5.

As is shown in Fig. 5, the maximum curvature difference has a sudden change in the location of the simulated damage. This shows the efficiency of the presented procedure in highlighting damage in a plate.

6. RESULTS AND DISCUSSION

In this section, firstly, the effect of various boundary conditions on the efficiency of the presented damage identification procedure is checked. Then, the efficiency of the presented procedure in highlighting multi-damages at different locations of the plate and with different severities, is evaluated. Finally, the performance of the presented procedure is assessed when only a limited number of measurements are available.

6.1. Boundary Condition Effects

Different boundary conditions can be used to evaluate the first mode shape of the structure. As demonstrated in previous section, the location of damage within the plate can accurately be determined when considering clamped boundary conditions. In some experimental setups, the suspended structure is excited by a hammer in order to measure the modal parameters. Such experimental approach is called free-free conditions and the edges of the plate are free and no degree of freedom is restricted in the vibration analysis. As another boundary conditions, simply supported plate is considered, in which the

Figure 4. First mode shape of a) intact and b) damaged plates; maximum curvatures of c) intact and d) damaged plates.

displacement of the plate edges is described by:

$$\begin{aligned} v(x, \pm a) = w(x, \pm a) = \varphi_2(x, \pm a) = \\ u(y, \pm b) = w(y, \pm b) = \varphi_1(y, \pm b) = 0. \end{aligned} \quad (6)$$

For the plate shown in Fig. 3, the first mode shape of the intact and damaged plate with free and also simply supported boundary conditions are determined and their maximum curvatures are obtained. For both boundary conditions the difference

Figure 5. The difference between the maximum curvature of the first mode shape of the intact and single-damage plates by considering clamped boundary conditions.

between the maximum curvatures are then calculated and are shown in Fig. 6.

As can be seen in Fig. 6, the maximum curvature difference for both boundary conditions has a sudden change in the location of the simulated damage. Nevertheless, by comparing Figs. 5 and 6, it can be concluded that the clamped plate has the largest change in the maximum curvature at the location of damage. Therefore, in detecting the location of inner damages, the presented procedure gives the best results for the plate with clamped edges.

6.2. Effect of Damage Location

In the previous section, the presented procedure has been successfully used to detect a single defect in the middle of plate. In this section, the performance of the proposed technique in highlighting a damage on the edge of the plate with different boundary conditions is evaluated. Fig. 7 illustrates the geometry of a damage located on the edge of a plate.

The first mode shape and consequently the maximum curvature of the intact and damaged plates with three different boundary conditions including clamped, free and simply supported, are determined and then the differences between their maximum curvatures were calculated and are shown in Fig. 8.

As can be seen in Fig. 8, there is a considerable difference in the maximum curvature at the location of damage in the plate with different boundary conditions. The best result is obtained for the plate with free edges. This is because the damaged region is free to vibrate. Therefore, although it is possible to some extent to highlight the location of damage at the edge of a plate with simply supported and clamped edges, but the best result can be obtained for a plate with free edges.

6.3. Detection of Several Damages

To evaluate the effectiveness of the presented method in localizing the location of multi-damages in the plate, two damages were simulated in the plate as is shown in Fig. 9.

The mode shape and consequently, the maximum curvature of the intact and damaged plates with free and also clamped boundary conditions were determined. Fig. 10 demonstrates the maximum curvature differences for both boundary conditions.

As is shown in Fig. 10, for both boundary conditions, there is a sudden change in the maximum curvature difference in

Figure 6. The difference between the maximum curvature of the first mode shape of the intact and single-damaged plates with: (a) free edges and (b) simply supported edges.

Figure 7. A plate with single damage located on the edge (dimensions in mm).

the damaged regions. Nevertheless, for each boundary condition, only one of the damages can be clearly identified. The edge damage is clearly detected in the plate with free edges, while the interior damage is clearly monitored in the plate with clamped boundaries.

The presented method was once again applied to a plate with four defects located on the edges of the plate with free edges (see Fig. 11).

The first mode shape of the intact and damaged plates, were determined and then the difference between their maximum curvature was calculated and is demonstrated in Fig. 12.

As can be seen in Fig. 12, the location of all damaged regions are clearly detected due to the abrupt change in maximum curvature difference. The proposed procedure was also applied to a clamped plate with four interior damages (see Fig. 13).

For the intact and damaged plate, the first mode shape was determined and subsequently, the difference between their maximum curvature was calculated and is shown in Fig. 14.

As is shown in Fig. 14, abrupt changes in the maximum curvature difference coincide well with the damaged regions. From the results of this section it can be concluded that the presented procedure is able to highlight the location of multi damages in the plate.

6.4. Critical Damage Identification

In the previous sections, it was shown that the presented procedure is able to highlight the location of multi damages with the same severity. It is also important to evaluate the performance of the presented method in highlighting damages with different severities. To this aim, two inner damages with the same size but different severities were introduced in a clamped plate as is shown in Fig. 15. In order to simulate damages with different severities, different material properties were assigned to the damaged regions. The material properties of the damages are listed in Table 1.

Figure 8. The difference between the maximum curvature of the first mode shape of the intact and single-edge damaged plates with: (a) free edges, (b) simply supported edges, and (c) clamped edges.

Table 1. Material properties of damaged regions.

Region	Density (kg/m^3)	Elastic Modulus (Pa)	Poisson Ratio
1	7800	207×10^7	0.33
2	1×10^{-7}	10	0.33

The first mode shape of the intact and damaged plates, were determined and then the difference between their maximum curvature was calculated and is demonstrated in Fig. 16.

As can be seen in Fig. 16, the maximum curvature change is different in the location of two simulated damages and there is more maximum curvature variation in the weaker damaged region. Therefore, in addition to highlighting the location of damages, the presented procedure is able to determine the severity of damages.

Figure 9. A plate with two damaged regions located on the edge and interior of the plate (dimensions in mm).

6.5. Diagonal Crack Identification

In the previous sections, all the simulated damages were cracks parallel to the edges of the plate. In this section, the efficiency of the proposed method in highlighting diagonal cracks is investigated. To this aim, a $20 \text{ mm} \times 1 \text{ mm}$ diagonal crack was simulated in the plate with clamped boundary conditions (see Fig. 17).

The first mode shape of the intact and damaged plates was determined and the difference between their maximum curvatures was calculated and is shown in Fig. 18.

As can be seen in Fig. 18, the abrupt change in the maximum curvature difference coincides well with the location of simulated diagonal crack. Hence, it can be concluded that the presented procedure is able to highlight damages regardless of their direction.

6.6. Applicability of the Proposed Method When a Limited Number of Measurements Are Available

In order to evaluate the performance of the proposed method when only a limited number of measurements are available, the presented procedure was again applied to the plate shown in Fig. 3, but this time, only the measurements of 121 nodes were used. In fact, the sides of the plate were divided into 10 segments, so that 100 elements with 121 nodes were considered to mesh the plate. The difference between the maximum curvature of the first mode shape of the intact and damaged plates is determined and is shown in Fig. 19.

As can be seen in Fig. 19, although the number of measurements are limited to 121 nodes, the proposed method is still able to highlight the location of the damage on the plate.

In order to evaluate the efficiency of the presented procedure in highlighting multi damages when only a limited number of measurements are available, two asymmetric damages with the

Figure 10. The difference between the maximum curvature of the first mode shape of the intact and two-damaged plates with: (a) free edges and (b) clamped edges.

Figure 11. A plate with four damaged regions located on the edges (dimensions in mm).

Figure 12. The difference between the maximum curvature of the first mode shape of the intact and multi-damaged plate with free edges.

same size (20×1 mm) and severity were introduced in a plate with clamped boundaries (see Fig. 20).

Once again, by only using 121 measurements, the difference between the maximum curvature of the first mode shape of the intact and damaged plates was determined and is shown in Fig. 21.

As can be seen in Fig. 21, the nodes with the maximum curvature difference coincide well with the location of damages. Therefore, in spite of using a limited number of measurements, the proposed procedure is still able to highlight the location of damages in the plate. However, by comparing Figs. 5 and 19, it can be concluded that when low dense measurements are used, the presented method fails to determine the exact size of damage.

7. EXPERIMENTAL VERIFICATION

The accuracy of the proposed damage identification method was experimentally assessed by a test on an aluminum plate of dimensions $60 \text{ cm} \times 60 \text{ cm}$ with 1 mm thickness. For the measurement of the responses the plate was divided by a grid

Figure 13. A plate with four interior damages (dimensions in mm).

Figure 14. The difference between the maximum curvature of the first mode shape of the intact and multi-damaged plate with clamped edges.

Figure 15. A plate with two different inner damages (dimensions in mm).

Figure 16. The difference between the maximum curvature of the first mode shape of the intact and damaged plate with two inner damages with different severities.

Figure 17. A plate with a diagonal crack (dimensions in mm).

Figure 18. The difference between the maximum curvature of the first mode shape of the intact and damaged plate with a diagonal crack.

Figure 19. The difference between the maximum curvature of the first mode shape of the intact and single-damage plates (using 121 measurements).

Figure 20. A plate with two asymmetric damages (dimensions in mm).

Figure 21. The difference between the maximum curvature of the first mode shape of the intact and multi-damage plates (using 121 measurements).

Figure 22. An aluminum plate with a hole (dimensions in cm).

Figure 23. Experimental setup.

of 10 cm $\times$ 10 cm corresponding to 5 $\times$ 5 nodes. The plate was tested before and after introducing damage. The damage was introduced by manufacturing a hole at the location shown in Fig. 22 with diameter of 1 cm.

The first mode shape of both the intact and damaged plate was determined using a PSV-500 Scanning Vibrometer (see Fig. 23).

Consequently, the difference between the maximum curvature of the first mode shape of the intact and damaged plates was determined and is shown in Fig. 24.

As can be seen in Fig. 24, the abrupt increase in the maximum curvature difference value coincides well with the location of the hole. This shows that the presented technique is able to localize damages in experimental condition. However, since the number of measurements are limited to 25 nodes, only the location of the hole can be highlighted and the presented method fails to determine the size of damage.

8. CONCLUSIONS

In this study, a procedure was developed for damage identification and characterization in plates based on the principal curvatures of their first mode shape. Since, the presented method only requires the first mode shape of the intact and damaged structures, it reduces the computational time compared to previous damaged identification procedures considering multiple mode shapes. Firstly, the efficiency of the presented procedure

Figure 24. The difference between the maximum curvature of the first mode shape of the intact and damaged aluminum plates.

was assessed using finite element analysis. It was shown that the location of damages coincides well with the regions having abrupt change in the maximum curvature. The performance of the presented method in highlighting the location of multi damages located in different locations of plate, was also demonstrated. It was shown that the curvature change is sensitive to the amount of stiffness reduction. Therefore, the presented method is able to determine the severity of damages as well. Moreover, the efficiency of the presented procedure was evaluated when a limited number of measurements is available. It was shown that the proposed method remains efficient in localizing damages even when a limited number of measurements is available. Finally, the accuracy of the proposed method was experimentally verified by test on an aluminum plate and it was shown that the proposed method is able to highlight damages in experimental condition. The main shortcoming of the developed method is that in experimental condition when only a limited number of measurements is available, it can only highlight the location of damages and it is not able to determine the exact size of damages. Furthermore, using the presented method, the interior damages can be clearly highlighted when the boundaries of the plate are clamped, while the boundary damages can be clearly monitored when the plate has free edges. Research is being conducted to overcome these shortcomings.

ACKNOWLEDGEMENTS

This work was financially supported by the National Natural Science Foundation of China (11872191 and 11520101001)

REFERENCES

- ¹ Nobahari, M., and Seyedpoor, S. M. An efficient method for structural damage localization based on the concepts of flexibility matrix and strain energy of a structure, *Struct. Eng. Mech.*, **46**, 231–244, (2013).
- ² Ubertini, F., Comanducci, G., and Cavalagli, N. Vibration-based structural health monitoring of a historic bell-tower using output-only measurements and multivariate statistical analysis, *Struct. Health Monit.*, **15**, 438–457, (2016). <https://dx.doi.org/10.1177/1475921716643948>
- ³ Weijtjens, W., Verbelen, T., Capello, E., and Devriendt C. Vibration based structural health monitoring of the substructures of five offshore wind turbines, *Proced. Eng.*, **199**, 2294–2299, (2017). <https://dx.doi.org/10.1016/j.proeng.2017.09.187>
- ⁴ Khodabandehlou, H., Pekcan, G., and Fadali M. S. Vibration-based structural condition assessment using convolution neural networks, *Struct. Control Health Monit.*, **26** (2), e2308, (2019). <https://dx.doi.org/10.1002/stc.2308>
- ⁵ Jayasundara, N., Thambiratnam, D., Chan, T., and Nguyen, A. Vibration-based dual-criteria approach for damage detection in arch bridges, *Struct. Health Monit.*, **18**, 2004–2019, (2019). <https://dx.doi.org/10.1177/1475921718810011>
- ⁶ Spanos, N. A., Sakellariou, J. S., and Fassois, S. D. Vibration-response-only statistical time series structural health monitoring methods: A comprehensive assessment via a scale jacket structure, *Struct. Health Monit.*, **19** (3), 736–750, (2019). <https://dx.doi.org/10.1177/1475921719862487>
- ⁷ Fan, W., and Qiao, P. Vibration-based damage identification methods: a review and comparative study, *Struct. Health Monit.*, **10**, 83–111, (2011). <https://dx.doi.org/10.1177/1475921710365419>
- ⁸ Cao, M., Sha, G., Gao, Y., and Ostachowicz, W. Structural damage identification using damping: a compendium of uses and features, *Smart Mater. Struct.*, **26**, 043001, (2017).
- ⁹ Rubio, L., Fernández-Sáez, J., and Morassi, A. Identification of two cracks in a rod by minimal resonant and antiresonant frequency data, *Mech. Syst. Signal Process.*, **60–61**, 1–13, (2015). <https://dx.doi.org/10.1016/j.ymssp.2015.01.025>
- ¹⁰ Lee, J-W. Crack identification method for tapered cantilever pipe-type beam using natural frequencies, *Int. J. Steel Struct.*, **16**, 467–476, (2016). <https://dx.doi.org/10.1007/s13296-016-6017-x>
- ¹¹ Yang, C., and Oyadiji S. O. Damage detection using modal frequency curve and squared residual wavelet coefficients-based damage indicator, *Mech. Syst. Signal Process.*, **83**, 385–405, (2017). <https://dx.doi.org/10.1016/j.ymssp.2016.06.021>
- ¹² Sha, G., Radzieński, M., Cao, M., and Ostachowicz, W. A novel method for single and multiple damage detection in beams using relative natural frequency changes, *Mech. Syst. Signal Process.*, **132**, 335–352, (2019). <https://dx.doi.org/10.1016/j.ymssp.2019.06.027>
- ¹³ Doebling, S. W., Farrar, C. R., and Prime, M. B. A summary review of vibration-based damage identification methods, *The Shock and Vibration Digest*, **30**, 91–105, (1998).

- ¹⁴ Vestroni, F., and Capecchi, D. Damage detection in beam structures based on frequency measurements, *Journal of Engineering Mechanics*, **126**, 761–768, (2000). [https://dx.doi.org/10.1061/\(ASCE\)0733-9399\(2000\)126:7\(761\)](https://dx.doi.org/10.1061/(ASCE)0733-9399(2000)126:7(761))
- ¹⁵ Cruz, P. J. S., and Salgado, R. Performance of vibration-based damage detection methods in bridges, *Comput.-Aided Civ. Infrastruct. Eng.*, **24**, 62–79, (2009). <https://dx.doi.org/10.1111/j.1467-8667.2008.00546.x>
- ¹⁶ Pau, A., Greco, A., and Vestroni, F. Numerical and experimental detection of concentrated damage in a parabolic arch by measured frequency variations, *Journal of Vibration and Control*, **17**, 605–614, (2010). <https://dx.doi.org/10.1177/1077546310362861>
- ¹⁷ Antonacci, E., De Stefano, A., Gattulli, V., Lepidi, M., and Matta, E. Comparative study of vibration-based parametric identification techniques for a three dimensional frame structure, *Struct. Control Health Monit.*, **19**, 579–608, (2012). <https://dx.doi.org/10.1002/stc.449>
- ¹⁸ Yuen, M. M. F. A numerical study of the eigenparameters of a damaged cantilever, *J. Sound Vib.*, **103**, 301–310, (1985). [https://dx.doi.org/10.1016/0022-460X\(85\)90423-7](https://dx.doi.org/10.1016/0022-460X(85)90423-7)
- ¹⁹ Ma, X., Azeez, M. F. A., Vakakis, A. F., and Azeez A. M. F. Non-linear normal modes and non-parametric system identification of non-linear oscillators, *Mech. Syst. Signal Process.*, **14**, 37–48, (2000). <https://dx.doi.org/10.1006/mssp.1999.1267>
- ²⁰ Caddemi, S., and Morassi, A. Multi-cracked Euler–Bernoulli beams: mathematical modeling and exact solutions, *International Journal of Solids and Structures*, **50**, 944–956, (2013). <https://dx.doi.org/10.1016/j.ijsolstr.2012.11.018>
- ²¹ Kerschen, G., Worden, K., Vakakis, A. F., and Golinval J.-C. Past, present and future of non-linear system identification in structural dynamics, *Mech. Syst. Signal Process.*, **20**, 505–592, (2006). <https://dx.doi.org/10.1016/j.ymssp.2005.04.008>
- ²² Deraemaeker, A., Reynders, E., and De Roeck, G. Vibration based SHM: comparison of the performance of modal features vs features extracted from spatial filters under changing environmental conditions, *Proceedings of the ISMA*, 849–863, (2006).
- ²³ Dilella, M., Morassi, A., and Perin, M. Dynamic identification of a reinforced concrete damaged bridge, *Mech. Syst. Signal Process.*, **25**, 2990–3009, (2011). <https://dx.doi.org/10.1016/j.ymssp.2011.05.016>
- ²⁴ Gorgin, R., Luo, Y., and Wu, Z. Environmental and operational conditions effects on Lamb wave based structural health monitoring systems: A review, *Ultrasonics*, **105**, 106114, (2020). <https://dx.doi.org/10.1016/j.ultras.2020.106114>
- ²⁵ Pandey, A. K., Biswas, M., and Samman, M. M. Damage detection from changes in curvature mode shapes, *J. Sound Vib.*, **145**, 321–332, (1991). [https://dx.doi.org/10.1016/0022-460X\(91\)90595-B](https://dx.doi.org/10.1016/0022-460X(91)90595-B)
- ²⁶ Ratcliffe, C. P. Damage detection using a modified laplacian operator on mode shape data, *J. Sound Vib.*, **204**, 505–517, (1997). <https://dx.doi.org/10.1006/jsvi.1997.0961>
- ²⁷ Abdel Wahab, M. M., and De Roeck, G. Damage detection in bridges using modal curvatures: application to a real damage scenario, *J. Sound Vib.*, **226**, 217–235, (1999). <https://dx.doi.org/10.1006/jsvi.1999.2295>
- ²⁸ Ismail, Z., Abdul Razak, H., and Abdul Rahman, A. G. Determination of damage location in RC beams using mode shape derivatives, *J. Eng. Struct.*, **28**, 1566–1573, (2006). <https://dx.doi.org/10.1016/j.engstruct.2006.02.010>
- ²⁹ Roy, K., and Ray-Chaudhuri, S. Fundamental mode shape and its derivatives in structural damage localization, *J. Sound Vib.*, **332**, 5584–5593, (2013). <https://dx.doi.org/10.1016/j.jsv.2013.05.003>
- ³⁰ Yazdanpanah, O., Seyedpoor, S. M., and Bengar, H. A. A new damage detection indicator for beams based on mode shape data. *Struct. Eng. Mech.*, **53**, 725–744, (2015). <https://dx.doi.org/10.12989/sem.2015.53.4.725>
- ³¹ Hansen, J. B., Brincker, R., López-Aenlle, M., Overgaard, C. F., and Kloborg, K. A new scenario-based approach to damage detection using operational modal parameter estimates, *Mech. Syst. Signal Process.*, **94**, 359–373, (2017). <https://dx.doi.org/10.1016/j.ymssp.2017.03.007>
- ³² Talaei, S., Beitollahi, A., Moshirabadi, S., and Fallahian, M. Vibration-based structural damage detection using Twin Gaussian Process (TGP), *Structures*, **16**, 10–19, (2018). <https://dx.doi.org/10.1016/j.istruc.2018.08.006>
- ³³ Gorgin, R., Wu, Z., Gao, D., and Wang, Y. Damage size characterization algorithm for active structural health monitoring using the A0 mode of Lamb wave, *Smart Mater. Struct.*, **23**, 035015, (2014).
- ³⁴ Gorgin, R. Damage identification technique based on mode shape analysis of beam structures, *Structures*, **27**, 2300–2308, (2020). <https://dx.doi.org/10.1016/j.istruc.2020.08.034>
- ³⁵ Limongelli, M. P. Health Monitoring of Partially Instrumented Structures. *Proceedings of the Twelfth International Conference on Civil, Structural and Environmental Engineering Computing*, Civil-Comp Press, Stirlingshire, Scotland, 157, (2009).