Component Mode Synthesis Method Applied to Acoustic Resonators in Ducts for Structural Vibration Analysis

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Tubular structural systems appear in many industrial applications, such as heating, ventilation, and air conditioning systems, which are responsible for making any enclosed environment remain within a temperature, humidity, and cleanliness range. This kind of system has its applications in the internal environmental comfort of industrial spaces, buildings, and vehicles. Several of these spaces have industrial processes that generate high sound frequencies and mechanical vibrations that need to be adequately controlled to meet both environmental and health norms. With the intention to analyze the structural vibration of tubular systems, the modal analysis technique is a classical methodology for the extraction of natural frequencies and vibration modes. Among the various techniques of modal analysis, numerical methodologies such as the finite element method, and also analytical methodologies such as the Component Mode Synthesis (CMS) can be found. CMS is one of the leading modeling tools for complex systems that are applied to large systems. The method uses a modal superset and consists of separately modeling individual components of a structure and coupling them into a single system. The objective of this work is to demonstrate the application of the CMS technique through the estimation of natural frequencies and vibration modes in a simplified tubular structural system formed by two substructures, using MATLAB® and ANSYS®. The validation of the results was done through numerical modeling using the finite element method using and ANSYS® software. The results obtained were satisfactory, thus demonstrating the feasibility of applying the CMS technique to an analysis of structural vibration in tubular structural systems.

1. INTRODUCTION

Currently, industry has a variety of challenges, one of which is how to exclude or reduce vibration to an acceptable level in mechanical systems, which can also produce noise affecting the comfortable environment.¹ Noise can affect the environment in diverse ways, from being a general disturbance, to reducing personal comfort (such as lack of privacy or difficulty in talking), and even leading to serious health problems such as heart failure, hearing difficulties, stress, and others disorders.^{2,3} Some tubular systems can be found in common spaces, where they perform their function such as heating, ventilating and air conditioning. Controlling and maintaining conditions of desirable temperature, humidity, and cleanliness for people or organisms within a closed environment is also an important function of tubular systems.⁴

The Tubular system design is a practice of mechanical engineering, based on the principles of thermodynamics, fluid mechanics, and heat transfer. This type of system applies to a wide variety of cases and situations with the objective of air-flow transport for temperature control.⁵

Duct systems can present different sizes, cross-sections, materials, etc. Dynamic analysis of convoluted industrial structures by the finite element method leads to many degrees of freedom (DOF). As the order of the model is reduced, the sys-

tem can be reduced by component mode synthesis (CMS).⁶ CMS is applied to perform an analysis of the dynamics of structures through a separation of the structure into substructures. These substructures are separately reduced and then connected. To achieve reasonably accurate results, the master's DOFs must be chosen carefully, or an imprecise reduced model may cause an inaccurate result.⁷

Substructuring routines diverge from the chosen Ritz representation basis for substructure motion which includes: the vibration normal modes, the rigid body modes, the static modes, the attachment modes, etc. They also diverge in terms of the assembling procedures, by elimination or by transformation.⁸ Free interface⁹ and hybrid interface CMS¹⁰ were proposed with various boundary conditions at the interface and many insights, variants and improvements have been proposed.^{11,12} Recently, the eigenvalue error estimating method and the iterations method for controlling accuracy of frequency and mode shape were proposed to ensure the quantitative accuracy of CMS.^{13,14}

The modal synthesis by residual flexibility (MSRF) is one of the CMS techniques¹⁵ that adopts the modal superset of residual flexibility. It has the advantage of being a blended routine that can be used in both numerical and experimental methods. To use this routine, it is fundamental that the modal base is

mass normalized. Although, when the problem is entirely experimental, the application of this routine becomes problematic because the physical matrices of the structure problem are unknown.

As is well known, the beams are used in many structural and mechanical applications such as in steel construction and machinery industries. It is the norm that structures must work properly during their lifetime without presenting a risk to anyone or the structural integrity. Nevertheless, in any circumstances, damages can appear as a failure or breach of the norms.¹⁶ Cantilever beams appear in many different mechanical and structural engineering examples around us such as buildings, bridges, and the wings of an airplane.

Research has been conducted by many scientists on the analysis of sound and vibration in a tubular system, although those physical problems are not part of the primary heating, cooling, and air conditioning functions because they are secondary factors which, if not properly handled, can destroy an otherwise successful tubular installation.⁴ All sounds and vibrations are forms of kinetic energy, and in the tubular and fluid transport world they are usually derived from moving equipment, moving air, pressure-reducing equipment, or other moving fluid. A problem arises when a tubular system component generates noise or vibration within, or adjacent to, an inhabited or process-sensitive space. If the generated sound or vibration level exceeds the local tolerance level, the tubular system is deemed unacceptable.

Looking to solve these sound and vibration failures, researchers have designed different geometries and physical characteristics for resonator mufflers. These mufflers can be reactive and have the purpose of providing high impedance that causes an incident acoustic wave that gets reflected inside the muffler geometry. This kind of muffler usually does not have acoustic absorption material, in contrast to an absorptive muffler that depends on the use of acoustic absorbing material to attenuate incident acoustic waves.

The objective of this paper is to implement the CMS using the MSRF for the appraisal of the natural frequencies and modal shapes of a full-beam model with a quarter-wavelength tube and a Helmholtz resonator geometry. The entire system is partitioned in two beams like a Helmholtz resonator with different geometry. This gives an advantage to work the procedure as a Timoshenko beam.¹⁷ The complete system and substructures will be calculated through FEM using the Hermitian beam element applied utilizing MATLAB® software. To validate the results, the complete system and the substructures will be modeled in ANSYS® software.

2. MODAL SYNTHESIS METHOD WITH RESIDUAL FLEXIBILITY

The process of modal synthesis using the MSRF method^{15,18} consists of three basic steps:

1. The definition of the substructures that were divided from the complete system;

Figure 1. The first substructure of the beam (11 nodes). b) the second substructure of the beam (11 nodes). c) the resulting beam discretized and interconnected by a common interface with 21 nodes.

2. The calculation of the modal superset (flexible modes and static modes) through an analytical or experimental formulation; and,
3. Assemble and solve the global equations of motions according to the connectivity imposed.

A further explanation of this methodology, would be to consider two clamped-free beams with those beams joined at its free boundary, Fig. 1(a) and Fig. 1(b). In this way, those beams will form a clamped-clamped beam (Fig. 1(c)). These beams will be discretized using physical coordinates and its boundaries (x , y , and rotation along the xy plane).

The procedure of analysis and calculation of the MSRF technique uses the modes of residual flexibility as a function of the inertia plots and rigidity of the dynamic matrix¹⁵.

$$[\delta_f] = [\theta_d][\Lambda_{dd}]^{-1}[\theta_{cd}]^t; \quad (1)$$

where: d and c – Unselected and boundary nodes, respectively. δ_f – Residual flexibility modes. Λ_{dd} – Unselected eigenvalues matrix. θ_{cd} – Unselected normal modes. Once the maintained modes and the eliminated modes are determined from the modal basis, normal modes are given:

$$\theta_{dc} = \{0 \quad f_c\}\theta_{nc}; \quad (2)$$

where: n – Total number of normal modes. f_c – Unitary forces applied to the boundary of the substructures.

Once the selected or maintained normal modes of the modal base are obtained, as φ_m , the next step is to calculate the modes that will be used in the equation of motion for the two interconnected substructures. The general connectivity matrix is obtained using some auxiliary variables:

$$[\hat{A}] = (\delta_{fc}^a + \delta_{fc}^b)^{-1}; \quad (3)$$

where a and b are respectively the substructures. The matrix δ_{fc} is the portion of residual flexibility modes relative to the interface between the two substructures connected. A global equation of motion that take into account the two connected substructures can be synthesized for:

$$[\bar{M}]\{\ddot{p}\} + [\bar{K}]\{p\} = \{0\}; \quad (4)$$

where:

$$\{\hat{q}\} = [S]\{p\}; \quad (5)$$

$$[S] = \begin{bmatrix} -\hat{A}\phi_{mc}^a & \hat{A}\phi_{mc}^b \\ \hat{A}\phi_{mc}^a & -\hat{A}\phi_{mc}^b \\ I_{mm}^a & 0 \\ 0 & I_{mm}^b \end{bmatrix}; \quad (6)$$

$$[\bar{M}] = \begin{bmatrix} I_{mm}^a + (\phi_{mc}^a)^t V \phi_{mc}^a & -(\phi_{mc}^a)^t V \phi_{mc}^b \\ -(\phi_{mc}^b)^t V \phi_{mc}^a & I_{mm}^b + (\phi_{mc}^b)^t V \phi_{mc}^b \end{bmatrix}; \quad (7)$$

$$[\bar{K}] = \begin{bmatrix} \Lambda_{mm}^a + (\phi_{mc}^a)^t \hat{A} \phi_{mc}^a & -(\phi_{mc}^a)^t \hat{A} \phi_{mc}^b \\ -(\phi_{mc}^b)^t \hat{A} \phi_{mc}^a & \Lambda_{mm}^b + (\phi_{mc}^b)^t \hat{A} \phi_{mc}^b \end{bmatrix}; \quad (8)$$

$$[V] = [\hat{A}]^t (\beta_{ff}^a + \beta_{ff}^b) [\hat{A}]; \quad (9)$$

$$[\beta_{ff}] = [\theta_{cd}] ([\Lambda_{dd}]^{-1})^2 [\theta_{cd}]^t. \quad (10)$$

Solving the reduced eigenproblem, Eq. (4), was obtained using eigenvalues and eigenvectors (Σ) that were representative of the problem in the kept (p) modal coordinates. To return to the system's original modal base, an inverse operation was used as follows:

$$[\zeta] = [\Psi][S][\Sigma]; \quad (11)$$

where:

$$[S] = \begin{bmatrix} -\hat{A}\phi_{mc}^a & \hat{A}\phi_{mc}^b \\ \hat{A}\phi_{mc}^a & -\hat{A}\phi_{mc}^b \\ I_{mm}^a & 0 \\ 0 & I_{mm}^b \end{bmatrix}; \quad (12)$$

$$[E] = \begin{bmatrix} \delta_{fi}^a & 0 & \phi_{mi}^a & 0 \\ \delta_{fc}^a & 0 & \phi_{mc}^a & 0 \\ 0 & \delta_{fi}^b & 0 & \phi_{mi}^b \\ 0 & \delta_{fc}^b & 0 & \phi_{mc}^b \end{bmatrix}. \quad (13)$$

(S) and (E) are the restriction matrices and original modal of substructures, respectively. The procedure of modal synthesis using two interconnected components generates equations of easy computational implementation by always using two substructures mutually connected. The first two connected substructures will be considered in the next stage as a new and the only substructure that is connected to a third. Thus, the substructures are connected, one to one, until the assembly of the global structure.¹⁹

When a modal synthesis is developed that uses the modal superset of residual flexibility, the researcher can define which modes he wants to remove from the modal basis. These modes are subtracted from the modal base, because they are used to define residual flexibility modes that improve the modal basis of the substructures, likewise depending on the number of modes maintained in the modal basis, they may be significant in identifying modes which can be removed. The methodology denominated, "Automatic Mode Elimination Criterion (AEC)", based on the energy level of the contour of the substructures, may be used for the selection of modes to be maintained.¹⁵

Figure 2. a) Quarter-wavelength tube b) Helmholtz resonator.²

Figure 3. Flow chart of the methodology.

3. METHODOLOGY

The resonator models considered in this work are shown in Fig. 2(a), which is a Quarter-wavelength tube and Fig. 2(b), a denominated Helmholtz resonator.

Ordóñez, 2017,²⁰ shows a schematic of a quarter-wavelength tube (QWT) resonator silencer attached to a circular main exhaust duct and its relevant parameters. For this research, a similar geometry and distribution of the nodes with the Helmholtz resonator geometry were used. The results with the routine proposed for Ordóñez, were compared and validated with the Helmholtz resonator geometry.²⁰

Figure 3 represents the flow chart of the methodology used. This methodology was used for both cases of study of resonator models (quarter-wavelength tube and Helmholtz resonator).

The modeling of the resonator silencer utilizing the quarter-wavelength tube and the Helmholtz resonator was done and implemented in MATLAB® using a Hermitian beam element as a planar (2-D) frame element including the axial and bending deformation, as shown in Eqs. (7) and (8). These equations represent the mass and the stiffness matrix configuration that is implemented in the routine. These matrices are considered for each beam represented in the Figures 5 and 6. The substructures were also modeled by this technique including axial and

Figure 4. A Linear Frame Element.²¹

bending stiffnesses:²¹

$$[K] = \frac{E}{l^3} \begin{bmatrix} Al^2 & 0 & 0 & -Al^2 & 0 & 0 \\ 0 & 12I & 6\Pi & 0 & -12I & 6\Pi \\ 0 & 6\Pi & 4\Pi^2 & 0 & -6\Pi & 2\Pi^2 \\ -Al^2 & 0 & 0 & Al^2 & 0 & 0 \\ 0 & -12I & -6\Pi & 0 & 12I & -6\Pi \\ 0 & 6\Pi & 2\Pi^2 & 0 & -6\Pi & 4\Pi^2 \end{bmatrix}; \quad (14)$$

where:

$$[M] = \frac{\rho Al}{420} \begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22l & 0 & 54 & -13l \\ 0 & 22l & 4l^2 & 0 & 13l & -3l^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13l & 0 & 156 & -22l \\ 0 & -13l & -3l^2 & 0 & -22 & 4l^2 \end{bmatrix}; \quad (15)$$

where: I : Moment of inertia of cross-section. E : Elastic Modulus. A : Cross-sectional area. ρ : Mass density per volume. l : Length of the element. K : Stiffness matrix. M : Mass matrix.

For the element DOF $\{u_1, v_1, \theta_1, u_2, v_2, \theta_2\}$ as seen in Fig. 4, the DOF are going to be used to represent the vibration mode in the result of the synthesis and also to be implemented in the MAC analyses.

The syntheses were made through the MSRF method and were evaluated using discrete masses.

Figure 5 shows the dimensions and the geometry of the Helmholtz resonator² and Fig. 6 shows the node distribution along the geometry that is composed for two beams, which is constituted of a bigger beam with 40 elements and 41 nodes and a more complex smaller beam for two different diameters with 15 elements and 16 nodes. This means that the complete system was composed for 55 elements and 56 nodes, the distribution of the nodes as can be seen in the Fig. 6 was used in both analyses, quarter-wavelength tube and Helmholtz resonator. The modeling method was implemented in MATLAB® software. Remembering that the Hermitian beam element has three DOF per node (displacement in u , v , and the rotation in the plane uv).

Table 1. shows the material properties of the beam used in the analysis in SAE 1010.²²

As seen in Fig. 6, the boundary conditions of the first substructure are clamped-clamped, which means that nodes 1 and 41 are constrained in their three DOF, while the boundary conditions of the second substructure are free-free.

Once the parameters for the FEM for the entire system and

Figure 5. Model of the Helmholtz Resonator.

Figure 6. The finite element model of the quarter-wavelength tube and the Helmholtz resonator.

the substructures were established, the natural frequencies and mode shapes were calculated. As a result for each DOF in all the nodes, it was established that there was one natural frequency and mode shape.

The next step in the numerical procedure was utilizing the component mode synthesis with the program MATLAB® to obtain the frequencies of the system by using the results obtained for the natural frequencies and mode shapes of each substructure. For this problem, the first 40 natural frequencies and mode shapes from substructure 1 and the first 15 natural frequencies and mode shapes from substructure 2 will be used, with a goal of maintaining a proportion in the size and quantity of the modes of each substructure in mind. In this case, the last frequency of each substructure was chosen to define residual flexibility modes. This frequency and mode shape are the higher value in this problem, and it is the frequency with less probability to appear in this kind of structural system.

To validate the result, this same model was simulated with the software ANSYS® - APDL 16.0, which is the most common technique, used to extract the natural frequencies and vibration modes. In the same way as the FEM in MATLAB® were used as the element type BEAM188 and the quantity of the element were described, the structure was divided into substructures to compare each result with the analytical result. Also, the entire system was modeled through numerical analysis to compare the result of the synthesis and the numerical result of the entire system with ANSYS®.

Table 1. Material properties of the beam.

Density (ρ)	7870 [Kg/m ³]
Modulus of Elasticity (E)	200 [GPa]
Poisson's Coefficient (ν)	0,3
Thickness (h)	0,002 [m]

Table 2. The first ten natural frequencies of the global system in ANSYS® and the synthesized system in MATLAB®.

Mode N°	Theoretical Frequency ANSYS® [Hz]	Frequency Synthesized MATLAB® [Hz]	Relative Error [%]	MAC
1	20,43	20,49	0,290	0,9972
2	31,76	32,08	1,017	0,9655
3	103,28	105,39	2,045	0,1202
4	133,32	135,56	1,683	0,7105
5	179,76	186,70	3,859	0,9627
6	323,48	339,52	4,958	0,0045
7	364,33	379,51	4,167	0,5418
8	428,88	464,15	8,224	0,9320
9	605,79	610,77	0,823	0,9258
10	650,72	709,69	9,063	0,0321

Table 3. The first ten natural frequencies of the entire system and the synthesized system.

Mode N°	Theoretical Frequency ANSYS® [Hz]	Frequency Synthesized MATLAB® [Hz]	Relative Error [%]	MAC
1	12,79	12,27	4,109	0,9996
2	26,65	26,65	0,010	0,9694
3	107,46	107,58	0,111	0,9376
4	172,31	174,25	1,124	0,9921
5	174,27	177,89	2,078	0,9451
6	353,69	353,77	0,022	0,8901
7	440,61	440,62	0,001	0,9036
8	614,95	615,86	0,148	0,9772
9	722,93	723,28	0,048	0,8385
10	797,74	798,36	0,077	0,8502
Mean Values			0,771	0,9304

4. RESULTS

4.1. Quarterwave-Length Tube

Table 2 shows the relationship between the first ten natural frequencies of the synthesized structure of the quarterwave-length tube using the FEM, Fig. 6, in MATLAB® and the fully calculated structure using ANSYS® - APDL 16.0 with BEAM188 as an element type.

It can be seen that if it improves the “mesh”, or in a simple way, if it increases the number of elements that describe the system, the modal base will be better described.²¹ The author researched this problem improving the number of elements that described the system; this gave a better representation of the system. In this order, this is the reason the low values in the MAC value resulted in frequencies 3, 6 and 10.

4.2. Helmholtz Resonator

Table 3 shows the relationship between the first ten natural frequencies of the synthesized structure and the fully calculated structure both using the FEM as the Hermitian beam element as a planar (2-D) frame element in MATLAB® for a Helmholtz resonator, Fig. 6.

Sixteen frequencies in the first substructure and 6 frequencies in the second substructure were synthesized, including the constrained DOF in both substructures. This means that the total frequencies obtained on the synthesized system were 16. Because one mode shape was eliminated - and defined

Table 4. First ten natural vibration frequencies of the entire system in ANSYS® and the synthesized system in MATLAB®.

Mode N°	Theoretical Frequency ANSYS® [Hz]	Frequency Synthesized MATLAB® [Hz]	Relative Error [%]	MAC
1	12,77	12,27	3,922	0,8797
2	26,64	26,65	0,032	0,9702
3	107,20	107,58	0,356	0,7972
4	170,42	174,25	2,244	0,9830
5	173,86	177,89	2,317	0,9546
6	351,40	353,77	0,675	0,8742
7	437,67	440,62	0,674	0,6960
8	614,27	615,86	0,260	0,8476
9	712,01	723,28	1,583	0,7821
10	785,75	798,36	1,605	0,8568

Figure 7. TMAC index behavior between the answers in MATLAB® and ANSYS®.

the residual flexibility modes and the first six frequencies that represented the constrained DOF of the system, the value was zero.

Table 4. shows the relationship between the first ten natural frequencies of the synthesized structure using the FEM in MATLAB® and the fully calculated structure using ANSYS® - APDL 16.0 with BEAM188 as an element type.

5. CONCLUSIONS

The Hermitian beam element as a planar (2-D) frame element including the axial and bending deformation presented an effectiveness to obtain the entire system frequencies and also the use of this element to obtain a synthesized system through the MSFR method, with a relative error near to zero and a MAC near to unity, as shown in Table 3 and Table 4.

The results of the identification of the modes of substructures using the MAC method showed that the MAC index is higher than 0.80 and that some of these values are near to unity between ANSYS® and MATLAB®, with a mean relative error less than 2%.

According to Howard and Cazzolato, 2015,² the results indicated that the acoustic resonance frequencies are approximately 58 Hz, 177 Hz and 290 Hz, which means that if the structural natural frequencies are near to these values, it will

appear noisy due to the vibration of the structural part. One way to avoid this phenomenon, would be to change the length of the bigger tube (substructure 1) or modify the position of the constraint.

For both cases (quarterwave-length tube and Helmholtz resonator geometry), the frequency of 177 Hz has to be avoided because the system can present acoustic and structural resonance.

REFERENCES

- ¹ Mardiana, A. and Riffat, S. B. Building Energy Consumption and Carbon Dioxide Emissions: Threat to Climate Change. *Journal of Earth Science & Climatic Change*, S3:001, (2015). <https://dx.doi.org/10.4172/2157-7617.s3-001>
- ² Howard, C.Q. and Cazzolato, B.S. Acoustic Analyses Using MATLAB® and ANSYS®. Taylor & Francis Group, LLC. CRC Press, (2015). <https://dx.doi.org/10.1201/b17825>
- ³ Nunes, M.A.A. and Duarte, M.A.V. The use of axial splitters inside ducts for active noise control of higher order acoustic modes, *Noise Control Engineering Journal*, **59**(4), pp. 347-362, (2011). <https://dx.doi.org/10.3397/1.3581378>
- ⁴ Haines R. W. and Lewis C. W. HVAC Systems Design Handbook. McGraw-Hill Professional, (2003).
- ⁵ American Society Of Heating, Refrigerating And Air-Conditioning Engineers, INC. HVAC Fundamentals Handbook. Atlanta: ASHRAE, (2001).
- ⁶ Craig JR, R. R. Coupling of substructures for dynamic analyses: an overview. *41st Structures, Structural Dynamics, and Materials Conference and Exhibit*, Austin, TX, AIAA-2000-1573. p. 12, (2000). <https://dx.doi.org/10.2514/6.2000-1573>
- ⁷ Zuo, Y. and Wang, J., A component mode synthesis method for 3-D finite element models of aero-engines, *Journal of Mechanical Science and Technology*, **29**(12) 5157-5166, (2015). <https://dx.doi.org/10.1007/s12206-015-1116-8>
- ⁸ Sasri, D. and Azrar, L. Dynamic analysis of large structures with uncertain parameters based on coupling component mode synthesis and perturbation method. *Ain Shams Engineering Journal*, **7**(1), 1-11, (2015). <https://dx.doi.org/10.1016/j.asej.2015.03.005>
- ⁹ Craig R. and Chang C. J., Free-interface methods of substructure coupling for dynamic analysis, *AIAA J.*, **14**(11) 1633-1635, (1976). <https://dx.doi.org/10.2514/3.7264>
- ¹⁰ MacNeal R. H., A hybrid method of component mode synthesis, *Computers & Structures*, **1**(4) 581-601, (1971). [https://dx.doi.org/10.1016/0045-7949\(71\)90031-9](https://dx.doi.org/10.1016/0045-7949(71)90031-9)
- ¹¹ Tournour M. A., Atalla, N., Chiello, O. and Sgard, F. Validation, performance, convergence and application of free interface component mode synthesis, *Computers and Structures*, **79**(20) 1861-1876, (2001). [https://dx.doi.org/10.1016/s0045-7949\(01\)00114-6](https://dx.doi.org/10.1016/s0045-7949(01)00114-6)
- ¹² Besset S. and Jezequel L., Dynamic substructuring based on a double modal analysis, *J. of Vibration and Acoustics*, **130**(1), (2008). <https://dx.doi.org/10.1115/1.2346698>
- ¹³ Bathe K. J. and Jian D., Component mode synthesis with subspace iterations for controlled accuracy of frequency and mode shape solutions, *Computers & Structures*, **139**, 28-32, (2014). <https://dx.doi.org/10.1016/j.compstruc.2014.03.003>
- ¹⁴ Kim J. G., Lee K. H. and Lee P. S., Estimating relative eigenvalue errors in the craig-bampton method, *Computers & Structures*, **139**, 54-64, (2014). <https://dx.doi.org/10.1016/j.compstruc.2014.04.008>
- ¹⁵ Nunes, M. A. A and Duarte, M. A. V. Component Mode Synthesis Method Applied to Two-Dimensional Acoustic Analysis in Ducts. *Journal of Vibration and Acoustics*, **135**(1), 011016, (2013). <https://dx.doi.org/10.1115/1.4007243>
- ¹⁶ Gawande, S.H. and More, R. R., 2016. Some Investigations on Effect of Notch on Dynamics Characteristics of Cantilever Beams. *International Journal of Acoustics and Vibration*, **24**(1), (2019). <https://dx.doi.org/10.20855/ijav.2019.24.11121>
- ¹⁷ Timoshenko PSP. On the transverse vibrations of bars of uniform cross-section. *Philosophical Magazine Series 6*, **43**(253), 125-131, (1922). <https://dx.doi.org/10.1080/14786442208633855>
- ¹⁸ Junqueira M. M., Uma Contribuição ao Método de Síntese Modal Experimental. Ph.D. Thesis. Universidade Federal de Uberlândia, Faculdade de Engenharia Mecânica, (2006).
- ¹⁹ Junqueira M. M. and Araújo C. A., Evaluation of an Experimental Modal Synthesis Method Using Mass Normalized Modes. *ABCM Symposium Series in Mechatronics*, **1**, 286-294, (2004).
- ²⁰ Ordóñez, J.M.B. and Nunes, M.A.A., 2017. HVAC duct system modeling through the component mode synthesis for structural vibration analysis. *Proceedings of the 24th ABCM International Congress of Mechanical Engineering*. Curitiba, Brazil, (2017). <https://dx.doi.org/10.26678/abcm.cobem2017.cob17-2738>
- ²¹ Kwon, Y.W. and Hyochoong, B. Acoustic Analyses Using MATLAB® and ANSYS®. Taylor & Francis Group, LLC. CRC Press, (2000).
- ²² Materials, E. K. S. O. Carbon Steel SAE 1010. http://www.substech.com/dokuwiki/doku.php?id=carbon_steel_sae_1010, (Accessed 15 August 2017).