Influence of Road Roughness Wavelengths on Bus Passengers' Oscillatory Comfort

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This paper analyses the effect of the waviness parameter w on the oscillatory comfort of bus passengers for one road in good condition. The analysis is carried out by a validated nonlinear oscillatory 16 degrees of freedom (DOF) bus model defined in the software Matlab/Simulink and procedure from ISO 2631/1997 standard. The root-mean-square (RMS) values of the weighted vertical accelerations from the passengers' seats increase with the increase in the parameter w . On the other hand, the RMS values of the weighted vertical accelerations from bus floor decreased with the increase in the parameter w . The best oscillatory comfort has a passenger in the middle part of the bus, and the worst has a passenger in the rear part of the bus. The intensities of the vertical accelerations from the bus floor are mainly concentrated into two frequency ranges, $0 \div 2.5$ Hz and $5 \div 10$ Hz. The passenger seats are dampened with vertical vibrations above 4 Hz. For the narrow range $w = 1.5 \div 2.3$, nominal value of the waviness parameter ($w = 2$) is sufficiently precise in assessment of the bus passengers' oscillatory comfort.

1. INTRODUCTION

The oscillatory movement of the bus negatively affects bus users' oscillatory comfort, road surface, and the vehicle itself. Although bus oscillatory motion is excited from a number of sources, the dominant excitation originates from road roughness. Longitudinal road roughness comprises of a number of different unevenness, wavelengths, and amplitudes. Spectrum composition and unevenness amplitude, and their effect on the vibration intensity, depend on the type/condition of the road roughness and on vehicle speed.

There are a considerable number of direct and indirect parameters for the description of the longitudinal road roughness.¹ Direct parameters come from vertical road elevation data processing. Two of these parameters refer to road elevation power spectral density (PSD) - unevenness index $\Phi_\zeta(n_0)$ and waviness parameter w .² Standard ISO 8608 analytically defines PSD and presents a classification of road roughness according to different unevenness index $\Phi_\zeta(n_0)$ values and one constant waviness w value ($w = 2$).

The unevenness index defines road roughness quality. Higher values of the unevenness coefficient indicate a worse state of the road. The waviness parameter expresses distribution of short/long wavelengths in road elevation spectrum. According to the ISO 8608 standard, the nominal value of parameter w is 2 and indicates a similar proportion of short/long wavelengths in the road spectrum. However, studies have shown that the values of the parameter w lie within the limits of 1.5 to 3.5.³ Standard shape of the PSD with waviness of $w = 2$ correspond to 60% of analysed road profiles. Long wavelengths ($w > 2$) prevailed in 20%, and for the remaining 20%, short wavelengths ($w < 2$) dominated.⁴

Můčka analysed influence of the longitudinal road unevenness on the dynamic tire forces using the quarter model of a commercial heavy vehicle with 2 DOF.⁴ Dynamic tyre forces in function of random road profiles with different waviness using longitudinal planar model of three axle heavy vehicle with 8 DOF were analysed by Můčka.⁵ Four different spatial mathematical models of heavy vehicles are used for simulation when analysing influence of different roads roughness and waviness

on dynamic load stress factor by Steinauer and Ueckermann.⁶ To the author's knowledge, influence of road waviness on the bus vertical dynamics, especially on passengers' ride comfort, is not a sufficiently investigated area.

The aim of this paper is to determine the influence of the waviness parameter w on bus passengers' whole-body vibration (WBV) considering vertical accelerations from their seats. The aim is also to determine the effect of the parameter w on vertical acceleration from bus floor acting on the passenger's legs. The main goal of this work is to determine passengers' ride comfort in the function of the parameter w . In order to achieve these goals, a validated nonlinear in-plane bus model with 16 DOF was considered. The model was defined in the Matlab/Simulink software. Longitudinal road roughness in good condition was modelled by road elevation PSD defined in ISO 8608 standard. Ride comfort was determined by procedure and criteria prescribed by ISO 2631/1997 standard. This standard is the most frequently used one when assessing human response to WBV.⁷

2. BUS OSCILLATORY MODEL

Figure 1 shows intercity bus IK-301 for passengers transport. Elementary geometry parameters are shown in Fig. 1(a). Position of the passengers' seats ($1 \div 3$), considered in the oscillatory comfort analysis are denoted in Fig. 1(a, b). Figure 1(b) also indicates layout of the bus seats.

A sixteen DOF plane model of the intercity bus IK-301 was used in the study (Fig. 2). The DOF included mass m_{hi} , ($i = 1 \div 3$) bouncing (z_{hi} , ($i = 1 \div 3$)), mass m_{ci} , ($i = 1 \div 3$) bouncing (z_{ci} , ($i = 1 \div 3$)), mass m_{lti} , ($i = 1 \div 3$) bouncing (z_{lti} , ($i = 1 \div 3$)), mass m_{tpi} , ($i = 1 \div 3$) bouncing (z_{tpi} , ($i = 1 \div 3$)), mass m (vehicle body) bouncing (z), mass m (vehicle body) pitching (θ), mass m_{t1} (front axle) bouncing (z_{t1}), and mass m_{t2} (rear axle) bouncing (z_{t2}).

When forming the bus oscillatory model, the next assumptions were considered:

- the bus body, front and rear axles are rigid bodies;

Figure 1. Bus IK-301 a) elementary geometry parameters (L —wheelbase; a —front overhang; b —rear overhang), b) position of the passengers' seats for ride comfort analysis.

- the bus is symmetrical relative to the longitudinal centre of gravity axis (x -axis);
- there are no angular movements around x -axis;
- the excitations on both left and right wheels are the same;
- contact between wheel and road is in one point;
- characteristics of all elastic elements are linear;
- the bus is moving along a straight line with constant speed.

Bus IK-301 had two wheels on the front axle and two twin wheels at the rear axle.⁸ The front axle suspension comprised of two air springs and four telescopic shock absorbers, while on the rear axle, it included four air springs and four telescopic shock absorbers.⁸

The differential equations of motion, by applying Lagrange's equations of motion of second kind, for oscillatory model are given by Eqs. (1–10):

$$m_{hi}\ddot{z}_{hi} + c_1\dot{z}_{hi} + k_1z_{hi} - c_1\dot{z}_{ci} - k_1z_{ci} = 0, \quad (i = 1 \div 3); \quad (1)$$

$$m_{ci}\ddot{z}_{ci} + (c_1 + c_2)\dot{z}_{ci} + (k_1 + k_2)z_{ci} - c_1\dot{z}_{hi} - k_1z_{hi} - c_2\dot{z}_{ti} - k_2z_{ti} = 0, \quad (i = 1 \div 3); \quad (2)$$

$$m_{ti}\ddot{z}_{ti} + (c_2 + c_3)\dot{z}_{ti} + (k_2 + k_3)z_{ti} - c_2\dot{z}_{ci} - k_2z_{ci} - c_3\dot{z}_{tpi} - k_3z_{tpi} = 0, \quad (i = 1 \div 3); \quad (3)$$

$$m_{tp1}\ddot{z}_{tp1} + (c_3 + c'_4)\dot{z}_{tp1} + (k_3 + k'_4)z_{tp1} - c_3\dot{z}_{ti} - k_3z_{ti} - c'_4\dot{z} - k'_4z - c'_4d_1\dot{\theta} - k'_4d_1\theta = 0; \quad (4)$$

$$m_{tp2}\ddot{z}_{tp2} + (c_3 + c'_4)\dot{z}_{tp2} + (k_3 + k'_4)z_{tp2} - c_3\dot{z}_{ti} - k_3z_{ti} - c'_4\dot{z} - k'_4z - c'_4d_2\dot{\theta} - k'_4d_2\theta = 0; \quad (5)$$

$$m_{tp3}\ddot{z}_{tp3} + (c_3 + c'_4)\dot{z}_{tp3} + (k_3 + k'_4)z_{tp3} - c_3\dot{z}_{ti} - k_3z_{ti} - c'_4\dot{z} - k'_4z - c'_4d_3\dot{\theta} - k'_4d_3\theta = 0; \quad (6)$$

$$m\ddot{z} + 3c'_4\dot{z} + (3k'_4 + k_f + k_r)z + (d_1 + d_2 - d_3)c'_4\dot{\theta} + (k'_4d_1 + k'_4d_2 - k'_4d_3 + k_fa - k_rb)\theta - c'_4\dot{z}_{tp1} - k'_4z_{tp1} - c'_4\dot{z}_{tp2} - k'_4z_{tp2} - c'_4\dot{z}_{tp3} - k'_4z_{tp3} - k_fz_{t1} - k_rz_{t2} + F_{f.eq}(\Delta\dot{z}_f) + F_{r.eq}(\Delta\dot{z}_r) = 0; \quad (7)$$

$$J_y\ddot{\theta} + (d_1^2 + d_2^2 + d_3^2)c'_4\dot{\theta} + (d_1^2k'_4 + d_2^2k'_4 + d_3^2k'_4 + a^2k_f + b^2k_r)\theta + (d_1 + d_2 - d_3)c'_4\dot{z} + (d_1k'_4 + d_2k'_4 - d_3k'_4 + ak_f - bk_r)z - d_1c'_4\dot{z}_{tp1} - d_1k'_4z_{tp1} - d_2c'_4\dot{z}_{tp2} - d_2k'_4z_{tp2} + d_3c'_4\dot{z}_{tp3} + d_3k'_4z_{tp3} - ak_fz_{t1} + bk_rz_{t2} + aF_{f.eq}(\Delta\dot{z}_f) - bF_{r.eq}(\Delta\dot{z}_r) = 0; \quad (8)$$

$$m_{t1}\ddot{z}_{t1} + c_{ft}\dot{z}_{t1} + (k_{ft} + k_f)z_{t1} - k_fz - ak_f\theta - F_{f.eq}(\Delta\dot{z}_f) = c_{ft}\dot{\zeta}_1 + k_{ft}\zeta_1; \quad (9)$$

$$m_{t2}\ddot{z}_{t2} + c_{rt}\dot{z}_{t2} + (k_{rt} + k_r)z_{t2} - k_rz + bk_r\theta - F_{r.eq}(\Delta\dot{z}_r) = c_{rt}\dot{\zeta}_2 + k_{rt}\zeta_2. \quad (10)$$

The sets $F_{f.eq}(\Delta\dot{z}_f)$ and $F_{r.eq}(\Delta\dot{z}_r)$ represented nonlinear equivalent damping forces of the shock absorbers on the front and rear axles of the bus (Eqs. (7–10)). The damping forces depended on the relative piston velocity in the shock absorber cylinder, (Eqs. (11) and (12)):

$$\Delta\dot{z}_f = v_f = \dot{z}_D - \dot{z}_{t1} = \dot{z} + a\dot{\theta} - \dot{z}_{t1}; \quad (11)$$

$$\Delta\dot{z}_r = v_r = \dot{z}_E - \dot{z}_{t2} = \dot{z} - b\dot{\theta} - \dot{z}_{t2}. \quad (12)$$

Figure 3 shows the piecewise linear curves that represented characteristics for one damper on front/rear axle, as well as their equivalent characteristics. Piecewise linear curves have been used for modelling of experimentally established nonlinear damper properties.⁹ Nonlinear characteristics of the individual shock absorber are described analytically (Eqs. (13) and (14)). The equivalent characteristics of the shock absorber were loaded into the oscillatory model using the Simulink function 'Lookup table'. The damping force was considerably greater in the rebound mode, compared to the compression mode of the shock absorber (Fig. 3).

$$F_f(v_f) = \begin{cases} c_{f1}v_f, & 0 \leq v_f < 0.16; \\ c_{f2}v_f + 1850, & 0.16 \leq v_f \leq 1; \\ c_{f3}v_f, & -1 \leq v_f < 0; \end{cases} \quad (13)$$

$$F_r(v_r) = \begin{cases} c_{r1}v_r, & 0 \leq v_r < 0.3; \\ c_{r2}v_r + 2800, & 0.3 \leq v_r \leq 1; \\ c_{r3}v_r, & -1 \leq v_r < 0. \end{cases} \quad (14)$$

Values of damping coefficients for each shock absorber, equivalent damping coefficients and transition speeds for both axles are given in Table 1.

The WBV bus passenger's exposure analysis was performed using a linear seat-human model (Fig. 2). Linear biodynamic human body 4 DOF model proposed by Boileau was used when defining a seat-human model.¹⁰ Connection between the passenger body and the seat was represented by an elastic-damping element with an equivalent coefficient of stiffness k'_4 and an equivalent coefficient of damping c'_4 . Equivalent coefficients were determined based on the serial connection between seat cushion and human buttocks oscillatory parameters, (Eqs. (15) and (16)).

$$k'_4 = \frac{k_1 \cdot k_4}{k_1 + k_4}; \quad (15)$$

$$c'_4 = \frac{c_1 \cdot c_4}{c_1 + c_4}. \quad (16)$$

The passenger seats did not have an elastic suspension system and were stiff-suspended on the bus body. Vibration

Figure 2. In-plane oscillatory bus model with 16 DOF.

Figure 3. Damper force-velocity characteristics for a) front axle, b) rear axle.

damping was performed by foam seat cushions. Seat cushion stiffness/damping coefficients originated from available literature.^{11,12} Table 3 shows the parameter values for the linear seat-human oscillatory model, according to Boileau.¹⁰ Figure 4 shows top level Simulink diagram of the bus oscillatory model. The Matlab built-in function 'ode45' with a variable step-size was used for numerical integration. The defined initial conditions for all variables were set to zero. The chosen simulation time was 7 seconds.

Parameters from differential equations and their values are shown in Table 2, and originated from the available literature.^{8,11}

2.1. Validation of the Bus Oscillatory Model

Bus model validation was based on acceleration signals measured in bus IK-301 during real exploitation condition.¹³ A total of 14 measurements were carried out on three different types of road (asphalt-concrete/macadam/cube pavement) in three different states (bad/medium/good) and at five different bus speeds (40, 60, 80, 100 and 130 km/h).

The bus model was validated by two different oscillatory

quantities from MEASUREMENT 11: vertical acceleration on the passenger seat surface positioned above rear axle and vertical accelerations on the floor below passenger's seat. The position of the passenger's seat was denoted with V in Fig. 1(a-b). The measurement of these signals was carried out on the empty intercity bus IK-301 on sections of the 'Airport-Ikarbus' asphalt-concrete road in a good condition.¹³ The bus was moving at a speed of 100 km/h. A driver, one passenger, and two persons for data collection participated in the measurements. The acceleration signal was registered on the surface of the passenger's seat positioned over the rear axle, with the passenger seated on the seat, below the passenger's body.

A WBV tri-axial seat pad accelerometer (B&K type 4321) registered vertical acceleration signal on passenger's seat (Fig. 5(a)). A B&K type 2635 amplifier was used to amplify the output signal from the accelerometer (Fig. 5(b)). An inductive accelerometer (HBM type B 12/200) registered vertical acceleration signals from the bus floor below passenger's seat (Fig. 5(c)). Signals were recorded during 13.65 s.

Real road roughness in good condition was introduced in the bus oscillatory model when performing simulation for the

Figure 6. Real road roughness of asphalt-concrete pavement in good condition registered at speed of 90 km/h.

Figure 7. Measured and simulated vertical acceleration signal for a) bus floor below passenger's seat, b) passenger's seat above rear axle.

Table 1. Damping coefficients and transitions speeds of shock absorbers for front/rear axle.

Front axle — Parameter	Value
Low speed rebound mode damping coefficient c_{f1} (Ns/m)	14062.5
Equivalent low speed rebound mode damping coefficient ($4 \cdot c_{f1}$) (Ns/m)	56250
High speed rebound mode damping coefficient c_{f2} (Ns/m)	2500
Equivalent high speed rebound mode damping coefficient ($4 \cdot c_{f2}$) (Ns/m)	10000
High speed rebound mode damping coefficient c_{f3} (Ns/m)	750
Equivalent high speed rebound mode damping coefficient ($4 \cdot c_{f3}$) (Ns/m)	3000
Transition velocity v_{fe} (m/s)	0.16
Rear axle — Parameter	Value
Low speed rebound mode damping coefficient c_{r1} (Ns/m)	13333.3
Equivalent low speed rebound mode damping coefficient ($4 \cdot c_{r1}$) (Ns/m)	53333.3
High speed rebound mode damping coefficient c_{r2} (Ns/m)	4000
Equivalent high speed rebound mode damping coefficient ($4 \cdot c_{r2}$) (Ns/m)	16000
High speed rebound mode damping coefficient c_{r3} (Ns/m)	500
Equivalent high speed rebound mode damping coefficient ($4 \cdot c_{r3}$) (Ns/m)	2000
Transition velocity v_{re} (m/s)	0.3

ferences, some simplifications referring to the bus oscillatory model might affect simulation results (such as using an in-plane and not space bus model, linear characteristics of elastic elements, point wheel-road contact, etc.).

3. BUS MODEL EXCITATION

Unevenness PSD defined in ISO 8608 was used for road roughness modeling (Eq. (17)).

$$\Phi_{\zeta}(n) = \Phi_{\zeta}(n_0) \cdot \left(\frac{n}{n_0} \right)^{-w}; \quad (17)$$

where $\Phi_{\zeta}(n)$ —PSD of road roughness ($\text{m}^2/(\text{cycles/m})$); $\Phi_{\zeta}(n_0)$ —unevenness index ($\text{m}^2/(\text{cycles/m})$); n_0 —reference spatial frequency ($n_0 = 0.1$ (cycles/m)); w —waviness (-). Figure 8 shows the PSD in the function of spatial frequency for different road roughness conditions.

ISO 8608 presented a classification of road roughness according to different unevenness index values and for one constant waviness value $w = 2$. The unevenness index denoted road roughness quality, and defined the ordinate position of the PSD function (Fig. 8). The value of the unevenness index for a road in good condition (A/B - ISO level class) is $32 \cdot 10^6 \text{ m}^2/(\text{cycles/m})$.² Although the PSDs in ISO 8608 were defined for standard constant waviness value, measurements in the United States, Germany, and Sweden revealed that waviness parameter frequently has a value in a rather wide range $1.5 \div 3.5$.^{3,15}

The connection between the temporal and spatial frequency as well as the corresponding PSDs can be represented by

Figure 8. PSD for different road roughness quality and constant waviness parameter value ($w = 2$).

Table 2. Bus IK-301 oscillatory model parameters used in simulation.

Geometric parameters of the bus	Values
Wheelbase l (m)	5.65
Distance from front axle to centre of gravity of loaded/empty bus a (m)	3.55/3.70
Distance from rear axle to centre of gravity of loaded/empty bus b (m)	2.10/1.95
Distance from passenger1's seat to centre of gravity d_1 (m)	4.60
Distance from passenger2's seat to centre of gravity d_2 (m)	0.90
Distance from passenger3's seat to centre of gravity d_3 (m)	4.98
Distance from passenger's seat above rear axle to centre of gravity of empty bus b (m)	1.95
Mass parameters of the bus	Values
Sprung mass of fully loaded/empty bus m (kg)	15399/9399.4
Front axle mass m_{t1} (kg)	745.6
Rear axle mass m_{t2} (kg)	1355
Suspended mass moment of inertia of fully loaded/empty bus relative to the y -axis J_y (kgm²)	150000/115000
Oscillatory parameters of the bus	Values
Front axle single air bag stiffness k_{s1} (N/m)	175000
Front axle equivalent air bags stiffness k_f (N/m)	350000
Rear axle single air bag stiffness k_{s2} (N/m)	200000
Rear axle equivalent air bags stiffness k_r (N/m)	800000
Single tyre stiffness on the front/rear axle k_{t1} (N/m)	1000000
Equivalent tyre stiffness on the front axle k_{ft} (N/m)	2000000
Equivalent tyre stiffness on the rear axle k_{rt} (N/m)	4000000
Single tyre damping on the front/rear axle c_{t1} (Ns/m)	150
Equivalent tyre damping on the front axle c_{ft} (Ns/m)	300
Equivalent tyre damping on the rear axle c_{rt} (Ns/m)	600

Eqs. (18) and (19):

$$f = nV; \quad (18)$$

$$\Phi_\zeta(f) = \frac{1}{V} \Phi_\zeta(n); \quad (19)$$

where f —temporal frequency of excitation (Hz); V —vehicle constant speed (m/s). The PSD in the function of temporal frequency of excitation can be expressed by Eq. (20):

$$\Phi_\zeta(f) = V^{w-1} \Phi_\zeta(n_0) \cdot \left(\frac{f}{n_0} \right)^{-w}; \quad (20)$$

where $\Phi_\zeta(f)$ —PSD of road roughness (m^2/Hz).

Figure 9 shows PSDs in the function of temporal frequency for different waviness values and for a constant vehicle speed of 100 km/h. The PSDs were shown for the temporal frequency range 0.5 – 50 Hz, since these mostly affected the vehicle's

Table 3. Parameters for seat-human model.

Seat-human model parameters	Values
Head and neck m_{hi} , ($i = 1 \div 3$) (kg)	5.31
Chest and upper torso m_{ci} , ($i = 1 \div 3$) (kg)	28.49
Lower torso m_{li} , ($i = 1 \div 3$) (kg)	8.62
Thighs and pelvis m_{pi} , ($i = 1 \div 3$) (kg)	12.78
Cervical spine damping c_1 (Ns/m)	400
Thoracic spine damping c_2 (Ns/m)	4750
Lumbar spine damping c_3 (Ns/m)	4585
Buttocks damping c_4 (Ns/m)	2064
Seat cushion damping c_{sc} (Ns/m)	220
Equivalent damping c'_4 (Ns/m)	198.81
Cervical spine stiffness k_1 (N/m)	310000
Thoracic spine stiffness k_2 (N/m)	183000
Lumbar spine stiffness k_3 (N/m)	162800
Buttocks stiffness k_4 (N/m)	90000
Seat cushion stiffness k_{sc} (N/m)	20000
Equivalent stiffness k'_4 (N/m)	16364

Table 4. Values of statistical parameters for vertical accelerations from measurements and simulation.

Statistical parameters	Bus floor (below pass. seat)		Passenger's seat	
	Measurement	Simulation	Measurement	Simulation
Maximum value	3.35	1.3606	1.80	1.2581
Minimum value	-3.69	-2.1108	-1.50	-1.4469
Mean value	0.04	-0.0040	0.00	-0.0052
Dispersion	0.32	0.2664	0.45	0.2526

oscillatory movement.¹⁶ The unevenness index $\Phi_\zeta(f_0)$ in the function of the reference temporal frequency f_0 is denoted in Fig. 9.

Values of the important parameters taken for longitudinal road roughness generation are shown in the Table 5.

3.1. Road Roughness Modelling

Road roughness in good condition for a vehicle speed of 100 km/h was modelled by PSD for A/B road class in ISO 8608. Single-track road roughness can be represented by an infinite sum of harmonic cosine functions of different amplitudes, circular frequencies, and phase angles. Sekulic et al. presented a detailed description of the road modelling process.¹⁷

Table 5. Parameters for longitudinal road roughness generation.

Reference frequency	Unevenness index		Waviness parameter
	n_0 (c/m)	f_0 (Hz)	
		$\Phi_\zeta(n_0)$ ($\text{m}^2/(\text{c/m})$)	$\Phi_\zeta(f_0)$ (m^2/Hz)
0.1	2.7778	$32 \cdot 10^{-6}$	$1.152 \cdot 10^{-6}$
			w (-)
			1.5; 2.0; 2.5; 3.0; 3.5

Figure 9. PSD for A/B road class in function of temporal frequency and different value of waviness w .

Figure 10 shows longitudinal road roughness in good condition for bus speed of 100 km/h in function of waviness w . The amplitude of roughness increased with the increase in the parameter w (Fig. 10). For a waviness value of 1.54, the road as a whole looked flat, but had pronounced short wavelengths with amplitudes in the range ± 0.010 m. On the other hand, a road with a high waviness value of 3.5 was uneven with maximum roughness amplitudes in the range from -0.04 m to $+0.03$ m. However, such road unevenness on short wavelengths were insignificant.

4. STANDARD ISO 2631/1997

The international standard ISO 2631/1997 “Mechanical vibration and shock—Evaluation of human exposure to whole-body vibration” described the methods to measure, quantify, and evaluate the effect of random vibrations on human health, comfort, perception, and motion sickness.¹⁸ This standard was applied to the vibrations acting on the human body in the standing/seated/recumbent positions.

The paper analysed the effect of vertical vibrations on the ride comfort of three passengers for two receiving points (seat and bus floor under the passenger’s seat). The relevant value for ride comfort assessing, according to ISO 2631/1997, was the overall vibration total value (Eq. (21)).

$$\ddot{z}_{w,RMS-total} = ((k_{z,seat} \cdot \ddot{z}_{w,RMS-seat})^2 + (k_{z,floor} \cdot \ddot{z}_{w,RMS-floor})^2)^{1/2}; \quad (21)$$

where $\ddot{z}_{w,RMS-total}$ was the overall RMS total value of the weighted vertical acceleration; $\ddot{z}_{w,RMS-seat}$ was the RMS of the weighted vertical acceleration acting on the whole body from the seat; $\ddot{z}_{w,RMS-floor}$ was the RMS value of the weighted vertical acceleration acting on the passenger’s feet from bus floor below the seat; $k_{z,seat}$, $k_{z,floor}$ were the multiplying factors, which values were 1 and 0.4, respectively.

RMS values of the weighted vertical accelerations ($\ddot{z}_{w,RMS-seat}$, $\ddot{z}_{w,RMS-floor}$) were calculated by Eqs. (22) and (23):

$$\ddot{z}_{w,RMS-seat} = \sqrt{\frac{1}{T} \int_0^T \ddot{z}_{w-seat}^2(t) dt}; \quad (22)$$

Table 6. Oscillatory comfort criteria in public means of transport.

Vibration intensity [m/s^2]	Comfort assessment
< 0.315	not uncomfortable
$0.315 - 0.63$	a little uncomfortable
$0.5 - 1.0$	fairly uncomfortable
$0.8 - 1.6$	uncomfortable
$1.25 - 2.5$	very uncomfortable
> 2.0	extremely uncomfortable

$$\ddot{z}_{w,RMS-floor} = \sqrt{\frac{1}{T} \int_0^T \ddot{z}_{w-floor}^2(t) dt}; \quad (23)$$

where $\ddot{z}_{w-seat}(t)$, $\ddot{z}_{w-floor}(t)$ were frequency weighted vertical accelerations for the seat and floor, respectively; and T was the simulation time.

Standard ISO 2631/1997 defined filter W_k for weighting of vertical accelerations acting on human body in seated position and on legs (Fig. 11). The analytical expressions for filter W_k can be found in annex A of this standard. According to these expressions, the function for acceleration weighting was defined in Matlab software.

Table 6 presents oscillatory comfort criteria in public means of transport according to ISO 2631/1997.

5. SIMULATOIN RESULTS AND DISCUSSION

In the paper, vertical acceleration of mass m_{tpi} , ($i = 1 \div 3$) were analysed since these accelerations presented an indicator of passengers exposure to WBV coming from the seats (Fig. 12(a,b,c)). The acceleration amplitude for each passenger increased with an increase in parameter w . Passengers in the rear part of the bus (passenger 3) had a greater value of vertical acceleration for $w = 3.5$ (peak minimum/maximum values approximately amount to $\pm 4 m/s^2$).

The oscillatory comfort was also influenced by the vibrations acting on the passengers’ legs from the bus floor. Acceleration amplitude from bus floor increased with an increase in parameter w (Fig. 12(d,e,f)). The greater values were for the rear part of the bus (peak minimum/maximum values approximately amounted to $\pm 3 m/s^2$ for $w = 3.5$).

Figure 13 shows values of equivalent damping forces for the front/rear axle used in the simulation. Max/min values of relative piston velocity was around $\pm 0.5 m/s^2$ for both bus axles.

Figure 10. Roughness of A/B road class according to ISO 8608 a) in function of time, b) effective values, for different parameter w .

Figure 11. Filter W_k for vertical acceleration weighting.

5.1. Analysis of Constrained Forces Between Passengers' Bodies and Their Seats

The constrained force between passenger body and the seat was comprised of a static and dynamic part, as follows:

$$\begin{aligned} F(t) &= F_{st} + F_{dyn}(t) \\ &= (m_{hi} + m_{ci} + m_{ltpi} + m_{tpi}) + k'_4(z_j - z_{tpi}); \\ &\quad (i = 1 \div 3; \quad j = A, B, C); \end{aligned} \quad (24)$$

where F_{st} was a static force (N), and $F_{dyn}(t)$ was a dynamic force (N). The dynamic part could have positive and negative values. Total constrained force was equal to zero when $F_{dyn}(t) = -F_{st}$ and there had been a loss of contact between the passenger body and the seat. Figure 14 shows constrained forces in function of time for every passenger. The total forces had positive values, meaning that contact had not been lost. Total forces were of the greatest values for the waviness parameter $w = 3.5$. Static force has also been denoted in the Fig. 14.

Figure 15(a) comparatively shows road roughness excitation for waviness parameter value $w = 3.5$ and two bus speeds (speed of 100 km/h and a maximum speed of 130 km/h). Figure 15(b-c) shows the constrained forces for the bus passengers. Although the magnitudes of the total forces for the

maximum bus speed noticeably changed compared to those for a speed of 100 km/h, forces were still positive values (Fig. 15(c)). For the maximum bus speed and road roughness in good condition, passenger bodies were in permanent contact with their seats. Permanent contact between passengers' bodies and their seats was a prerequisite for using the differential equations of motion (Eqs. (1–10)) for the defined bus oscillatory model.

5.2. Analysis of Passengers' Oscillatory Comfort

Table 7 shows *RMS* values of raw vertical acceleration for bus floor and passengers' seats in the function of parameter w . The *RMS* acceleration values for every seat increased with an increase in parameter w . The *RMS* acceleration values for bus floor first decreased, and then increased with the increase in parameter w . The *RMS* acceleration values for seats were greater compared to the bus floor for $w = 2.5 \div 3.5$.

Figure 16 shows PSDs of vertical accelerations acting on passengers' bodies. The PSDs were obtained according to the Welch method implemented by the "pwelch" function in the Matlab Signal Processing Toolbox.¹⁹ Intensities of PSDs increased with an increase in parameter w . Passenger 3 had the greatest PSD value, approximately to 1.6 (m/s²)²/Hz for

Figure 12. Vertical acceleration for a) passenger 1's seat, b) passenger 2's seat, c) passenger 3's seat, d) floor below passenger 1's seat, e) floor below passenger 2's seat, f) floor below passenger 3's seat, in function of parameter w .

Figure 13. Equivalent damping forces used in simulation for a) front, b) rear bus axle.

$w = 3.5$. Vibration intensities were distributed in the frequency range $0 \div 4$ Hz. The maximum values of PSDs for all passengers were at 1.2 Hz. The seats attenuated vertical vibrations at frequencies above 4 Hz.

Figure 17 shows the PSDs of vertical accelerations from the bus floor below the passengers' seats. Vibrational intensities were mainly distributed into two frequency ranges, $0 \div 2.5$ Hz and $5 \div 10$ Hz. The higher vibrational intensities corresponded to a lower frequency range. The intensities of the vertical acceleration increased in the lower frequency range, whereas in the higher frequency range intensities decreased with an increase in parameter w .

Figure 18(a) shows RMS values of the weighted vertical acceleration for passengers' seats and bus floor in function of parameter w . The RMS values for the seats increased with an increase in parameter w . The greater RMS values corresponded to parameter $w = 3.5$, and the lowest to $w = 1.5$. On the other hand, the RMS values from bus floor decreased with an increase in parameter w . The lowest RMS values corresponded to waviness $w = 3$, and the greatest to waviness $w = 1.5$.

Figure 18(b) shows the change in the total RMS value of

the weighted vertical acceleration in function of parameter w . For every passenger, the total RMS value increased with an increase in parameter w . For passenger 1, the RMS value was greater than 0.5 m/s^2 for the parameter w above 2.5. For passenger 2, the RMS value was greater than 0.315 m/s^2 for the parameter w above 2.7. For passenger 3, the RMS value was greater than 0.8 m/s^2 for the parameter w above 3.3.

Table 8 shows total RMS values and comfort assessment for every passenger in function of parameter w . Passenger 2 had the best comfort, and the worst was passenger 3. For the unevenness parameter values of $w = 3, 3.5$, comfort for passenger 2 corresponded to a 'little uncomfortable'. The comfort assessment for passenger 3 for $w = 3.5$ corresponded to 'uncomfortable'. The passenger in the front part of the bus had better oscillatory comfort than the passenger in the rear part of the bus.

These results indicated that the waviness parameter w affected the oscillatory comfort of the bus users. This fact should be considered when designing buses, and especially oscillatory parameters of the systems that have significant impact on the bus ride comfort (e.g. elastic bus suspension system, passenger seat cushion, etc.).

Figure 14. Constrained forces between passenger's body and the seat in function of time for a) passenger 1, b) passenger 2, c) passenger 3.

Figure 15. Change in a) road roughness in good condition for different bus speeds, b) constrained forces for speed of 100 km/h, c) constrained forces for speed of 130 km/h.

Figure 19(a) shows the change in parameter $SEAT_{RMS}$ in the function of the parameter w for the bus passengers. The parameter $SEAT_{RMS}$ indicated an isolation characteristic of the seats equipped with different suspension systems when excited with the same input signals.²⁰ Although all passenger seats have the same seat cushion oscillatory parameters, the bus floor excitations are not the same. In this paper, the parameter $SEAT_{RMS}$ assessed the isolation characteristics of the users' seats in the function of their position on the bus platform.

$SEAT_{RMS}$ parameter can be calculated by following Eq. (25):

$$SEAT_{RMS} [\%] = \frac{\ddot{z}_{RMS}(w, seat)}{\ddot{z}_{RMS}(w, floor)} \cdot 100 = \frac{\left(\frac{1}{T} \int_0^T \ddot{z}_{w, seat}^2(t) dt \right)^{1/2}}{\left(\frac{1}{T} \int_0^T \ddot{z}_{w, floor}^2(t) dt \right)^{1/2}} \cdot 100; \quad (25)$$

where $\ddot{z}_{RMS}(w, seat)$ and $\ddot{z}_{RMS}(w, floor)$ were RMS values of the weighted vertical accelerations on the seat and the bus floor below the seat.

A $SEAT_{RMS}$ value greater than 100% indicates that the seat amplified bus floor vibrations. If the $SEAT_{RMS}$ value was equal to 100%, the seat did not isolate floor vibration. The

seat attenuated some vibration if $SEAT_{RMS}$ value was less than 100%.²¹

As the parameter w increased, the value of the $SEAT_{RMS}$ parameter increased. The passenger seats attenuated less vibration with the increase in the parameter w . Passenger 2 had the lowest values of the $SEAT_{RMS}$. The seat increased the intensities of vertical vibrations for the passenger 1, passenger 2 and passenger 3 for the values of parameters w greater than 2.7, 2.8 and 2.4, respectively.

Figure 19(b) shows change in percentage differences PD_{RMS_total} (%) for passengers in the function of parameter w , according to Eq. (26):

$$PD_{RMS_total} [\%] = \frac{\ddot{z}_{w, RMS_total}(W) - \ddot{z}_{w, RMS_total}(W=2)}{\ddot{z}_{w, RMS_total}(W=2)} \cdot 100; \quad (26)$$

where $\ddot{z}_{w, RMS_total}(W)$ was the total RMS value of the weighted vertical acceleration in the function of parameter w ; $\ddot{z}_{w, RMS_total}(W=2)$ was the total RMS value of the weighted vertical acceleration for the standard value of the waviness parameter ($w = 2$).

The percentage differences between the RMS value for the waviness $w = 1.5 \div 2.3$ and RMS value for nominal waviness $w = 2$ was lower than 5% for each passenger. This indicated

Table 7. *RMS* of raw vertical acceleration for bus floor and passengers' seat in function of parameter w .

Waviness parameter		$w = 1.5$	$w = 2$	$w = 2.5$	$w = 3$	$w = 3.5$
<i>RMS</i> value of raw vertical acceleration (m/s^2)	Floor below passenger 1 seat	0.737	0.706	0.734	0.820	0.975
	Passenger 1—mass m_{tp1}	0.662	0.723	0.822	0.959	1.149
	Floor below passenger 2 seat	0.459	0.446	0.471	0.535	0.647
	Passenger 2—mass m_{tp2}	0.392	0.444	0.525	0.629	0.776
	Floor below passenger 3 seat	0.967	0.916	0.946	1.049	1.224
	Passenger 3—mass m_{tp3}	0.961	1.025	1.129	1.278	1.485

Figure 16. PSDs of the seat vertical acceleration for a) passenger 1, b) passenger 2, c) passenger 3, in function of parameter w .

that the parameter w in the specified narrow range for A/B road class had similar effect on passengers' comfort as for $w = 2$. The standard value of the parameter $w = 2$ was precise enough for estimating the comfort of the bus users.

6. CONCLUSIONS

In this paper, bus passengers' ride comfort is analysed in the function of the waviness parameter w from range $1.5 \div 3.5$. Vertical acceleration amplitude on the passengers' seats as well as that on the bus floor under the seat increase with the increase in parameter w . The intensities of vertical acceleration for passengers' seats are concentrated in the frequency range $0 \div 4$ Hz. The acceleration intensities from the bus floor are mainly distributed into two frequency ranges, $0 \div 2.5$ Hz and $5 \div 10$ Hz, with higher acceleration intensities in the lower frequency range.

The *RMS* values of the weighted vertical acceleration from seats increase with the increase in the parameter w , and reach the greatest values for $w = 3.5$. On the other hand, the *RMS* values from the bus floor decrease, and reach the lowest values for $w = 3$.

The total *RMS* values of the weighted vertical acceleration increase with the increase in the parameter w . The passenger in the rear part of the bus has the worst comfort level, whereas passenger in the middle part has the best. The comfort assessment for the passenger in the bus's middle part for the unevenness parameter values $w = 3, 3.5$ is 'a little uncomfortable'. The passenger in the rear part of the bus has the greatest total *RMS* value of 0.835 m/s^2 for parameter $w = 3.5$, which corresponds to an 'uncomfortable' comfort assessment.

As the parameter w increases, *SEAT_{RMS}* parameter value increases. Passenger seats attenuate less vibration with the in-

crease in the parameter w . The lowest values of the parameter *SEAT_{RMS}* have a passenger in the bus's middle part. The percentage differences between the *RMS* of the weighted vertical acceleration for the parameter w in the range $1.5 \div 2.3$, and the standard value $w = 2$ is less than 5% for each passenger. This indicates that the parameter w in the specified narrow range has similar effect on passengers' comfort as for $w = 2$.

Although many real road profiles have a non-standard PSD, in the vehicle industry standard PSD with waviness $w = 2$ is often used for simulation in the process of dynamic vehicle behaviour investigation. The results of the analysis indicate that wavelengths distribution of the longitudinal road roughness affects oscillatory comfort of the bus users. This fact should be considered when designing buses, and especially the oscillatory parameters of the systems that have significant impact on the bus ride comfort (e.g. elastic bus suspension system, passenger seat cushion, etc.).

The developed oscillatory model in this paper could be used for an investigation of oscillatory comfort not only for intercity buses, but also for the other types of the buses (e.g. city/intercity/tourist buses). Apart from the oscillatory comfort, this model could be useful for analysis of different problems in the area of the bus vertical dynamics, such as analysis of pavement load, deformation of the bus suspension system, and the like.

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Figure 17. PSDs of the floor vertical accelerations below a) passenger 1's seat, b) passenger 2's seat, c) passenger 3's seat, in function of parameter w .

Figure 18. Change in a) RMS value of the weighted vertical acceleration for passengers' seats and bus floor, b) total RMS value of the weighted vertical acceleration, in function of parameter w .

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Table 8. Comfort assessment for the bus passengers in function of parameter w .

Waviness parameter		$w = 1.5$	$w = 2$	$w = 2.5$	$w = 3$	$w = 3.5$
Passenger 1	Total RMS value (m/s^2)	0.449	0.459	0.495	0.556	0.648
	Comfort assessment	a little uncomfortable	a little uncomfortable	a little uncomfortable	fairly uncomfortable	fairly uncomfortable
Passenger 2	Total RMS value (m/s^2)	0.263	0.271	0.299	0.343	0.411
	Comfort assessment	not uncomfortable	not uncomfortable	not uncomfortable	a little uncomfortable	a little uncomfortable
Passenger 3	Total RMS value (m/s^2)	0.643	0.640	0.673	0.736	0.835
	Comfort assessment	fairly uncomfortable	fairly uncomfortable	fairly uncomfortable	fairly uncomfortable	uncomfortable

Figure 19. Change in a) parameter $SEAT_{RMS}$, b) percentage differences, for bus passengers in function of parameter w .

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