Optimum Tuning of a Gyroscopic Vibration Absorber for Vibration Control of a Vertical Cantilever Beam with Tip Mass

F. Ünker

Department of Mechanical Engineering, Gümüşhane University, 29100 Gümüşhane, Turkey.

O. Çuvalcı

Department of Mechanical Engineering, Karadeniz Technical University, 61080 Trabzon, Turkey.

(Received 8 October 2015; accepted 26 August 2016)

In this paper, a symmetric gyro with coupling to the tip mass of a beam mounted on a vibrating base is considered. Taking advantage of the angular momentum of the rotating gyroscope, gyrostabilizer systems are expected to become more widely actualized as they provide an effective means of motion control with several significant advantages for various structures. This paper mainly focused on finding optimized stiffness and damping values that minimize the vibration responses with the derivation of the frequency equations of a special combined gyro-beam system. Correctness of the analytical results is verified by numerical simulations. The comparison with the results from the derivation of the corresponding frequency equations shows that the optimized stiffness and damping values are very accurate.

1. INTRODUCTION

Beams carrying a tip mass with (or without) rotary inertia can find application in the modeling of many engineering fields such as tall buildings, towers, piles, antennas, robotic manipulators, appendages of aircrafts, spacecrafts or vehicles, etc. Hence, there has been a lot of research work on the nonlinear dynamics of such beams subject to various boundary and load conditions over the past few decades due to both theoretic and practical demands. In some previous studies, a Euler-Bernoulli beam model has been used to investigate the problem of a vertical cantilever beam having a concentrated mass and various springs along the beam.¹⁻³

Currently, a number of researchers studied and tried to solve the vibration problem due to low stiffness of the lightweight and flexible structures by applying one or more passive dynamic vibration absorbers. The most widely used passive vibration absorbers in engineering are tuned mass absorbers, beam absorbers, pendulum-type absorbers, and liquid absorbers. Cuvalcı investigated the dynamic behavior of a beam-tip mass-pendulum system subjected to a periodic excitation in the presence of autoparametric interaction between the beam and pendulum.⁴ Ertas et al. investigated experimentally an orientable flexible beam with a tip mass and pendulum.⁵ The passive vibration absorbers can be quite effective and desirable due to their simplicity and low cost; but, since passive vibration absorbers have their limitations, one needs to develop the active vibration absorbers. The passive control method is activated by the structural motion without requiring external force or energy to reduce structural responses and utilizes the motion of the structure at the location of the device.⁶ The active control method requires a considerable amount of external power to operate actuators that supply a control force to the structure. Due to requiring sensors and controller equipments, active control devices are more complex, and they do not have the reliability and robustness compared to passive control techniques.⁷

In recent years, several researchers have conducted studies

investigating the complicated motions which appear in a gyrostabilizer, of which there are various types and arrangements. One engineering application of the gyroscopic effect is as an effective means of motion control in many situations, for example stabilization of bicycles, cars, monorails, building wind induced vibrations, and ships.⁸⁻¹² Compared to conventional active mass dampers for wind vibration suppression, gyrostabilizers represent a weight and volume saving. The mechanism does not require any other external source of energy, in which the rotor speed is produced by an electric motor in a rotating gimbal. Therefore, this can be classified as a passive control device in a variation of passive vibration control systems. The gyrostabilizer is effective for bending moments rather than shear forces because the gyrostabilizer utilizes the gyroscopic moment to reduce or eliminate the undesirable motions. This paper considers the motion of a symmetric gyro with coupling to the free end of a beam subjected to a harmonic base excitation, which is supported with a torsional spring and damper. The optimal values of the rotor speed, torsional spring, and damper are theoretically analyzed and derived for a cantilever beam. The relations for the optimal values of the gyrostabilizer are so tuned that this system is more adaptable and has a smaller mass than other conventional control devices with the same ability to be employed for vibration control.

2. EQUATIONS OF MOTION OF THE GYRO-BEAM SYSTEM FOR SMALL VIBRATION

Figure 1 shows the beam as a vertical cantilever of length L with an end mass M_t to which an additional gyro system was attached at the free end and subjected to a harmonic base excitation at the other end of the beam. The horizontal displacement of base was z . The beam was assumed to be initially straight, of length L . The horizontal and vertical elastic displacements at the free end were v and u , respectively. Due to elastic deformation of the beam, s represented the distance along the arc-length of the beam.

The gyro system consisted of a disk mass m , which can

Figure 1. Cantilever model with end mass, gyroscope, and horizontal base excitation.

Table 1. Physical properties of the system.

Symbol	Numerical values	Description
E	$17 \times 10^9 \text{ N/m}^2$	Young's modulus
h	200 mm	beam thickness
b	300 mm	beam width
L	5 m	length of the beam
ρ	2300 kg/m ³	density of the beam
M_t	2000 kg	tip mass
I_t/M_t	0.2 m ²	ratio of tip mass moment of inertia
I_t	400 kg·m ²	tip mass moment of inertia
I	$2 \times 10^{-4} \text{ m}^4$	geometrical moment of inertia
g	9.81 m/s ²	gravitational acceleration
$A = b \cdot h$	0.06 m ²	area of cross section
c	80–160 N·m·s/rad	damping coefficient
k	$0.25I_o\omega_n^2 - 4I_o\omega_n^2$	stiffness of torsion spring
m	100 kg	disk mass of gyroscope
r	0.25 m	radius of disk
$I_p = m \cdot r^2/2$	3.125 kg·m ²	rotary inertia of disk
$I_o = I_p/2$	1.5625 kg·m ²	mass moment of inertia of disk
Ω	0–20000 rpm	rotating speed of disk

spin freely about its geometric axis (gimbal) mounted to the tip mass of the beam as shown in Fig. 2. The disk mass, m of gyro at the free end was assumed to have angular speed Ω , and the rotation of the gimbal of this disk was also resisted by torsional spring and damping torque defined by $k\theta$ and $c\dot{\theta}$, respectively.

Equations of the beam and gyro have been expressed in Ünker and Çuvalcı (see Eqs. (16) and (17)) as Eq. (1) and Eq. (2).¹³

After the rearrangement, Eq. (1) became Eq. (3) in which:

Figure 2. Gyro model with end mass M_t , massless torsional spring and damper.

$$\begin{aligned}
 \alpha_1 &= \rho A G_1 + m + M_t + I_t G_5^2; \\
 \alpha_2 &= \rho A G_3 + (m + M_t) G_4^2 + I_t G_5^4; \\
 \alpha_3 &= \frac{1}{4} I_t G_5^6; \\
 \alpha_4 &= E I G_6 - \rho A g G_9 - (m + M_t) g G_4; \\
 \alpha_5 &= 2 E I G_7; \\
 \alpha_6 &= \frac{3}{4} E I G_8; \\
 \gamma &= \rho A G_2 + m + M_t;
 \end{aligned} \quad (4)$$

where the constants from G_1 to G_9 are given in Table 2.

Neglecting the terms of higher power for small vibrations ($v \approx 0$), Eqs. (2) and (3) were reduced to:

$$I_0 \ddot{\theta} - \frac{1}{2} (I_p - I_o) (\dot{v} G_5)^2 \sin 2\theta - I_p \Omega \dot{v} G_5 \cos \theta + c \dot{\theta} + k \theta = 0; \quad (5)$$

$$\begin{aligned}
 &[\alpha_1 + G_5^2 (I_o \cos^2 \theta + I_p \sin^2 \theta)] \ddot{v} \\
 &+ [\alpha_2 + G_5^4 (I_o \cos^2 \theta + I_p \sin^2 \theta)] v \dot{v} + \alpha_4 v \\
 &+ G_5 [G_5 (I_p - I_o) \dot{v} \theta \sin 2\theta + I_p \Omega \dot{\theta} \cos \theta] = -\gamma \ddot{z}. \quad (6)
 \end{aligned}$$

3. OPTIMAL TUNING ABOUT THE EQUILIBRIUM POSITION

The easiest way to approach this problem was from the point of view of energy. We chose the kinetic to be zero when the tip mass was at the bottom of its swing. Setting $\dot{v} \approx 0$ and $\dot{\theta} \approx 0$ about the equilibrium position, Eqs. (5) and (6) reduced to:

$$I_o \ddot{\theta} - I_p \Omega \dot{v} G_5 \cos \theta + c \dot{\theta} + k \theta = 0; \quad (7)$$

$$\begin{aligned}
 &[\alpha_1 + G_5^2 (I_o \cos^2 \theta + I_p \sin^2 \theta)] \ddot{v} \\
 &+ \alpha_4 v + G_5 I_p \Omega \theta \cos \theta = -\gamma \ddot{z}. \quad (8)
 \end{aligned}$$

The natural frequencies of gyro and beam were determined as follows, respectively:

$$\omega_g^2 = \frac{k}{I_o}; \quad (9)$$

$$\omega_n^2 = \frac{\alpha_4}{\alpha_1 + G_5^2 (I_o \cos^2 \theta + I_p \sin^2 \theta)}. \quad (10)$$

$$\left\{ \begin{array}{l} \rho A G_1 + m + M_t + I_t G_5^2 + \\ [\rho A G_3 + (m + M_t) G_4^2 + I_t G_5^4] v^2 \\ + \frac{1}{4} I_t G_5^6 v^4 \end{array} \right\} \ddot{v} + \left[\rho A G_3 + (m + M_t) G_4^2 + I_t G_5^4 + \frac{1}{2} I_t G_5^6 v^2 \right] v \dot{v}^2 + \left[E I G_6 - \rho A g G_9 - (m + M_t) g G_4 + 2 E I G_7 v^2 + \frac{3}{4} E I G_8 v^4 \right] v + \left(G_5 + \frac{1}{2} v^2 G_5^3 \right) \left[\begin{array}{l} (I_p - I_o) \left(\dot{v} G_5 + \frac{1}{2} v^2 \dot{v} G_5^3 \right) \dot{\theta} \sin 2\theta \\ + I_o v \dot{v}^2 G_5^3 \cos^2 \theta \\ + I_p v \dot{v}^2 G_5^3 \sin^2 \theta \\ + I_p \Omega \dot{\theta} \cos \theta \end{array} \right] = -(\rho A G_2 + m + M_t) \ddot{z}; \quad (1)$$

$$I_o \ddot{\theta} - \frac{1}{2} (I_p - I_o) \left(\dot{v} G_5 + \frac{1}{2} v^2 \dot{v} G_5^3 \right)^2 \sin 2\theta - I_p \Omega \left(\dot{v} G_5 + \frac{1}{2} v^2 \dot{v} G_5^3 \right) \cos \theta + c \dot{\theta} + k \theta = 0; \quad (2)$$

$$\left[\alpha_1 + \alpha_2 v^2 + \alpha_3 v^4 + \left(G_5 + \frac{1}{2} v^2 G_5^3 \right)^2 (I_o \cos^2 \theta + I_p \sin^2 \theta) \right] \ddot{v} + \left[\alpha_2 + 2 \alpha_3 v^2 + G_5^3 \left(G_5 + \frac{1}{2} v^2 G_5^3 \right) (I_o \cos^2 \theta + I_p \sin^2 \theta) \right] v \dot{v}^2 + (\alpha_4 + \alpha_5 v^2 + \alpha_6 v^4) v + \left(G_5 + \frac{1}{2} v^2 G_5^3 \right) \left[(I_p - I_o) \dot{v} \left(G_5 + \frac{1}{2} v^2 G_5^3 \right) \dot{\theta} \sin 2\theta + I_p \Omega \dot{\theta} \cos \theta \right] = -\gamma \ddot{z}. \quad (3)$$

Table 2. Constants of the equation of motion of the gyro-column system in SI units.

G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9
$= L \left(\frac{3}{2} - \frac{4}{\pi} \right)$	$= \frac{L}{\pi} (\pi - 2)$	$= \frac{\pi^2}{384L} (2\pi^2 - 9)$	$= \frac{\pi^2}{8L}$	$= \frac{\pi}{2L}$	$= \frac{\pi^4}{32L^3}$	$= \frac{\pi^6}{512L^5}$	$= \frac{\pi^8}{4096L^7}$	$= \frac{1}{16} (\pi^2 - 4)$
1.1338	1.8169	0.0552	0.2467	0.3142	0.0244	6.0087e-4	2.9652e-5	0.3669

Assuming a harmonic disturbing base excitation $z = \bar{z} e^{j\omega t}$, the responses were written as angular frequency ω :

$$\theta = \bar{\theta} e^{j\omega t} \quad \text{and} \quad v = \bar{v} e^{j\omega t}. \quad (11)$$

Therefore, Eqs. (7) and (8) were rewritten in the matrix form:

$$\bar{\theta} \begin{bmatrix} j\omega G_5 I_p \Omega \cos \theta \\ -I_o \omega^2 + j c \omega + k \end{bmatrix} + \bar{v} \begin{bmatrix} -\omega^2 [\alpha_1 + G_5^2 (I_o \cos^2 \theta + I_p \sin^2 \theta)] + \alpha_4 \\ -j\omega I_p \Omega G_5 \cos \theta \end{bmatrix} = \begin{bmatrix} \omega^2 \gamma \bar{z} \\ 0 \end{bmatrix}. \quad (12)$$

Solving these equations yielded Eqs. (13) and (14). The complex Eqs. (13) and (14) were transformed to the form of Eqs. (15) and (16)

3.1. The Undamped Gyroscopic Vibration Absorber

In order to reach the minimum tip mass displacement $|\bar{v}|$, the natural frequency of the added gyro-spring system itself should have been the same as the excitation frequency ($\omega^2 = \frac{k}{I_o} = \omega_g^2$), and by eliminating term of damping, Eq. (16) was then:

$$|\bar{\theta}| = \gamma \bar{z} \omega^3 \sqrt{\frac{(I_p \Omega G_5 \cos \theta)^2}{[\omega^2 (I_p \Omega G_5 \cos \theta)^2]^2}}. \quad (17)$$

Autoparametric resonance occurred when ($\omega^2 = \omega_n^2 = \omega_g^2$). Upon assuming $\theta = \bar{\theta} e^{j\omega t} = \bar{\theta}_{max}$ for the equation of a stable motion at $\omega \leq \omega_n$, Eq. (17) corresponded to:

$$|\bar{\theta}_{max}| = \frac{\gamma \bar{z}_n \omega}{I_p \Omega G_5 \cos \bar{\theta}_{max}}. \quad (18)$$

The minimum disk speed Ω_{min} was chosen carefully to eliminate the instability at $\omega \leq \omega_n$. Since a minimum disk speed Ω_{min} is required for a stable motion, Eq. (18) was modified into the following form:

$$\bar{\theta}_{max} \cos \bar{\theta}_{max} = \pm \frac{\omega_n \gamma \bar{z}}{I_p G_5 \Omega_{min}}. \quad (19)$$

Supposing that a minimum disk speed was required for a stable motion, this transformed to a minimum disk speed equation:

$$\Omega_{min} = \pm \frac{\omega_n \gamma \bar{z}}{I_p G_5 \bar{\theta}_{max} \cos \bar{\theta}_{max}}. \quad (20)$$

Figure 3 shows that the theoretical limit of the amplitude at resonance is one when absolute value of the rotation $\bar{\theta}_{max}$ approaches 0.86 rad. Supposing that $|\bar{\theta}_{max} \cos \bar{\theta}_{max}| = 0.5611$ and $|\bar{\theta}_{max}| = 0.86$ rad for $-\pi/2 \leq \theta \leq \pi/2$, Eq. (20) was rewritten as:

$$\Omega_{min} = 1.7822 \frac{\omega_n \gamma \bar{z}}{I_p G_5}; \quad (21)$$

where Eq. (10) was then:

$$\omega_n = \sqrt{\frac{\alpha_4}{\alpha_1 + G_5^2 (0.426 I_o + 0.574 I_p)}}. \quad (22)$$

$$\bar{v} = \frac{-\omega^2 \gamma \bar{z} [(-I_o \omega^2 + k) + j c \omega]}{\left\{ \begin{array}{l} \omega^2 (I_p \Omega G_5 \cos \theta)^2 - (-I_o \omega^2 + k) \{-\omega^2 [\alpha_1 + G_5^2 (I_o \cos^2 \theta + I_p \sin^2 \theta)] + \alpha_4\} \\ -j c \omega \{-\omega^2 [\alpha_1 + G_5^2 (I_o \cos^2 \theta + I_p \sin^2 \theta)] + \alpha_4\} \end{array} \right\}}; \quad (13)$$

$$\bar{\theta} = \frac{-j \omega^3 \gamma \bar{z} I_p \Omega G_5 \cos \theta}{\left\{ \begin{array}{l} \omega^2 (I_p \Omega G_5 \cos \theta)^2 - (-I_o \omega^2 + k) \{-\omega^2 [\alpha_1 + G_5^2 (I_o \cos^2 \theta + I_p \sin^2 \theta)] + \alpha_4\} \\ -j c \omega \{-\omega^2 [\alpha_1 + G_5^2 (I_o \cos^2 \theta + I_p \sin^2 \theta)] + \alpha_4\} \end{array} \right\}}; \quad (14)$$

$$|\bar{v}| = \gamma \bar{z} \omega^2 \sqrt{\frac{(-I_o \omega^2 + k)^2 + (c \omega)^2}{\left\{ \begin{array}{l} \omega^2 (I_p \Omega G_5 \cos \theta)^2 - (-I_o \omega^2 + k) \{-\omega^2 [\alpha_1 + G_5^2 (I_o \cos^2 \theta + I_p \sin^2 \theta)] + \alpha_4\} \\ + \{-c \omega \{-\omega^2 [\alpha_1 + G_5^2 (I_o \cos^2 \theta + I_p \sin^2 \theta)] + \alpha_4\}\}^2 \end{array} \right\}^2}}; \quad (15)$$

$$|\bar{\theta}| = \gamma \bar{z} \omega^3 \sqrt{\frac{(I_p \Omega G_5 \cos \theta)^2}{\left\{ \begin{array}{l} \omega^2 (I_p \Omega G_5 \cos \theta)^2 - (-I_o \omega^2 + k) \{-\omega^2 [\alpha_1 + G_5^2 (I_o \cos^2 \theta + I_p \sin^2 \theta)] + \alpha_4\} \\ + \{-c \omega \{-\omega^2 [\alpha_1 + G_5^2 (I_o \cos^2 \theta + I_p \sin^2 \theta)] + \alpha_4\}\}^2 \end{array} \right\}^2}}. \quad (16)$$

Figure 3. The theoretical limit curves: natural frequency rad/s (top), gyro response rad (bottom).

Figure 4. The theoretical limit curves: minimum disk speed Ω_{min} rad/s (top), Optimal damper tuning Nms/rad (bottom).

3.2. The Damped Gyroscopic Vibration Absorber

With the help of Eq. (16), it was rewritten as Eq (23).

After setting $\omega^2 = \omega_n^2 = \omega_g^2$ and $\Omega = \Omega_{min}$ in order to reach the optimum tuning, Eq. (23) was expressed as Eq. (24) Since Eq. (24) was an undefined form, with the help of Eq. (14) and using L'Hospital's rule, optimum damping was obtained Eq. (25).

As seen from Fig. 4, minimum disk speed was generated at the angle of 0.86 rad, which achieved a stable motion of its swing. Throughout the next entire analysis, the angle

of 0.86 rad was preferred as an allowable maximum precession. Hence, setting $\bar{\theta}_{opt} = |\bar{\theta}_{max}| = 0.86$ rad and $\Omega_{min} = 1.7822 \frac{\omega_n \gamma \bar{z}}{I_p G_5}$ to eliminate the instability due to the natural frequency of the added gyroscope resulted in optimal damping coefficient. Figure 5 shows the possible maxima and minima of the theoretical frequency responses of a beam and gyroscope obtained from Eqs. (15) and (16), respectively. In the figure, it was fairly proven that optimal damping and spring coefficients do not allow the precession grow over 0.86 rad for a stable motion.

Figure 5. The analytical frequency response curves of tip of beam with $\bar{z} = 0.05$ m; for (dotted) $\Omega = \Omega_{min}$, $k = I_0\omega_n^2$ and $c = 0$, for (solid) $\Omega = \Omega_{min}$, $k = I_0\omega_n^2$ and $c = c_{opt}$, and for (dash-dot) $\Omega = 0$.

4. NUMERICAL SIMULATIONS

In the following calculations, a rectangle-cross beam was considered with thickness $h = 200$ mm, width $b = 300$ mm, length $L = 5$ m, density $\rho = 2300$ kg/m³, and Young's modulus along the axial direction $E = 17 \times 10^9$ N/m². Equations (1) and (2) were solved by using a MATLAB software tool that involves the fourth-order Runge-Kutta method. The parameters of the numerical example are given in Table 1. In order to identify the dynamical behavior, the time history was simulated with the time step size of 0.05 s and zero initial conditions.

4.1. The Undamped Gyroscopic Vibration Absorber

Figure 6 shows the rotational speed curves for cases: (blue) $0.25\omega_n^2 = \omega_g^2$, (green) $\omega_n^2 = \omega_g^2$, and (red) $\omega_n^2 = 0.25\omega_g^2$, respectively. The results presented in this figure all indicated that the gyro system had significantly reduced the vibration of a beam at the free end by increasing the rotating speed Ω of a disk of a gyro. The tip response of the beam for case (red) $\omega_n^2 = 0.25\omega_g^2$ was effectively reduced over minimum rotational speed (Ω_{min}). However, optimal performance was achieved for the case of (green) $\omega_n^2 = \omega_g^2$, which generated less displacement than the response of (red) $\omega_n^2 = 0.25\omega_g^2$ as seen from Fig. 7. Consequently, the case of (green) $\omega_n^2 = \omega_g^2$ was preferred as an optimum frequency rate, which agrees with the analytical solutions.

As the disk speed was increased, the location of the maximum frequency of the stable motion changed, as seen from Fig. 8. By the damping effect of the angular momentum of

Figure 6. Rotational speed response curves: tip response of beam (top), gyro response (bottom) for (blue) $0.25\omega_n^2 = \omega_g^2$, (green) $\omega_n^2 = \omega_g^2$, and (red) $\omega_n^2 = 0.25\omega_g^2$ with the base excitation of $z = 0.05 \cos(\omega_n t)$.

the gyro, the reachable frequency of a stable motion increased compared to the case for the minimum rotational speed in Fig. 7. However, the peak with low frequency was found to increase the amplitude of displacement as the disk speed was increased.

4.2. The Damped Gyroscopic Vibration Absorber

In the case of the base excitation with $z = 0.05 \cos(\omega t)$, Figs. 9 and 10 show frequency responses of the beam (top) and the gyro (bottom) at the free end for the damping coefficients of (∇) $c = 80$ N·m·s/rad, ($\square$) $c = 120$ N·m·s/rad, and ($\circ$) $c = 160$ N·m·s/rad, respectively. For different rotational speeds, it was seen that the case of $c = 80$ N·m·s/rad was approximately optimum damping coefficient obtained from Fig. 4 corresponding to θ_{opt} (0.86 rad). When damping coefficient was increased, the frequency response of the beam increased compared to the optimum damping $c = 80$ N·m·s/rad; however, the frequency response of the gyro for $c = 80$ N·m·s/rad had high values and the frequency response of the beam jumped into an unstable zone at low frequencies when the rotational speed was increased, as seen in Fig. 10. It may be fairly concluded and proven that optimizing the coefficients of damper and spring would be able to suppress the undesired motions of beam and gyro.

5. CONCLUSIONS

This paper provides some analytical equations to investigate the optimum damping and spring coefficient of a gyro-beam system. Optimum spring and damper parameters for minimum

$$c^2 = \frac{(\gamma \bar{z} \omega^3)^2 \frac{(I_p \Omega G_5 \cos \theta)^2}{(\bar{\theta})^2} - \left\{ \frac{\omega^2 (I_p \Omega G_5 \cos \theta)^2 - (-I_o \omega^2 + k)}{-\omega^2 [\alpha_1 + G_5^2 (I_o \cos^2 \theta + I_p \sin^2 \theta)] + \alpha_4} \right\}^2}{\{-\omega \{-\omega^2 [\alpha_1 + G_5^2 (I_o \cos^2 \theta + I_p \sin^2 \theta)] + \alpha_4\}\}^2}; \quad (23)$$

$$c^2 = \frac{(\gamma \bar{z} \omega_n^3)^2 \frac{(I_p \Omega_{min} G_5 \cos \theta)^2}{(\bar{\theta})^2} - [\omega_n^2 (I_p \Omega_{min} G_5 \cos \theta)^2]^2}{\{-\omega_n \{-\omega_n^2 [\alpha_1 + G_5^2 (I_o \cos^2 \theta + I_p \sin^2 \theta)] + \alpha_4\}\}^2} = \frac{0}{0}; \quad (24)$$

$$c_{opt} = \frac{\sqrt{9(\gamma \bar{z} \omega_n)^2 \frac{(I_p \Omega_{min} G_5 \cos \bar{\theta}_{opt})^2}{(\bar{\theta}_{opt})^2} + 4(I_p \Omega_{min} G_5 \cos \bar{\theta}_{opt})^4}}{2\omega_n [\alpha_1 + G_5^2 (I_o \cos^2 \bar{\theta}_{opt} + I_p \sin^2 \bar{\theta}_{opt})]}. \quad (25)$$

Figure 7. Frequency response curves for $\Omega = 15000$ rpm: tip response of beam (top), gyro response (bottom) for (blue) $0.25\omega_n^2 = \omega_g^2$, (green) $\omega_n^2 = \omega_g^2$, and (red) $\omega_n^2 = 0.25\omega_g^2$ with the base excitation of $z = 0.05 \cos(\omega_n t)$.

response are obtained from frequency equations of the system. The displacement response of case (green) $\omega_n = \omega_g$ is effectively reduced up to the resonant frequency. The gyroscope can jump into unstable zone when the stiffness is not taken properly. Hence, the optimum spring coefficient ($k = I_o \omega^2$) is chosen to reach the optimum relations for the minimum rotational speed and an optimum damping. Then, setting $\omega = \omega_n = \omega_g$ and $\Omega = \Omega_{min}$ to eliminate the instability due to the natural frequency of the added gyro-spring system results in optimal damping at the resonant frequency.

The study clearly indicates that the tip response of a beam is influenced with the disk speed of the gyro but also depends fairly on the optimum damping coefficient. The numerical results agree with the analytical solution of the damping coefficient corresponding to the minimum disk speed. Therefore, the cooperation of the optimum damping coefficient with the

Figure 8. Frequency response curves for $\Omega = 20000$ rpm: tip response of beam (top), gyro response (bottom) for (blue) $0.25\omega_n^2 = \omega_g^2$, (green) $\omega_n^2 = \omega_g^2$, and (red) $\omega_n^2 = 0.25\omega_g^2$ with the base excitation of $z = 0.05 \cos(\omega_n t)$.

disk speed of the gyro is not negligible, and the minimum disk speed at the resonant frequency should always be considered in such calculations.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

ACKNOWLEDGEMENTS

This study was mainly supported by the Scientific and Technological Research Council of Turkey (TUBITAK) under Grant no: 114M760.

Figure 9. Frequency response curves for $\omega_n^2 = \omega_g^2$ and $\Omega = 10000$ rpm: tip response of beam (top), gyro response (bottom) for (∇) $c = 80$ N·m·s/rad, ($\square$) $c = 120$ N·m·s/rad, and ($\circ$) $c = 160$ N·m·s/rad with the base excitation of $z = 0.05 \cos(\omega_n t)$.

Figure 10. Frequency response curves for $\omega_n^2 = \omega_g^2$ and $\Omega = 15000$ rpm: tip response of beam (top), gyro response (bottom) for (∇) $c = 80$ N·m·s/rad, ($\square$) $c = 120$ N·m·s/rad, and ($\circ$) $c = 160$ N·m·s/rad with the base excitation of $z = 0.05 \cos(\omega_n t)$.

REFERENCES

- Kim, C. S., and Dickinson, S. M. On the analysis of laterally vibrating slender beams subject to various complicating effects, *Journal of Sound and Vibration*, **122** (3), 441–455, (1988). [https://dx.doi.org/10.1016/S0022-460X\(88\)80093-2](https://dx.doi.org/10.1016/S0022-460X(88)80093-2)
- Liu, W. H., and Huang, C.-C. Free vibration of restrained beam carrying concentrated masses, *Journal of Sound and Vibration*, **123** (1), 31–42, (1988). [https://dx.doi.org/10.1016/S0022-460X\(88\)80075-0](https://dx.doi.org/10.1016/S0022-460X(88)80075-0)
- To, C. W. S. Vibration of a cantilever beam with a base excitation and tip mass, *Journal of Sound and Vibration*, **83** (4), 445–460, (1982). [https://dx.doi.org/10.1016/S0022-460X\(82\)80100-4](https://dx.doi.org/10.1016/S0022-460X(82)80100-4)
- Cuvalci, O. The effect of detuning parameters on the absorption region for a coupled system: a numerical and experimental study. *Journal of Sound and Vibration*, **229** (4), 837–857, (2000). <https://dx.doi.org/10.1006/jsvi.1999.2526>
- Ertas, A., Cuvalci, O., and Ekworo-Osire S. Performance of pendulum absorber for a nonlinear system of varying orientation, *Journal of Sound and Vibration*, **229** (4), 913–933, (2000). <https://dx.doi.org/10.1006/jsvi.1999.2521>
- Soong, T. T., and Dargush, G. F. *Passive Energy Dissipation System in Structural Engineering*, Wiley, New York, NY, USA, (1997).
- Soong, T. T., and Spencer, B. F., Jr. Supplemental energy dissipation: state-of-the-art and state-of-the practice, *Engineering Structures*, **24** (3), 243–259, (2002). [https://dx.doi.org/10.1016/S0141-0296\(01\)00092-X](https://dx.doi.org/10.1016/S0141-0296(01)00092-X)
- Beznos, A., Formal'sky, A., Gurfinkel, E., Jicharev, D., Lensky, A., Savitsky, K., and Tchesalin, L. Control of Autonomous Motion of Two-Wheel Bicycle With Gyroscopic Stabilization, *Proceedings of the IEEE International Conference on Robotics and Automation*, Leuven, Belgium, **3**, 2670–2675, (1998). <https://dx.doi.org/10.1109/ROBOT.1998.680749>
- Schilovski, P. *Gyrocar*, (1909), Patent No. GB 12,021.
- Tomlinson, N. *Louis Brennan: Inventor Extraordinaire*, John Halliwell: Chatham, (1980).
- Higashiyama, H., Yamada, M., Kazao, Y., and Namiki, M. Characteristics of Active Vibration Control System Using Gyro-Stabilizer, *Eng. Struct.*, **20** (3), 176–183, (1998). [https://dx.doi.org/10.1016/S0141-0296\(97\)00076-X](https://dx.doi.org/10.1016/S0141-0296(97)00076-X)
- Schlick, E. The Gyroscopic Effect of Flywheels on Board Ship, *Transactions of the Institute of Naval Architects*, **23**, 117–134, (1904).
- Ünker, F., and Çuvalcı, O. Vibration Control of a Column Using a Gyroscope, *Procedia - Social and Behavioral Sciences*, **195**, 2306–2315, (2015). <https://dx.doi.org/10.1016/j.sbspro.2015.06.182T>