Robust Inverse Dynamic Control of a Maneuvering Smart Flexible Satellite with Piezoelectric Layers

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In this paper, a satellite with two flexible appendages and a central hub is considered. The piezoelectric layers are attached to both side of the appendages and used as actuators. The governing equations of motion are derived based on Lagrange method. Using Rayleigh-Ritz technique ordinary differential equations of motion are obtained. A robust inverse dynamic control is applied to the system to not only control the three axes manoeuvre of the satellite but also suppress the vibrations of the flexible appendages. Finally, the system is simulated and simulation results show good performance of this controller.

1. INTRODUCTION

The large angle attitude rotation of a spacecraft is one of its major desirable advantages. The modern flexible spacecrafts need to re-point and track manoeuvres. The dynamics of the spacecraft with large rotational manoeuvre is time varying and nonlinear. Moreover, the manoeuvring introduces certain levels of vibration to flexible appendages.

Most studies on spacecrafts consist of the manoeuvre control of spacecraft around single axis rotation. Song and Kotejshyer¹ studied vibration reduction of flexible structures during slew operations such as attitude control of spacecraft with large solar panels. Ge et al.² applied an energy-based robust control approach for a flexible spacecraft attitude regulating control, as well as the vibration suppression of the flexible appendages. A robust adaptive sliding mode control was presented by Hu³ to control the attitude manoeuvring of a flexible spacecraft, and linear quadratic regulator (LQR) was utilized for suppressing the induced vibration in the appendages. Erdong and Zhaowei⁴ investigated an attitude regulation control problem for a flexible spacecraft. Azadi et al.^{5,6} studied the hybrid controller to suppress the appendage vibrations and manoeuvring control of a one axis rotating flexible satellite.

Recently, some researchers have benefitted from using smart materials like piezoelectrics for sensors/actuators. Hu and Ma,⁷ using the theory of variable structure control, presented an approach to reduce the vibration of the flexible spacecraft during attitude manoeuvre. Hu⁸ also proposed a composite control scheme for a single angle rotational spacecraft with bounded piezoelectric material on its flexible appendages, satisfying the actuator constraints in the presence of mismatched perturbation. Neto et al.⁹ presented a general methodology for active vibration control of high-speed flexible linkage mechanisms with piezoelectric actuators and sensors. Further, equations of motion for flexible multi-body systems and for the relevant finite elements used in the modelling of piezoelectric materials were introduced. Sales et al.¹⁰ considered the appli-

cation of shunted piezoelectric transducers to control the vibrations of a simple satellite composed of a hub, a reaction wheel, which is a device traditionally used for active attitude control, and two identical flexible panels, which contain piezoelectric patches symmetrically bonded to their surfaces.

Some scientists have studied three axes rotational manoeuvre control of flexible satellites. Hu¹¹ applied a semi-globally input-to-state control technique to a flexible spacecraft attitude manoeuvre in the presence of parameter uncertainties and external disturbances. A fault-tolerant adaptive backstepping sliding mode control scheme was developed for flexible spacecraft attitude manoeuvring using redundant reaction wheels by Jiang et al.¹² Xin and Pan¹³ investigated a nonlinear optimal control to approach and align a spacecraft to a target. Azadi et al.¹⁴ applied an adaptive robust controller to control the three axes manoeuvre of a satellite while suppressing the appendages vibrations using piezoelectric actuators.

In some studies, the satellite has been considered as a rigid body and the flexibility effects have been neglected. Verbin and Lappas¹⁵ presented a new attitude control method for agile rigid spacecrafts, which was based on combining single gimbal control moment gyros together with reaction wheels. The method was expected to suit remote sensing spacecrafts that are required to perform multiple rapid retargeting of their line of sight. They¹⁶ also presented a nonlinear state feedback design approach for large rotational manoeuvring of a rigid satellite with reaction wheels. Huang et al.¹⁷ studied the dynamic and control hovering of a rigid spacecraft. The dynamical model, which they considered, was built to describe the orbital motion of the spacecraft, and a control scheme was derived for different hovering configuration.

In this study a robust inverse dynamic control scheme is used to control the three axes manoeuvre of a satellite and suppress the vibrations of the appendages. The satellite is considered as a central hub with two flexible appendages. The piezoelectric layers are attached to both sides of the appendages as actuators. Each appendage is considered as an Euler-Bernoulli

Figure 1. Schematic of the satellite with coordinate systems.

beam. The nonlinear governing equations of motion are derived and the system is simulated to measure the controller robustness.

2. SYSTEM DYNAMICS

Consider a satellite with flexible appendages, which is shown in Fig. 1. This satellite moves in the space with constant velocity and manoeuvres around three axes. The appendages of this satellite are considered as Euler-Bernoulli beams, which have width b , thickness h , length L , density ρ , and Young's modulus E , and the piezoelectric layers are attached to them. Each piezoelectric layer has thickness h_p , density ρ_p , Young's modulus E_p , and the equivalent piezoelectric coefficient e_{31} . $w_i(x, t)$ represents the lateral deflection of the i th satellite appendage. Knowing the kinetic and potential energy of the system is necessary for using Lagrange method to derive the governing equations of motion of the system.

The kinetic energies of the n th appendage, piezoelectric layers, and hub are as:

$$\begin{aligned} T_a &= \frac{1}{2} \sum_{i=1}^2 \int_{\text{appendage}_i} \text{trace}\{\dot{r}_R(\dot{r}_R)^T\} dm; \\ T_p &= \frac{1}{2} \sum_{i=1}^2 \sum_{n=1}^{N_i} \int_{PZT_n} \text{trace}\{\dot{r}_R(\dot{r}_R)^T\} dm; \\ T_h &= \frac{1}{2} \int_{\text{hub}} \text{trace}\{\dot{r}_R(\dot{r}_R)^T\} dm; \end{aligned} \quad (1)$$

where $\dot{r}_R$ is the velocity vector of an element with respect to reference frame. N_i is the number of piezoelectric layers in i th appendage. The potential energies of the appendages and piezoelectric layers are:

$$U_a = \frac{1}{2} \sum_{i=1}^2 \int_{\text{appendage}_i} E z^2 \left(\frac{\partial^2 w_i}{\partial x^2} \right)^2 dv;$$

$$\begin{aligned} U_p &= \frac{1}{2} \sum_{i=1}^2 \left\{ \sum_{n=1}^{N_i} \int_{PZT_n} E_p z^2 \left(\frac{\partial^2 w_{pn}}{\partial x^2} \right)^2 dv \right. \\ &\quad \left. + \sum_{n=1}^{N_i} \int_{PZT_n} z e_{31} E_{z_n} \frac{\partial^2 w_{pn}}{\partial x^2} dv + \frac{1}{2} \sum_{n=1}^{N_i} \int_{PZT_n} E_{z_n} d_n dv \right\}; \end{aligned} \quad (2)$$

wherein z is the axis perpendicular to the piezoelectric surface, and d_n is the electric displacement for the n th patch of piezoelectric layers. The last term in Eq. (2) is the electric energy stored in the piezoelectric materials. E_{z_n} is electric field of the n th piezoelectric layer.

Considering w_j as:

$$w_j(x, t) = \sum_i \varphi_i^j q_i^j = (\Phi^j)^T \mathbf{q}^j \quad j = 1, 2; \quad (3)$$

and substituting Eqs. (1) and (2) in Lagrange equations, the governing equations of motion are obtained as:

$$\begin{aligned} \begin{bmatrix} M_{\theta\theta} & M_{\theta q} \\ M_{q\theta} & M_{qq} \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{\mathbf{q}} \end{Bmatrix} + \begin{bmatrix} C_{\theta\theta} & C_{\theta q} \\ C_{q\theta} & C_{qq} \end{bmatrix} \begin{Bmatrix} \dot{\theta} \\ \dot{\mathbf{q}} \end{Bmatrix} \\ + \begin{bmatrix} 0 & 0 \\ 0 & K_{qq} \end{bmatrix} \begin{Bmatrix} \theta \\ \mathbf{q} \end{Bmatrix} = \begin{Bmatrix} \tau \\ -K_{\text{pelaslecta}} v_a \end{Bmatrix}. \end{aligned} \quad (4)$$

Here, ϕ_i^j is a series of trial mode shapes satisfying the boundary conditions, and q_i^j is the time dependent generalized coordinate for the j th appendage. Next, $\theta = [\theta_1 \theta_2 \theta_3]^T$, where θ_i is the i th ($i = 1, 2, 3$) rotation angle of the satellite, namely around the z_1 , x_2 , and y_3 axes, respectively. The vector of generalized coordinates of the j th appendage is $\mathbf{q} = [(q_1)^T (q_2)^T]^T$ and q_j , which is used in Rayleigh-Ritz technique. Further, τ and v_a are the vectors controller input torques and actuator voltages, respectively. The parameters of the mass matrix are determined as:

$$\begin{aligned} (M_{\theta\theta})_{ij} &= \text{trace} \sum_{k=1}^2 \frac{\partial T_{a_k}^o}{\partial \theta_j} \int_{\text{appendage}_k} r_{a_n} r_{a_n}^T dm \left(\frac{\partial T_{a_k}^o}{\partial \theta_i} \right)^T \\ &\quad + \text{trace} \sum_{k=1}^2 \sum_{n=1}^{N_k} \frac{\partial T_{a_k}^o}{\partial \theta_j} \int_{PZT_n} r_{a_n} r_{a_n}^T dm \left(\frac{\partial T_{a_k}^o}{\partial \theta_i} \right)^T \\ &\quad + \text{trace} \frac{\partial T_h^o}{\partial \theta_i} \int_{\text{hub}} r_h r_h^T dm \left(\frac{\partial T_h^o}{\partial \theta_j} \right)^T; \\ (M_{\theta q})_{ij} &= (M_{q\theta})_{ji} = \text{trace} \sum_{k=1}^2 \frac{\partial T_{a_k}^o}{\partial \theta_j} \\ &\quad \cdot \int_{\text{appendage}_k} \frac{\partial \dot{r}_{a_n} r_{a_n}^T}{\partial \dot{q}_i} dm (T_{a_k}^o)^T \\ &\quad + \text{trace} \sum_{k=1}^2 \sum_{n=1}^{N_k} \frac{\partial T_{a_k}^o}{\partial \theta_j} \int_{PZT_n} \frac{\partial \dot{r}_{a_n} r_{a_n}^T}{\partial \dot{q}_i} dm (T_{a_k}^o)^T; \\ (M_{qq})_{ij} &= \frac{1}{2} \text{trace} \sum_{k=1}^2 T_{a_k}^o \int_{\text{appendage}_k} \frac{\partial^2 (\dot{r}_{a_n} \dot{r}_{a_n}^T)}{\partial \dot{q}_i \partial \dot{q}_j} dm (T_{a_k}^o)^T \end{aligned}$$

$$+ \frac{1}{2} \text{trace} \sum_{k=1}^2 \sum_{n=1}^{N_k} T_{a_k}^o \int_{PZT_n} \frac{\partial^2 (\dot{r}_{a_n} \dot{r}_{a_n}^T)}{\partial \dot{q}_i \partial \dot{q}_j} dm (T_{a_k}^o)^T. \quad (5)$$

where T_{ai}^o and T_h^o are the rotational/transitional matrix from the i th appendage coordinate and hub frame to the base frame, respectively, and $r(\cdot)$ is the position vector of the element coordinate $(\cdot)$. So r_h is the position vector of an element in the hub coordinate. The matrix centrifugal and Coriolis effects, C , in Eq. (4) is determined as:¹⁸

$$C_{ij} = \sum_k c_{ijk} \dot{\xi}_k; \\ c_{ijk} = \frac{1}{2} \left(\frac{\partial M_{ij}}{\partial \xi_k} + \frac{\partial M_{ik}}{\partial \xi_j} - \frac{\partial M_{jk}}{\partial \xi_i} \right); \\ \zeta = [\theta^T \quad q^T]^T. \quad (6)$$

The stiffness matrix is $K_{qq} = \text{diag}(K_{qq1}, K_{qq2})$ and:

$$K_{qqk} = \int_{\text{appendage}_k} EI \frac{\partial^2 \Phi^k(x)}{\partial x^2} \frac{\partial^2 (\Phi^k(x))^T}{\partial x^2} dx \\ + \sum_{n=1}^{N_k} \int_{PZT_n} E_p I_p \frac{\partial^2 \Phi^k(x)}{\partial x^2} \frac{\partial^2 (\Phi^k(x))^T}{\partial x^2} dx \\ k = 1, 2. \quad (7)$$

where I_p is the moment of inertia of the piezoelectric layers. The elastic-electric effect of the piezoelectric actuators is illustrated in $K_{pelastelecta}$.

$$K_{pelastelecta} = \begin{bmatrix} K_{pelastelecta}^1 & 0 \\ 0 & K_{pelastelecta}^2 \end{bmatrix}; \quad (8)$$

where 0 is the zero matrix and

$$K_{pelastelecta}^i = \begin{bmatrix} K_{pee_1}^i & K_{pee_2}^i & \cdots & K_{pee_{N_i}}^i \end{bmatrix}. \quad (9)$$

Here, $K_{pee_j}^i$ is the vector of the j th column of the elastic-electric matrix and is defined as:

$$K_{pee_j}^i = \frac{e_{31j}}{h_{p_j}} \int_{PZT_j} z \frac{\partial^2 \Phi^i}{\partial x^2} dv. \quad (10)$$

Equation (4) can be rewritten as:

$$M(\psi) \ddot{\psi} + C(\psi, \dot{\psi}) \dot{\psi} + K\psi = Y\rho = u; \quad (11)$$

where $u = [\tau^T (-K_{pelastelecta} v_a)^T]^T$ is the controller input, $\psi = [\theta^T q^T]^T$, and ρ is the parameters vector of the system.

3. ROBUST CONTROLLER DESIGN

If the bounds on the parameters of the system are known but these parameters are not exactly known, then the robust control scheme can be used. Considering the following controller vector:¹⁸

$$u = \hat{M} \cdot [\ddot{\psi}_d - K_D \cdot \dot{\psi} - K_P \cdot \psi] + \hat{C} \cdot \dot{\psi} + \hat{K} \psi + u_o; \quad (12)$$

where $\hat{M}$, $\hat{C}$, and $\hat{K}$ represent nominal values vis-à-vis to parameter uncertainty of M , C , and K . Also, ψ_d is the bounded,

twice-differentiable desired vector of generalized coordinates, the tracking error is $\tilde{\psi} = \psi - \psi_d$, u_o is an additional control input that will be defined later, and K_D and K_P are constant and positive definite diagonal matrices. Substituting the control input (12) into the dynamic model (11) gives the following error equation:

$$M \cdot [\ddot{\psi} + K_D \cdot \dot{\psi} + K_P \cdot \psi] = Y \cdot \tilde{\rho} + u_o; \quad (13)$$

using the property of linearity of dynamic equation with respect to a set of constant physical parameters ρ .

Equation (13) can be rewritten in state space form as follows:

$$\dot{\xi} = \begin{bmatrix} 0 & I \\ -K_P & -K_D \end{bmatrix} \xi + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} (Y(\cdot) \tilde{\rho} + u_o). \quad (14)$$

Here, $\xi = [\xi_1^T \quad \xi_2^T]^T$ is the state vector, and $\xi_1 = \tilde{\psi}$ and $\xi_2 = \dot{\tilde{\psi}}$ are the state variables.

The compact form of Eq. (14) is:

$$\dot{\xi} = A\xi + BM^{-1}u_o(\xi) + Be(\xi). \quad (15)$$

Here,

$$A = \begin{bmatrix} 0 & I \\ -K_P & -K_D \end{bmatrix}; B = \begin{bmatrix} 0 \\ I \end{bmatrix}; e(\xi) = M^{-1}Y(\cdot) \tilde{\rho}. \quad (16)$$

Consider the following positive definite Lyapunov function:¹⁸

$$V = \xi^T P \xi. \quad (17)$$

Here, P is a symmetric positive definite matrix satisfying $A^T P + PA = Q$, with Q being symmetric and positive definite too. Tacking the time derivative of V along the trajectories of the error system (13) yields:

$$\dot{V} = -\xi^T Q \xi + 2\xi^T PBM^{-1}(Y(\cdot) \tilde{\rho} + u_o). \quad (18)$$

From Eq. (18) it follows that

$$\dot{V} \leq -\xi^T Q \xi + 2 \|\xi^T P B\| \|M^{-1}Y(\cdot) \tilde{\rho}\| + 2\xi^T PBM^{-1}u_o. \quad (19)$$

We now make the assumptions that there exists a known function $\beta(\cdot): R^{2n} \times R \rightarrow R^l$ and a constant vector $a^* \in R^l$ such that:

$$\beta_i(\xi, t) \geq 0;$$

$$\alpha_i^* \geq 0 \quad 1 \leq i \leq l;$$

$$\|M^{-1}Y(\cdot) \tilde{\rho}\| \leq \beta^T(\xi, t) \alpha^*. \quad (20)$$

for all $(\xi, t) \in R^{2n} \times R$.

From properties of the dynamic model described above, a possible choice for $\beta^T(\xi, t) \alpha^*$ is:

$$\beta^T(\xi, t) \alpha^* = \alpha_1^* + \alpha_2^* \|\psi\| + \alpha_3^* \|\dot{\psi}\| + \alpha_4^* \|\ddot{\psi}\|^2. \quad (21)$$

Choosing the additional control input u_o as:

$$u_o = -\frac{(\beta^T(\xi, t) \alpha^*)^2 \lambda}{\epsilon} B^T P \xi; \quad (22)$$

Table 1. Characteristics of the piezoelectric layers and flexible appendages.¹⁴

Piezoelectric layer	Flexible appendage
Modulus, $E_p = 2 \times 10^9 \text{ N/m}^2$	Modulus, $E = 76 \times 10^9 \text{ N/m}^2$
Piezoelectric constant, $d_p = 22 \times 10^{-12} \text{ mV}^{-1}$	Length, $L = 5 \text{ m}$
Thickness, $h_p = 1 \text{ mm}$	Thickness, $h = 8 \text{ mm}$
Density, $\rho_p = 1800 \text{ kg/m}^3$	Width, $b = 0.5 \text{ m}$
	Density, $\rho = 2840 \text{ kg/m}^3$
	Radius of hub, $l = 1 \text{ m}$

and using Eq. (20), Eq. (19) can be rewritten as:

$$\dot{V} \leq -\xi^T Q \xi + \frac{2}{\epsilon} \|\xi^T P B\| \beta^T(\xi, t) \alpha^*(\epsilon - \|\xi^T P B\| \beta^T(\xi, t) \alpha^*). \quad (23)$$

Here, we have used the fact that $\lambda_{\min}(M^{-1}) = 1/\lambda_M$, wherein $\epsilon > 0$, $\lambda > \lambda_M$, and λ_M is the greatest eigenvalue of the mass matrix, M . Therefore, as long as the following inequality is verified:

$$\|\xi^T P B\| \beta^T(\xi, t) \alpha^* \geq \epsilon. \quad (24)$$

We get:

$$\dot{V} < -\xi^T Q \xi. \quad (25)$$

This inequality is strict; indeed, we can show that if $\xi = 0$ then:

$$\dot{V} \leq -\xi^T Q \xi + 2\epsilon. \quad (26)$$

This shows the state ξ is globally ultimately uniformly bounded with respect to a compact set around the origin $\xi = 0$.

4. SIMULATION RESULTS

In order to numerically solve the governing equation of motion and observe the response of the closed-loop control system, the governing equation of motion is solved using Newmark integration. The piezoelectric layers and satellite parameters are given in Table 1.

We consider large angles trajectory tracking around the x_2 , y_3 , and z_1 axes as $\pi/3$, $\pi/4$, and $\pi/6$, respectively. The desired trajectory angles of the satellite have been defined as the following function:

$$\theta_d = \left[6 \left(\frac{t}{t_f} \right)^5 - 15 \left(\frac{t}{t_f} \right)^4 + 10 \left(\frac{t}{t_f} \right)^3 \right] (\theta_f - \theta_0) + \theta_0.$$

Herein t_f is the total time needed to complete a manoeuvre, and θ_f and θ_0 are the initial and final angular positions, respectively.

In order to show high effectiveness and performances of the robust controller, nonzero initial conditions, which lead to greater amplitude appendages vibrations, are considered. The controller gains have been selected as: $K_p = \text{diag}[500 \ 500 \ 500 \ 250 \ 200 \ 100 \ 250 \ 200 \ 100]$ and $K_D = \text{diag}[460 \ 460 \ 460 \ 100 \ 80 \ 100 \ 80 \ 80]$. Six pairs of piezoelectric layers are considered to be attached to both the appendages. The length of all piezoelectric layers is same, 0.5 m. The piezoelectric actuators are located at the base, middle, and the end of the appendages. Figure 2 shows the angular trajectory tracking of the manoeuvring satellite around x_2 , y_3 , and z_1 axes and its desired one. It can be seen that the desired trajectory is effectively tracked by satellite. Figure 3 illustrates the tip deflection of the right appendage when voltage is applied to the piezoelectric actuators and without applying voltage to them. This figure shows that this robust controller can

Figure 2. Rotational trajectory tracking of the satellite. (a) around x_2 axis, (b) around y_3 axis, (c) around z_1 axis.

suppress the appendage vibrations rapidly by applying voltage to piezoelectric actuators. Since there is a symmetry in the system, the tip deflection of the left appendage is similar to Fig. 3. Figure 4 compares the generalized coordinates of the controlled system with uncontrolled one. This figure shows that all three generalized coordinates of the right appendage converge to zero by applying the voltages to the piezoelectric actuators. This means that the vibrations of this appendage have been damped and converge to zero, too. The simulation

Figure 3. Tip deflection of the right appendage with and without piezoelectric actuators.

results for the left appendage are similar to the right one. Voltages which are applied to the actuators are shown in Fig. 6. According to Fig. 6, the controller applies the admissible voltages to the actuators.¹⁹ The mid vibrations of the right appendage have been shown in Fig. 5. The simulation results show the high performance and capability of the controller.

5. CONCLUSION

A robust control scheme was used to control the manoeuvre and vibration of a flexible satellite. A satellite with two flexible appendages was considered as a central hub. Piezoelectric layers attached to appendages were used as actuators to suppress the vibration of the appendages. Further, nonlinear governing equations of motion were derived. The robust control scheme was defined and applied to the system. Finally, the system was simulated and the simulation results showed the high capability of this robust controller.

REFERENCES

- ¹ Song, G. and Kotejshyer, B. Vibration reduction of flexible structures during slew operations, *International Journal of Acoustics and Vibration*, **7** (2), 105–109, (2002).
- ² Ge, S.S., Lee, T.H., Hong, F., and Goh, C.H. Energy-based robust controller design for flexible spacecraft, *Journal of Control Theory and Applications*, **2**, 27–34, (2004). <https://dx.doi.org/10.1007/s11768-004-0020-z>
- ³ Hu, Q. Robust adaptive sliding mode attitude maneuvering and vibration damping of three-axis stabilized flexible spacecraft with actuator saturation limits, *Nonlinear Dynamics*, **55**, 301–321, (2009). <https://dx.doi.org/10.1007/s11071-008-9363-1>
- ⁴ Erdong, J., and Zhaowei, S. Passivity-based control for a flexible spacecraft in the presence of disturbances, *International Journal of Nonlinear Mechanics*, **45**, 348–356, (2010). <https://dx.doi.org/10.1016/j.ijnonlinmec.2009.12.008>
- ⁵ Azadi, E., Eghtesad, M., Fazlzadeh, S. A., and Azadi, M. Vibration suppression of smart nonlinear flexible appendages of a rotating satellite by using hybrid adaptive sliding mode/Lyapunov control, *Jour-*

Figure 4. Time history of the right appendage generalized coordinates. (a) first generalized coordinate, (b) second generalized coordinate, (c) third generalized coordinate.

Figure 5. Mid deflection of the right appendage with and without piezoelectric actuators.

Figure 6. Input voltages of piezoelectric actuators bonded to the right appendage. (a) first piezoelectric layer, (b) second piezoelectric layer, (c) third piezoelectric layer.

nal of Vibration and Control, **19** (7), 975–991, (2013). <https://dx.doi.org/10.1177/1077546312439913>

⁶ Azadi, E., Fazelzadeh, S. A., Eghtesad, M., and Azadi, M. Active vibration suppression and maneuver control of an orbiting smart flexible satellite, *IJST, Transactions of Mechanical Engineering*, **38** (M1), 119–133, (2014).

⁷ Hu, Q. and Ma, G. Variable structure control and active vibration suppression of flexible spacecraft during attitude

maneuver, *Aerospace Science and Technology*, **9**, 307–317, (2005). <https://dx.doi.org/10.1016/j.ast.2005.02.001>

⁸ Hu, Q. A composite control scheme for attitude maneuvering and elastic mode stabilization of flexible spacecraft with measurable output feedback, *Journal of Aerospace and Technology*, **13**, 81–91, (2009). <https://dx.doi.org/10.1016/j.ast.2007.06.007>

⁹ Neto, M. A., Ambrsio, J. A., Roseiro, L. M., Amaro, A., and Vasques, C. M. A. Active vibration control of spatial flexible multibody systems, *Multibody System Dynamics*, **30** (1), 13–35, (2013). <https://dx.doi.org/10.1007/s11044-013-9341-3>

¹⁰ Sales, T. P., Rade, D. A., and de Souza, L. C. G. Passive vibration control of flexible spacecraft using shunted piezoelectric transducers, *Aerospace Science and Technology*, **29** (1), 403–412, (2013). <https://dx.doi.org/10.1016/j.ast.2013.05.001>

¹¹ Hu, Q. Semi-globally input-to-state stable controller design for flexible spacecraft attitude stabilization under bounded disturbances, *Acta Astronautica*, **66**, 567–576, (2010). <https://dx.doi.org/10.1016/j.actaastro.2009.07.012>

¹² Jiang, Y., Hu, Q., and Ma, G. Adaptive backstepping fault-tolerant control for flexible spacecraft with unknown bounded disturbances and actuator failures, *ISA Transactions Journal*, **49**, 57–69, (2010). <https://dx.doi.org/10.1016/j.isatra.2009.08.003>

¹³ Xin, M., and Pan, H. Nonlinear optimal control of spacecraft approaching a tumbling target, *Aerospace Science and Technology*, **15** (2), 79–89, (2011). <https://dx.doi.org/10.1016/j.ast.2010.05.009>

¹⁴ Azadi, M., Fazelzadeh, S.A., Eghtesad, M., and Azadi, E. Vibration suppression and adaptive-robust control of a smart flexible satellite with three axes maneuvering, *Acta Astronautica*, **69**, 307–322, (2011). <https://dx.doi.org/10.1016/j.actaastro.2011.04.001>

¹⁵ Verbin, D., and Lappas V.J. Rapid rotational maneuvering of rigid satellites with hybrid actuators configuration, *Journal of Guidance, Control, and Dynamics*, **36** (2), 532–547, (2013). <https://dx.doi.org/10.2514/1.56405>

¹⁶ Verbin, D., and Lappas V.J. Rapid rotational maneuvering of rigid satellites with reaction wheels, *Journal of Guidance, Control, and Dynamics*, **36** (5), 1538–1544, (2013). <https://dx.doi.org/10.2514/1.58155>

¹⁷ Huang, X., Yan, Y., and Zhou, Y. Dynamics and control of spacecraft hovering using the geomagnetic Lorentz force, *Advances in Space Research*, **53** (3), 518–531, (2014). <https://dx.doi.org/10.1016/j.asr.2013.11.040>

¹⁸ Canudas, C., Siciliano, B., and Bastin, G. *Theory of Robot Control*, Springer-Verlag, (1997).

¹⁹ Li, Z., and Bainum, P. M. Vibration control of flexible spacecraft integrating a momentum exchange controller and a distributed piezoelectric actuator, *Journal of Sound and vibration*, **177** (4), 539–553, (1994). <https://dx.doi.org/10.1006/jsvi.1994.1450>