Emission of Train-Induced Ground Vibration — Prediction of Axle-Load Spectra and its Experimental Verification

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Train passages induce forces on the track, train-induced vibrations propagate through the soil and excite neighbouring buildings. The emission, which is the first part of the prediction of vibrations near railway lines, is presented by focusing on the dynamic axle loads. The calculation of the axle loads is based on the vehicle-track-soil interaction. This interaction calculus utilises the dynamic stiffness of the vehicle (the inertia of the wheelset) and the dynamic stiffness of the track-soil system. Based on various time consuming finite-element boundary-element calculations, an approximate track-soil model has been established. The vehicle-track-soil analysis yields several transfer functions between the various geometric or stiffness irregularities and the axle loads of the train. Geometric irregularities of the vehicle (the wheels) and the track (rail surface and track alignment) are the simplest components. Geometric irregularities of the subsoil (trackbed irregularities) have to be transferred to effective irregularities at rail level. The bending stiffness of the track is filtering out the short-wavelength contribution. Stiffness irregularities occur due to random variations in the ballast or the subsoil, which must also be transferred to effective track irregularities, and due to the discrete rail support on sleepers. All necessary transfer functions for the prediction of axle-load spectra are presented as general formula and as specific graphs for differing vehicle and track parameters. The prediction method is applied to a ballast track and a slab track and compared with corresponding axle-box measurements. Moreover, ground vibration measurements at numerous sites are exploited for the axle-load spectra and the validation of the prediction method. All theoretical and experimental results confirm that the dynamic axle-load spectra have an approximate value of 1 kN per third of octave and increase with train speed, track stiffness and around the vehicle-track resonance.

1. INTRODUCTION

Train passages induce static and dynamic forces on the track, the train-induced vibrations propagate through the soil and excite neighboring buildings. The problem of train vibrations is divided into the parts, which is outlined in Fig. 1, emission, which is the excitation by railway traffic (the present contribution), transmission, which is the wave propagation through the soil,¹ and immission, which is the transfer into a building.² The present contribution deals with the prediction of the emission part so that now all parts of the prediction of train-induced ground vibration are published in a series of articles in the *International Journal of Acoustics and Vibration*. The link between the emission and transmission parts of the prediction is the excitation force between the track and the soil. This excitation force, also denoted by axle load (or axle-load spectrum), is the topic of this contribution.

In the literature on train induced ground vibration, only a few contributions deal explicitly with excitation forces, for example.^{3–9} In two-step prediction methods,^{10,11} which can be wrong if the soil is missing in the emission part, the excitation force is transferred from the emission to the transmission part, but it is rarely presented.¹² Early and mainly experimental contributions such as^{13,14} consider the vibration of the track, or at a measuring point 3 or 8 m away from the track as the emission of the train. These vibrations include already the stiffness of the soil at the specific site, so that they cannot be regarded as an emission quantity. The current prediction methods mostly hide the excitation forces in a black-box prediction from track irregularities.¹⁵ The presentation of excitation forces is advantageous for the prediction of ground vibration as well as for the

evaluation of the reduction effect of elastic track elements.^{16,17}

It can be observed that the soil is often disregarded if the vehicle-track interaction is in focus,¹⁸ and that the excitation force is not usually analyzed if the wave propagation through the soil is in focus. Obviously, the prediction and analysis of railway induced ground vibration needs both, and the analysis of the vehicle-track interaction and a detailed soil model for the wave propagation. This comprehensive approach has been used and propagated by the author since the analysis of the sleeper passage component of the ICE test runs.¹⁹ The same approach has been presented in Auersch.^{1,16,20,21} Forward and backward transmission analysis is used in this contribution to improve the knowledge on the excitation process and the excitation forces of railway induced ground vibration.

2. METHODS OF VEHICLE-TRACK-SOIL INTERACTION

2.1. Vehicle and Track Irregularities

The main causes of train-induced ground vibrations are irregularities of the vehicle and the track, which are most simply described as geometric irregularities s at the interface between wheel and rail. The irregularities are described as functions of the wavelength λ , the wavenumber $\xi = 2\pi/\lambda$, the circular frequency $\omega = v_T \xi$ or the frequency $f = v_T/\lambda$. These are all related by the speed v_T of the train. The amplitudes s of the different irregularities depend on the condition of the vehicle and track, but the following general rules for the third-of-octave spectra have been found to be²³

Figure 1. The prediction of emission, transmission and immission of ground-borne vibration.

wheel out-of-roundness

$$s \sim \lambda^{0.5 \dots 1} \quad 3 \text{ m} > \lambda > 0.3 \text{ m} \\ s(\lambda = 1 \text{ m}) = 0.005 \dots 0.015 \text{ mm};$$

wheel roughness

$$s \sim \lambda^{0.5} \quad \lambda < 0.3 \text{ m} \\ s(\lambda = 0.25 \text{ m}) = 0.002 \dots 0.007 \text{ mm};$$

rail roughness

$$s \sim \lambda^{0.5 \dots 1} \quad \lambda < 0.5 \text{ m} \\ s(\lambda = 0.2 \text{ m}) = 0.001 \dots 0.005 \text{ mm};$$

track alignment

$$s \sim \lambda^{1.5 \dots 2} \quad \lambda < 1.2 \text{ m} \\ s(\lambda = 2 \text{ m}) = 0.017 \dots 0.17 \text{ mm}. \quad (1)$$

The irregularities s increase with the wavelength λ , strongest for the track alignment, and this means a decrease with frequency f . The wheel irregularities start with isolated peaks for the first and second out-of-roundness and yield a continuous spectrum for higher frequencies (short wavelengths). The track yields low-frequency (long-wavelength) alignment irregularities and high-frequency (short-wavelength) roughness irregularities. There is a wavelength gap between these track irregularities, mainly due to the vanishing alignment irregularities for $\lambda < 1.2 \text{ m}$.

Figure 2. The finite-element boundary-element model and the simplified multi-beam on Winkler-support model of the ballasted track-soil system consisting of the rail, rail pads, sleepers, sleeper pads, ballast, under-ballast mat, base plate, and soil.

2.2. Simple Vehicle and Multi-Beam on Winkler-Support Track Models

The dynamic stiffness K_V of the vehicle can be approximated by the mass m_W of the wheelset

$$K_V(\omega) = -m_W \omega^2. \quad (2)$$

The minor effects of more detailed vehicle models have been discussed by Auersch.²⁴

Many track systems as seen in Fig. 2 have been calculated by the combined finite-element boundary-element method,²⁵ and the results have been used to derive a faster multi-beam on Winkler-support track model. The track model consists of n beams which represent the rails and track slabs and are described by the bending stiffness EI_j , the mass per length m'_j , and by support elements representing rail pads, sleepers, the ballast and any isolation element. The multi-beam system fulfils the set of differential equations for the displacements $\mathbf{u}$ and the load $\mathbf{F}_T$:

$$\mathbf{EI} \mathbf{u}^{IV} + \mathbf{M} \ddot{\mathbf{u}} + \mathbf{C} \dot{\mathbf{u}} + \mathbf{K} \mathbf{u} = \mathbf{F}_T. \quad (3)$$

This equation transformed into the frequency-wavenumber domain reads

$$(\xi^4 \mathbf{EI} - \omega^2 \mathbf{M} + i\omega \mathbf{C} + \mathbf{K}) \mathbf{u} = (\xi^4 \mathbf{EI} + \mathbf{K}_{TS}(\omega)) \mathbf{u} = \mathbf{F}_T(\omega); \quad (4)$$

where $\mathbf{K}_{TS}(\omega)$ is the dynamic track-soil stiffness matrix which is calculated by the transfer matrices for each support element. The dynamic track stiffness $K_T(\omega)$ at the wheel rail contact can be calculated as the wavenumber integral by:

$$\frac{1}{K_T(\omega)} = \frac{u_R}{F_T}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathbf{e}_1^T (\xi^4 \mathbf{EI} + \mathbf{K}_{TS}(\omega))^{-1} \mathbf{e}_1 d\xi; \quad (5)$$

(where the base vectors $\mathbf{e}_1$ indicate the components of the matrix) or by modal superposition.²²

2.3. Vehicle-Track Transfer Functions

The dynamic stiffness $K_T(\omega)$ of the track is coupled with the dynamic stiffness $K_V(\omega)$ of the vehicle to analyse the combined vehicle-track system. The force F_T that acts on the track is calculated as:

$$F_T(\omega) = -\frac{K_V(\omega) K_T(\omega)}{K_V(\omega) + K_T(\omega)} s(\omega) = H_V(\omega) s(\omega); \quad (6)$$

for the geometric vehicle-track irregularities s . In case of a varying track stiffness $K_T(x)$ (see Section 3.2), a similar transfer function H_V^* for the effective track irregularities s^* yields:

$$F_T(\omega) = -\frac{K_V(\omega)K_{T0}}{K_V(\omega) + K_T(\omega)}s^*(\omega) = H_V^*(\omega)s^*(\omega). \quad (7)$$

The only difference is that the static track stiffness K_{T0} is used in the numerator.²⁰ Equation (7) holds also for the effective irregularity s_D^* due to the stiffness variation on and between the sleepers.

The influence of the track mass must be introduced by the force transfer function H_{FT} from top to bottom of the track which can be simply evaluated from the support chain.²² Finally, the force F_S on the soil, which excites the ground vibration, can be calculated as:

$$F_S(\omega) = H_{FT}(\omega)F_T(\omega). \quad (8)$$

3. THE TRACK FILTERING OF TRACKBED IRREGULARITIES

3.1. Geometric Trackbed Irregularities

The first problem of track behavior is a track excited by a harmonic irregularity of the soil surface:

$$s_S = s_S e^{i\xi x} \mathbf{e}_n. \quad (9)$$

The response $\mathbf{u}$ to this excitation is also a harmonic function which can be calculated in wavenumber domain according to

$$(\mathbf{EI}\xi^4 + \mathbf{K}_T)\mathbf{u} + \mathbf{K}_S(\mathbf{u} - s_S) = 0; \quad (10)$$

$\mathbf{K}_T$ is the static support stiffness inside the track beams and $\mathbf{K}_S$ is the static support stiffness under the track beams. No external force is present and only the difference $\mathbf{u} - s_S$ yields a force $\mathbf{K}_S(\mathbf{u} - s_S)$ between the track and the last support section. The solution is:

$$\begin{aligned} \mathbf{u} &= (\mathbf{EI}\xi^4 + \mathbf{K}_T + \mathbf{K}_S)^{-1} \mathbf{K}_S s_S; \\ \mathbf{u} &= (\mathbf{EI}\xi^4 + \mathbf{K}_{TS})^{-1} k_S^* \mathbf{e}_n \mathbf{e}_n^T s_S; \\ \mathbf{u} &= (\mathbf{EI}\xi^4 + \mathbf{K}_{TS})^{-1} \mathbf{e}_n k_S^* s_S; \end{aligned} \quad (11)$$

and the effective track irregularity s_R at the rail level is

$$s_R = \mathbf{e}_1^T \mathbf{u} = \mathbf{e}_1^T (\mathbf{EI}\xi^4 + \mathbf{K}_{TS})^{-1} \mathbf{e}_n k_S^* s_S = H_{sT}(\omega = v\xi) s_S. \quad (12)$$

The stiffness k_S^* of the last support section can be lower than the soil stiffness k_S if additional elements with stiffness k_A are included in series to the soil

$$k_S^* = \frac{k_A}{k_A + k_S} k_S. \quad (13)$$

3.2. Random Stiffness Variation

Another possible excitation of railway induced ground vibration is due to the variation of the sub-soil stiffness under the track²⁷ as

$$k_S = k_0 + k_1 \exp(i\xi_V x); \quad (14)$$

with a constant mean stiffness k_0 . The stiffness variation k_1 yields a track response in combination with the static train load F_0 , and

$$\mathbf{EI}u^{IV} + (\mathbf{K}_0 + \mathbf{K}_1(x))\mathbf{u} = F_0 \mathbf{e}_1 \delta(x); \quad (15)$$

Table 1. Parameters of the standard ballast and the standard slab track.

Component	Parameter	Value
wheelset	mass	$m_W = 1500 \text{ kg}$
UIC60 rail	flexural stiffness	$EI_R = 6.4 \times 10^6 \text{ Nm}^2$
UIC60 rail	mass per length	$m'_R = 60 \text{ kg/m}$
rail pads	stiffness per support	$k_R = 300 \text{ MN/m}$
ballast track	stiffness per support	$k_R = 60 \text{ MN/m}$
rail pads	stiffness per support	$k_R = 60 \text{ MN/m}$
slab track	stiffness per support	$k_R = 60 \text{ MN/m}$
sleeper	mass	$m_S = 340 \text{ kg}$
sleeper pad	stiffness per sleeper	$k_U = 40 \text{ MN/m}$
ballast	mass density	$\rho_B = 2000 \text{ kg/m}^3$
ballast	modulus of elasticity	$E_B^* = 4.4 \times 10^8 \text{ N/m}^2$
ballast	height	$h_B = 0.35 \text{ m}$
concrete slab	modulus of elasticity	$E_P = 3 \times 10^{10} \text{ N/m}^2$
concrete slab	mass density	$\rho = 2500 \text{ kg/m}^3$
concrete slab	thickness	$t = 0.2 \text{ m}$
soil	stiffness per half-sleeper	$k_S = 80 \times 10^7 \text{ N/m}$
soil	damping per half-sleeper	$c_S = 7.5 \times 10^5 \text{ Ns/m}$
elastic elements	hysteretic damping	$D = 0.1$

is the differential equation for the displacements along the track. The solution is established by a two-step linear perturbation analysis as $u = u_0 + u_1$.^{26,28} Finally, the effective track irregularity s^* at the rail level is achieved by the transfer function:

$$H_{kT}^*(\omega) = \frac{s^*}{k_1/k_0}(\omega) = -\frac{F_0 k_0}{2\pi} \int_{-\infty}^{\infty} \mathbf{e}_1^T (\mathbf{EI}\xi^4 + \mathbf{K}_0)^{-1} \mathbf{e}_n \cdot \mathbf{e}_n^T (\mathbf{EI}(\xi - \xi_V)^4 + \mathbf{K}_0)^{-1} \mathbf{e}_1 d\xi. \quad (16)$$

If the last support section includes elastic elements in addition to the soil spring, the soil stiffnesses k_S , k_0 , and k_1 in Eqs. (12) and (16) have to be modified.²⁶

3.3. Deterministic Stiffness Variation Due to the Discrete Support on Sleepers

The sleeper-distance effect is expressed as an effective track irregularity as:

$$s_D^*(\omega_D) = s_{D1}^*(K_{T0}/10^9 \text{ N/m})^{0.4} \quad \text{with } s_{D1}^* = 0.012 \text{ mm and } \omega_D = 2\pi v/d; \quad (17)$$

according to the results which have been calculated in Auersch.²⁹ These values hold for a static axle load of $F_0 = 100 \text{ kN}$.

4. TRANSFER FUNCTIONS FOR DIFFERENT VEHICLE AND TRACK PARAMETERS

4.1. Final Formula for the Axle-Load Spectrum and Basic Parameters

The final formula for the dynamic axle load due to the independent random contributions is Eq. (18).

It includes the geometric and stiffness induced trackbed irregularities s_S , k_1/k_0 , the corresponding track filters H_{sT} and H_{kT} , and the vehicle-track-soil transfer functions $H_{FT}H_V$ and $H_{FT}H_V^*$ for the geometric wheel and track irregularities and the stiffness induced effective track irregularities. The basic parameters of the vehicle and track are listed in Tab. 1.

$$F_S(\omega) =$$

$$\sqrt{H_{FT}(\omega)^2 \left(H_V(\omega)^2 \left(s(\omega)^2 + H_{sT}(\omega)^2 s_S(\omega)^2 \right) + H_V^*(\omega)^2 \left(s_D^*(\omega = \omega_D)^2 + H_{kT}(\omega)^2 k_1/k_0(\omega)^2 \right) \right)}. \quad (18)$$

Figure 3. The vehicle-track transfer function of a ballast track and the wheelset masses of $m_W = \square$ 3000, $\circ$ 2000, $\triangle$ 1500, and $+$ 1000 kg.

4.2. Transfer Functions of the Vehicle-Track-Soil Interaction

At first, the most important vehicle-track-soil transfer function $F_S/s = H_{FT}H_V$ is presented for some vehicle and track parameters. The vehicle-track-soil transfer function is strongly influenced by the unsprung mass of the vehicle: the wheelset mass Fig. 3. The dynamic axle loads are proportional to the wheelset mass for frequencies up to 60 Hz or even higher. For this frequency range, the transfer function increases with $F_S/s \sim f^2$. The higher frequency range is determined by the stiffness of the track and soil which is demonstrated by the variation of the ballast stiffness (the shear wave velocity of the ballast) in Fig. 4a. The stiffer ballast results in a higher vehicle-track resonance frequency up to 125 Hz. The high-frequency transfer function is almost constant where the highest values are reached for the stiffest ballast. Similar observations are found for the modified vehicle-track-soil transfer function $F_S/s^* = H_{FT}H_V^*$ for the effective irregularities s^* due to a stiffness variation in Fig. 4b. The high-frequency transfer values are smaller and the resonance is clearer for the stiffness variation. The increasing effect of the track stiffness is the same for both vehicle-track-soil transfer functions.

If soft track elements are introduced in the track, the vehicle-track resonance frequency is lower and the high-frequency transfer values are correspondingly reduced. Soft rail pads yield resonance frequencies down to 40 Hz Fig. 5a, soft sleeper pads have resonance frequencies between 16 and 64 Hz Fig. 5b. The axle-loads can thus be reduced to less than one tenth of the values of a standard ballast track.

4.3. Transfer Functions of the Track Filtering

The track filtering functions for the geometric and stiffness induced trackbed irregularities are evaluated for the three different track systems, the standard ballast track, a slab track and a ballast track with sleeper pads. Figure 6a shows that the low-

Figure 4. The vehicle-track-soil transfer function (a) and modified vehicle-track-soil transfer function and (b) of a wheelset ($m_W = 1500$ kg) on a track with a ballast of shear wave velocity $v_S = \square$ 100, $\circ$ 150, $\triangle$ 200, and $+$ 300 m/s.

frequency geometric irregularity is passed directly through the track and that means a transfer value of $s_R/s_S = 1$. Frequencies higher than 12 Hz (in case of a standard ballast track and train speed of $v_T = 160$ km/h) are reduced considerably because of a strong cut-off effect. The ballast track with sleeper pads has a lower cut-off frequency at 10 Hz and therefore a stronger track filtering effect with lower effective track irregularities. The slab track with its high bending stiffness has the lowest cut-off frequency of 8 Hz and, a stronger decay of amplitudes than the ballast tracks. Therefore, the effective track irregularities at higher frequencies are much lower for the slab track.

Figure 5. The vehicle-track-soil transfer function of a wheelset ($m_W = 1500$ kg) on a ballast track (a) with elastic rail pads of $k_R = 160, \circ 80, \triangle 40$, and $+ 20$ kN/mm, and (b) with soft sleeper pads of $k_S = 160, \circ 80, \triangle 40, + 20$, and $\times 10$ kN/mm.

Similar observations are made for the stiffness induced trackbed irregularities in Fig. 6b, but the cut-off frequencies are somewhat higher at about 20, 16 and 12 Hz, and the low-frequency transfer values depend on the stiffness of the track. The force amplitudes decrease for the soft sleeper pad and strongest for the high bending stiffness of the slab track.

5. EFFECTIVE VEHICLE AND TRACK IRREGULARITIES AND RESULTING AXLE-LOAD SPECTRA

The axle-load spectra are calculated for three different track types by using the transfer functions of the preceding section and the following irregularities:

$$s_S = s_{S1}(\lambda/\lambda_1)^2$$

with $s_{S1} = 0.02$ mm at $\lambda_1 = 1.2$ m;

Figure 6. The track-filter transfer functions for (a) the geometric track-bed error and (b) the stiffness variation, $\square$ ballast track, and $\circ$ a ballast track with soft sleeper pads ($k_S = 40$ kN/mm), and $\triangle$ a slab track with elastic rail pads ($k_R = 60$ kN/mm).

$$s_W = s_{W1}(\lambda/\lambda_1)$$

with $s_{W1} = 0.01$ mm at $\lambda_1 = 1.2$ m; (19)

in agreement to the general rules of Eq. (1). The wheel and track irregularities are combined in one irregularities s_W . The stiffness variation is assumed to be constant at $k_1/k_0 = 20\%$, and the sleeper distance excitation is a rather high $s_D^* = 0.012$ mm.

5.1. Ballast Track

The effective irregularities at rail level corresponding to these input parameters are shown in Fig. 7a for five train speeds. The following components can be observed where the frequencies correspond to the train speed of 100 km/h (curve $\times$): at low frequencies, the trackbed irregularities are dominating, but also strongly decreasing with $s_R \sim f^{-2}$ or stronger. The transfer function H_{ST} for the trackbed irregularity has a cut-off at about 8 Hz and therefore, the effective irregularity

s^* from the stiffness variation becomes dominating at about 10 Hz. This component starts constant in frequency and the corresponding cut-off of H_{kT} is at about 16 Hz, from where the stiffness component is also decreasing until the wheel and track irregularity s_W becomes dominant at 25 Hz. The following decrease is weaker, as $s_W \sim f^{-1}$ until 100 Hz. The sleeper-passage component can be clearly found in this range with a peak of $s_D^* = 0.012$ mm constant for all train speeds.

At the next step, the decreasing irregularities have to be multiplied by the increasing vehicle-track transfer function. The resulting axle-load spectra shown in Fig. 7c are almost constant around $F = 1$ kN. At frequencies below 25 Hz for $v_T = 100$ km/h, the forces are below 1 kN whereas for higher frequencies they slightly increase. The sleeper distance excitation yields relative maxima at 32, 40, 50, 64 and 80 Hz for the different train speeds. Finally the highly damped vehicle-track resonance can be found at 100 Hz followed by a weak decrease. The amplitudes increase with increasing train speed where this increase is stronger at low frequencies.

5.2. Slab Track

The effective irregularities of the slab track in Fig. 7b are considerably smaller than those of the ballast track (Fig. 7a). Both, the high bending stiffness of the slab and the softer rail pads are responsible for this strong reduction. The irregularities due to the trackbed irregularities can only be found up to 16 Hz (for $v_T = 160$ km/h), and for higher frequencies they are smaller than the geometric wheel and track irregularities.

The force spectra of the slab track shown in Fig. 7d look quite different compared to the ballast track shown in Fig. 7c. The vehicle-track resonance at 64 Hz is very strong and dominates the spectra. The sleeper-passage component is hidden in the amplification around this resonance. At low frequencies, a strong reduction can be observed for the trackbed irregularity s_S . As a consequence, the trackbed irregularity is exceeded by the wheel irregularity at frequency higher than 25 Hz (for $v_T = 160$ km/h). The stiffness-variation component s^* is completely hidden under the wheel irregularity due to the strong filter effect of the slab track combined with the rail pad.

5.3. Comparison of Three Track Types

Figure 8 shows the effective track irregularities and the axle load spectra for the three track types ballast track, slab track and ballast track with sleeper pads. The effective track irregularities are lowest for the stiff slab track and highest for the standard ballast track. The axle-load spectra clearly show the different vehicle-track resonances, a weak resonance at 100 Hz for the ballast track, a strong resonance at 64 Hz for the slab track with elastic rail pads, and a medium strong resonance at 32 Hz for the ballast track with soft sleeper pads. The differing resonance frequencies result in strongly differing axle loads at high frequencies. These are the dynamic reduction effects of soft track elements. An additional reduction of axle loads can be observed at a certain low-frequency range where the slab track yields the lowest amplitudes due to the track filter effect.

5.4. Comparison with Axle-Box Measurements

The calculated irregularities and forces are confronted with results from axle-box measurements. The measured wheelset accelerations a_W are divided by ω^2 to yield displacements (see

Figure 7. Irregularity spectra (a),(b) and force spectra (c),(d) of a ballast track (a),(c) and a slab track with elastic rail pads ($k_R = 60$ kN/mm, (b),(d)), $v_T = \square 160, \circ 125, \triangle 100, + 80$, and $\times 63$ km/h.

Figure 8. Irregularity spectra (a) and force spectra (b) of a $\square$ ballast track, $\circ$ a ballast track with soft sleeper pads ($k_S = 40$ kN/mm), and $\triangle$ a slab track with elastic rail pads ($k_R = 60$ kN/mm), for a train speed of 160 km/h.

Figs. 9a to 9b) which are almost the same as the effective irregularities (with differences around the vehicle-track resonance, mainly for the slab track). The accelerations a_W multiplied by the wheelset mass of 1500 kg are almost identical with the excitation forces acting on the track (see Figs. 9c to 9d), with small errors at low frequencies due to bogie contributions.²⁴ Measurements and calculations are in good agreement with all the details. The smaller measured slab track irregularities below 10 Hz can be attributed to the fact that the bending stiffness of the slab reduces not only the transfer but also the generation of trackbed irregularities. At all higher frequencies, the amplitudes, resonance frequencies and excitation peaks of the measurements are well represented in the calculated axle-load spectra.

6. AXLE-LOAD SPECTRA FROM GROUND VIBRATION MEASUREMENTS

The ground vibration from rail traffic can be predicted with the excitation force spectra and the specific transfer functions of a site.¹ The following results are calculated in the other direction and the excitation forces are calculated from the measured ground vibration and the transfer functions of the site.²³

6.1. Back-Calculated Axle-Load Spectra from Fifteen Measuring Sites

The results from fifteen different sites are presented in Fig. 10. All force spectra show some similarities. First, on average the forces are close to 1 kN within the third octave spectrum as predicted in the theoretical section. Second, char-

Figure 9. Irregularity spectra $s = a_W/\omega^2$ (a),(b) and force spectra $F = m_W a_W/\omega^2$ (c),(d) of a ballast track (a),(c) and a slab track (b),(d) from axle-box measurements (wheelset accelerations a_W), $v_T = \square$ 160, $\circ$ 125, $\triangle$ 100, $+$ 80, and $\times$ 63 km/h.

acteristic minima and maxima can be found which depend on the speed of the train. Two minima are at 8 and 25 Hz for the trains at a standard speed of 160 km/h (see Figs. 10d and 10e), and they are clearly shifted with the train speed (see Figs. 10a to 10c). The mid-frequency range between the two minima has higher amplitudes, typically 3 kN is reached in most cases and even 8 kN can be found for a few cases. Usually three thirds of octaves below $f/\text{Hz} = v_T/10$ km/h constitute this elevated mid-frequency region. The slowest trains in Fig. 10f have smoother force spectra with a wider amplified region. It should be noted that the maximum does not increase with increasing train speed in the ICE tests (see Figs. 10a to 10c). Especially in Fig. 10b, the three mid-frequency regions at 10–16 Hz for 160 km/h, 12–20 Hz for 200 km/h and 16–25 Hz for 250 km/h come close together at the force maximum of 3 kN at 16 Hz. Both minima consist of two or three thirds of octaves with clearly reduced forces. For the slowest trains (Fig. 10f) the second region of reduced forces is even wider. These force spectra hold for a variety of soils, soft and stiff, homogeneous and layered soils, and for different passenger trains whereas freight trains have not been included so far.

Finally, Fig. 11 shows predicted and measured ground vibration for two layered sites. The characteristics of the soil can be clearly seen, the low-frequency low amplitudes of the stiff underlying soil, the amplitudes of the soft layer at high frequencies, and a strong increase in between. At the end of this increase the strong mid-frequency component can be clearly observed for all measuring points up to 64 m distance from the track.

6.2. Discussion of the Dominant Mid-Frequency Component

The axle-load spectra are modified by the axle sequence of the passing train^{20,29} where similar two minima and the maximum typically appear due to the bogie configuration. The amplification and the second reduction are inferior, and only the first strong reduction can be explained by the axle sequence. The second strong reduction could be explained by the cut-off of the trackbed transfer functions H_{sT} or H_{kT} ,^{26,28,30} but the high mid-frequency amplitudes of up to 10 kN indicate that there is a different excitation mechanism. The nearly constant amplitudes for different train speeds suggest the axle impulses on the track as a possible vibration source, see also the arguments of Auersch.³¹ Once again, the scattering of the axle impulses by a randomly varying soil seems to be the only explanation of all these observations. This kind of excitation can be introduced in the prediction by equivalent forces.²⁹

7. CONCLUSION

The emission part for the prediction of railway induced ground vibration has been presented. The different irregularity components have been specified and all necessary transfer functions have been given as equations and graphs for different vehicle and track parameters. Finally, the dynamic axle-load spectra are calculated as almost constant spectra of about 1 kN per third of octave. These results have been verified in detail by axle-box measurements on a ballast and a slab track and in general for the force level by the evaluation of ground vibration measurements of normal and high-speed passenger trains at fifteen different sites. An elevated mid-frequency region has been observed for all ground vibration measurements and explained by scattered axle impulses. The good agreement be-

tween the prediction, the axle-box and ground-vibration measurements demonstrates that the emission of railway vibrations is well understood and represented.

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Figure 10. Force spectra back calculated from ground vibration, (a) site W_1 , ICE, $v_T = \square 250, \circ 200, \triangle 160$ km/h; (b) site W_3 , ICE, $v_T = \square 250, \circ 200, \triangle 160$ km/h; (c) site W_2 , ICE, $v_T = \square 300, \circ 250, \triangle 200$, and $+ 150$ km/h; (d) IC, $v_T \approx 160$ km/h, site $\square H, \circ L, \triangle N, + W$; (e) IC, $v_T = 100 - 160$ km/h, site $\square A, \circ Z, \triangle X_1$; (f) IC, $v_T \approx 125$ km/h, site $\square D, \circ X_2, \triangle X_3, + U_2$, and $\times Y$.

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Figure 11. Train induced ground vibration at site X_1 (a),(b), $r = \square, \circ, \triangle, +, \times$ 6, 8, 16, 32, 64 m, and site Z (c),(d), $r = \square, \circ, \triangle, +, \times$ 16, 24, 32, 48, and 60 m. The prediction with back-calculated force spectra are shown at the top (a),(c), and the measurements of IC trains at the bottom (b),(d).

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