Robust H_∞ Control of Building Structures with Time Delay

Kun Liu, Long-xiang Chen and Guo-ping Cai

Department of Engineering Mechanics, State Key Laboratory of Ocean Engineering, Shanghai Jiaotong University, Shanghai 200240, P. R. China

Wei-dong Zhang

Department of Automation, Shanghai Jiaotong University, Shanghai 200240, P. R. China

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In this paper, a robust H_∞ controller for a building structure with a time delay is studied. Firstly, the motion equation of the structural system with an explicit time delay is introduced. Then, the state space representation of a dynamic equation without any explicit time delay is deduced by a specific integral transformation to the time-delay equation. Secondly, a robust control method is applied to the H_∞ time-delay controller design and, due to the compensation of the time delay, the proposed controller contains not only the state term of the current step, but also a linear combination of some former steps of the control. Finally, numerical simulations and comparisons of a six-story building using the proposed time-delay controller are carried out. The simulation results indicate that the control performance will deteriorate if the time delay is not taken into account in the control design. The proposed H_∞ time-delay controller in this paper can effectively compensate for a time delay in order to get better control effectiveness. Besides, this delay controller is robust to both structural parameter and time delay uncertainties.

1. INTRODUCTION

The idea of using an active controller to reduce the response of civil engineering structures has been studied since it was proposed by Yao.¹ Various control methods in the automation field have been introduced into the studies of structural vibration control. The research results indicate that active control methods can effectively reduce the response of flexible structures and its control performance is much better than that of passive control. Active controllers are mainly designed based on the physical model established for the structures. The more precise the physical model is, the more effective the controller is. However, errors may exist inevitably between the physical model and practical structures due to some uncertainties, such as the structural parameters and boundary conditions of the structures et cetera. Besides, signal noise and external disturbance may also degrade control efficiency in control implementation. Hence, the designed controller is desired to have strong robustness for the uncertainties so as to eliminate the negative effect of the uncertainty factors on control performance. In modern control systems, the robust control method is robust to the variances of structural parameters and external disturbance, so this method has received a lot of attention and many studies have been done on it.^{2–5}

However, a time delay exists inevitably in active control systems for many reasons, such as online data acquisition from sensors at different locations of the structure, data processing and active control force calculation of the computer, and control force signals transmission to the actuators to build up required control force. Various research results indicate that even a small time delay may cause actuators to apply energy to the control system when energy is really not needed, which may cause degradation in control efficiency and even make the system unstable.^{6–8} So far, some methods have been proposed

to handle time delay problems in active control systems, such as the Taylor series expansion,⁹ phase shift technique,¹⁰ state pre-estimation,¹¹ and two direct design methods for time-delay controller.^{7,12} The first three methods work well with some small time delay problems but cannot deal with large time delay ones. Two direct design methods^{7,12} are to design a time-delay controller directly from a time-delay differential equation and no assumption is made in the entire design process and they are suitable for both small and large time delays. Cai and Chen^{8,13} verified these two methods by conducting an experiment using several flexible structures as research objects. Nowadays, time delay problems in robust H_∞ control have come to the attention of many researchers. For example, Du and Zhang¹⁴ investigated an H_∞ controller design approach for vibration attenuation of seismic-excited building structures with an uncertain time-invariant time delay in the control input. Zhang et al.¹⁵ studied the robust stability for a class of uncertain neutral systems with time-varying delay and nonlinear uncertainties, and using the Lyapunov method, put forward a new delay-dependent stability criteria. Zhao et al.¹⁶ discussed the robust H_∞ state-feedback controller design for a class of semi-active seat suspension systems with norm-bounded parameter uncertainties, time-varying input delay, and actuator saturation. The desired controller is derived by solving the LMIs and the corresponding closed-loop system is asymptotically stable with a guaranteed H_∞ performance. Chen et al.¹⁷ considered Takagi-Sugeno (T-S) fuzzy systems with both state and input time delays, robust H_∞ fuzzy controller is designed based on the Lyapunov-Krasovskii functional method and numerical simulations are given to illustrate the effectiveness and feasibility of the proposed controller. Since the robust control method is robust to the variances of structural parameters, it prompts the following question: “will the designed time-delay

controller using a robust control method be robust for time delay (the magnitude of delay does not have to be a little delay but varying within a limited range)?” In actual control systems, the magnitude of a time delay tends not to be significant and might vary within a limited range. Even for the control system with a significant delay, such as a space-craft controller, the delay also appears to be in a small range. It surely will make a difference if a robust controller over a time delay varying within a small range could be developed.

In this paper, we are interested in the problem of robust H_∞ control of a six-story building with a time delay and a method to deal with a time delay is also proposed. The robustness of the H_∞ controller against structural intrinsic parameter and time delay uncertainties is numerically investigated. This paper is organized as follows: Section 2 briefly introduces motion equation of the structural system with an explicit time delay and specific transformation for standard state space representation without any explicit time delay; the robust H_∞ controller design with a time delay is given in Section 3, including the controller design and control implementation; numerical simulations of a six-story building using the proposed time-delay controller are carried out in Section 4 and concluding remarks are given in Section 5.

2. MOTION EQUATION

Consider an n -story building, the structure undergoes an one-dimensional earthquake ground acceleration $w(t)$. The time delay in control is τ and the motion equation of the structural system is written as:

$$\mathbf{M}\ddot{\mathbf{Z}}(t) + \mathbf{C}\dot{\mathbf{Z}}(t) + \mathbf{K}\mathbf{Z}(t) = \mathbf{H}U(t - \tau) - \mathbf{M}_0w(t); \quad (1)$$

where $\mathbf{Z}(t) = [z_1, z_2, \dots, z_n]^T$ is the interstory drift of each story unit of the building structure; $\mathbf{M}$ is an $n \times n$ mass matrix, all elements of $\mathbf{M}$ are zero except $\mathbf{M}(i, j) = m_i$ for $i = 1, \dots, n$ and $j = 1, \dots, i$; $\mathbf{K}$ is an $n \times n$ elastic stiffness matrix, all elements of $\mathbf{K}$ are zero except $\mathbf{K}(i, i) = k_i$ for $i = 1, \dots, n$ and $\mathbf{K}(i, i + 1) = -k_{i+1}$ for $i = 1, \dots, n - 1$; $\mathbf{C}$ is an $n \times n$ damping matrix, all elements of $\mathbf{C}$ are zero except $\mathbf{C}(i, i) = c_i$ for $i = 1, \dots, n$ and $\mathbf{C}(i, i + 1) = -c_{i+1}$ for $i = 1, \dots, n - 1$; $\mathbf{M}_0$ is the vector whose elements are the mass of each story unit; $\mathbf{H}$ represents the location of active control force; and $U(t - \tau)$ is the active control force.

In the state space representation, Eq. (1) becomes:

$$\dot{\mathbf{X}}(t) = \mathbf{A}\mathbf{X}(t) + \mathbf{B}U(t - \tau) + \mathbf{B}_w w(t); \quad (2)$$

$$\text{where } \mathbf{X}(t) = \begin{bmatrix} \mathbf{Z}(t) \\ \dot{\mathbf{Z}}(t) \end{bmatrix}, \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{H} \end{bmatrix}, \mathbf{B}_w = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}^{-1}\mathbf{M}_0 \end{bmatrix}.$$

By the following transformation of Eq. (2):¹⁸

$$\bar{\mathbf{H}}(t) = \mathbf{X}(t) + \boldsymbol{\Gamma}(t) = \mathbf{X}(t) + \int_{-\tau}^0 e^{-\mathbf{A}(\eta+\tau)} \mathbf{B}U(t + \eta) d\eta; \quad (3)$$

then Eq. (2) becomes:

$$\dot{\bar{\mathbf{H}}}(t) = \mathbf{A}\bar{\mathbf{H}}(t) + \bar{\mathbf{B}}U(t) + \mathbf{B}_w w(t); \quad (4)$$

where $\bar{\mathbf{B}} = e^{-\mathbf{A}\tau}\mathbf{B}$.

Figure 1. The structural model of a six-story building.

Figure 2. The time history of the El Centro earthquake.

3. ROBUST H_∞ CONTROL

3.1. Design of Controller

The control system with the observation equation can be described as:

$$\begin{cases} \dot{\bar{\mathbf{H}}}(t) = \mathbf{A}\bar{\mathbf{H}}(t) + \bar{\mathbf{B}}U(t) + \mathbf{B}_w w(t) \\ z(t) = \mathbf{C}_{11}\bar{\mathbf{H}}(t) + \mathbf{D}_{12}U(t) \end{cases}; \quad (5)$$

where $z(t)$ is the controlled output and $\mathbf{C}_{11}$ and $\mathbf{D}_{12}$ are known constant matrices, $\mathbf{D}_{12}^T \mathbf{D}_{12} = \mathbf{I}$, $\mathbf{C}_{11}^T \mathbf{D}_{12} = \mathbf{0}$.

Let the robust H_∞ controller be described as $U(t) = \bar{\mathbf{K}}\bar{\mathbf{H}}(t)$, then the following closed-loop control system should be asymptotically stable:

$$\begin{cases} \dot{\bar{\mathbf{H}}}(t) = (\mathbf{A} + \bar{\mathbf{B}}\bar{\mathbf{K}}) \bar{\mathbf{H}}(t) + \mathbf{B}_w w(t) \\ z(t) = (\mathbf{C}_{11} + \mathbf{D}_{12}\bar{\mathbf{K}}) \bar{\mathbf{H}}(t) \end{cases}. \quad (6)$$

The norm of transfer function from $w(t)$ to $z(t)$ satisfies:⁵

$$\|\mathbf{T}_{zw}(s)\| = \left\| (\mathbf{C}_{11} + \mathbf{D}_{12}\bar{\mathbf{K}}) [s\mathbf{I} - (\mathbf{A} + \bar{\mathbf{B}}\bar{\mathbf{K}})]^{-1} \mathbf{B}_w \right\| < \gamma; \quad (7)$$

where γ is a constant and $\gamma > 0$. From Jia,⁵ we know that if matrices $\bar{\mathbf{X}}^T = \bar{\mathbf{X}} > 0$ and $\mathbf{Y}$ exist and satisfies the matrix inequality

$$\begin{bmatrix} \mathbf{A}\bar{\mathbf{X}} + \bar{\mathbf{B}}\mathbf{Y} + (\mathbf{A}\bar{\mathbf{X}} + \bar{\mathbf{B}}\mathbf{Y})^T & \mathbf{B}_w & (\mathbf{C}_{11}\bar{\mathbf{X}} + \mathbf{D}_{12}\mathbf{Y})^T \\ \mathbf{B}_w^T & -\mathbf{I} & \mathbf{0} \\ \mathbf{C}_{11}\bar{\mathbf{X}} + \mathbf{D}_{12}\mathbf{Y} & \mathbf{0} & -\gamma^2 \mathbf{I} \end{bmatrix} < 0; \quad (8)$$

Figure 3. The time histories of the system response with a no-delay controller: (a) the first story unit; (b) the sixth story unit; and (c) the active control force.

the closed-loop system in Eq. (5) is asymptotically stable with the H_∞ performance index γ , and $\bar{\mathbf{K}} = \mathbf{Y}\bar{\mathbf{X}}^{-1}$ is the state-feedback gain matrix of the robust H_∞ controller.

Based on Eq. (8), the following optimization problem can be obtained:

$$\begin{aligned} \min \quad & \rho \\ \text{s.t.} \quad & \begin{bmatrix} \mathbf{A}\bar{\mathbf{X}} + \bar{\mathbf{B}}\mathbf{Y} + (\mathbf{A}\bar{\mathbf{X}} + \bar{\mathbf{B}}\mathbf{Y})^T \mathbf{B}_w & (\mathbf{C}_{11}\bar{\mathbf{X}} + \mathbf{D}_{12}\mathbf{Y})^T \\ \mathbf{B}_w^T & -\mathbf{I} & \mathbf{0} \\ \mathbf{C}_{11}\bar{\mathbf{X}} + \mathbf{D}_{12}\mathbf{Y} & \mathbf{0} & -\rho\mathbf{I} \end{bmatrix} < \mathbf{0}; \\ & \bar{\mathbf{X}} > \mathbf{0}; \end{aligned} \quad (9)$$

where $\rho = \gamma^2$, the H_∞ state-feedback controller can be expressed as $U(t) = \bar{\mathbf{K}}\mathbf{H}(t) = \bar{\mathbf{H}}[\mathbf{X}(t) + \mathbf{\Gamma}(t)]$.

3.2. Control Implementation

The computation of the integral term $\mathbf{\Gamma}(t)$ in Eq. (3) is introduced in this section. The data sampling period $\bar{T}$ is chosen to be identical with the computing time step Δt , i.e., $\bar{T} = \Delta t$. Assuming that the time delay τ can be written as

$$\tau = l\bar{T} - \bar{m}; \quad (10)$$

where l is a positive integral number, $l > 0$, $0 \leq \bar{m} < \bar{T}$. In Sun,¹⁹ it is pointed out that a time delay has a small effect on

Figure 4. The maximum response quantity and the maximum active control force varying with the time delay when using the no-delay controller to control the system with the time delay.

control performance and can be ignored in the control design if it is smaller than data sampling period $\bar{T}$. A time delay can only affect a control system when it is larger than $\bar{T}$. Hence, this paper only considers the situation of $\tau > \bar{T}$ with the condition of $\bar{m} = 0$ (i.e., time delay is integer times of sampling period). For the case of $\bar{m} \neq 0$, refer to Chen⁸ and Sun.¹⁹

Between any two adjoining sampling points, the control force exerted on the structure can be considered as a constant if the data sampling period is small enough, that is,

$$U(t) = U(k\bar{T}), \quad k\bar{T} \leq t < (k+1)\bar{T}. \quad (11)$$

Since numerical computation for the control system is carried out on every sampling point, when $\bar{m} = 0$, the integral term $\Gamma(t)$ in Eq. (3) can be written as¹⁸

$$\begin{aligned} \Gamma(t) = & \mathbf{I}\mathbf{G}(\Delta t)U(t-l\Delta t) + \\ & \mathbf{F}(-\Delta t)\mathbf{G}(\Delta t)U[t-(l-1)\Delta t] + \\ & \mathbf{F}(-2\Delta t)\mathbf{G}(\Delta t)U[t-(l-2)\Delta t] + \dots + \\ & \mathbf{F}[-(l-1)\Delta t]\mathbf{G}(\Delta t)U(t-\Delta t); \end{aligned} \quad (12)$$

where

$$\begin{cases} \mathbf{F}(\bar{\xi}) = e^{\mathbf{A}\bar{\xi}} \\ \mathbf{F}(\bar{\xi}) = \int_0^{\bar{\xi}} e^{\mathbf{A}\theta} d\theta \cdot \mathbf{B} \end{cases} \quad (13)$$

We can observe from Eq. (12) that when a time delay exists in the system, every step of control implementation contains

not only the state term of the current step, but also a linear combination of the former l steps of control.

$\mathbf{F}(\bar{\xi})$ and $\mathbf{G}(\bar{\xi})$ can be determined by the following equations^{8,19}

$$\begin{cases} \mathbf{F}(\bar{\xi}) = e^{\mathbf{A}\bar{\xi}} = \sum_{p=0}^{\infty} \frac{\mathbf{A}^p \bar{\xi}^p}{p!} \\ \mathbf{F}(\bar{\xi}) = \int_0^{\bar{\xi}} e^{\mathbf{A}\theta} d\theta \cdot \mathbf{B} = \sum_{p=0}^{\infty} \frac{(-\mathbf{A})^{p-1} \bar{\xi}^p}{p!} \cdot \mathbf{B} \end{cases} \quad (14)$$

When $\bar{\xi}$ is given, $\mathbf{F}(\bar{\xi})$ and $\mathbf{G}(\bar{\xi})$ will both converge to constant matrices in limited steps of iterative computation.

4. NUMERICAL SIMULATION

A six-story building adopted in Du and Zhang paper¹⁴ is considered in this section as the structural model, as shown in Fig. 1. The El Centro earthquake with a maximum ground acceleration of $0.4g$, as shown in Fig. 2, was used as the external excitation and the earthquake episode was 8 s. An active brace system (ABS) was installed in the third floor. The mass, damping, and elastic stiffness of each story unit were given as $m_i = 345.6$ ton, $c_i = 2793$ kNs/m, and $k_i = 3.404 \times 10^5$ kN/m, respectively, for $i = 1, \dots, 6$. The data sampling period $\bar{T}$ and the computation time step Δt are both taken by 10^{-3} s, that is $\bar{T} = \Delta t = 10^{-3}$ s. The initial value of vector $\mathbf{Z}$ was zero. In Eq. (9), $\mathbf{C}_{11} = \text{diag}([1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0])$, $\mathbf{D}_{12} = \mathbf{0}$, and $\gamma = 0.9$.

Figure 5. The maximum response quantity and the maximum active control force varying with the time delay when the proposed controller is used.

4.1. Nominal System

4.1.1. Control with No Time Delay

Consider the case with no time delay in control, that is $\tau = 0$, the controller designed in the case of no time delay (called no-delay controller in this paper) is used for the building structure without a time delay. Figure 3 shows the results against time of the interstory drift and the absolute acceleration of the first and sixth story units, as well as the active control force with no-delay controller, denoted by the solid line. As observed from Fig. 3, the responses of building structure can

be evidently reduced by the proposed H_∞ controller in this paper.

4.1.2. Control with a Time Delay

Here we consider the case with a time delay. Firstly, the effect of a time delay on control performance is checked. Figure 4 shows, against a time delay, the maximum interstory drift and the maximum absolute acceleration of the first and sixth story units, as well as the maximum active control force using the no-delay controller to control the system with a time delay. It is shown that if a time delay is neglected in the control design, system responses would be unstable with an increase of it.

Figure 6. Time histories of the system response with the time-delay controller ($\tau = 0.1$ s): (a) the first story unit; (b) the sixth story unit; and (c) the active control force.

Figure 5 shows the variations of maximum responses against the time delay when the proposed time-delay controller is used. It is observed that, due to the compensation of the time delay, the controlled structure still remains stable even when the time delay increases to 0.25 s.

Secondly, the effectiveness of the designed time-delay controller is verified. Two cases are considered: $\tau = 0.1$ s and $\tau = 0.214$ s. Figures 6 and 7 show the results against the time of the interstory drift and the absolute acceleration of the first and sixth story units. They also show the active control force with the proposed time-delay controller when time delays are $\tau = 0.1$ s and $\tau = 0.214$ s, respectively, denoted by the solid

line. It is observed from Figs. 6 and 7 that the time delay is compensated effectively by the proposed time-delay controller and excellent effectiveness can be obtained, which proves the proposed controller works well when a time delay exists in the control system.

4.2. Robustness against Elastic Stiffness Uncertainties

It is well known that the H_∞ controller has a strong robustness against the variances of structural parameters and external disturbance. In this section, the robustness of the H_∞

Figure 7. Time histories of the system response with the time-delay controller ($\tau = 0.214$ s): (a) the first story unit; (b) the sixth story unit; and (c) the active control force

controller against the elastic stiffness uncertainties of building structure is studied.

Firstly, we consider the case with no time delay in control. In this section, we take $\mathbf{K}$ as the elastic stiffness matrix of nominal system and $\tilde{\mathbf{K}} = \mathbf{K} \times \tilde{X}\%$ as that of the uncertain system. When using the no-delay controller designed by $\mathbf{K}$ to control the building structure, Figs. 8 and 9 show the simulation results for the cases $\tilde{X} = 80$ and $\tilde{X} = 120$, respectively. It is observed from Figs. 8 and 9 that, in spite of the changes on elastic stiffness of each story unit, the designed H_∞ controller can robustly stabilize the system and remarkably suppress the responses of building structure. Extensive

simulation results indicate that the H_∞ controller is always applicable when $61 \leq \tilde{X} \leq 370$, and is not good when $\tilde{X}$ is larger than 370 or smaller than 61.

Secondly, we consider the case with the time delay in the control. Time delay $\tau = 0.05$ s is introduced into the control system and proposed time-delay controller is used for the vibration attenuation. Figures 10 and 11 show the simulation results for the cases $\tilde{X} = 70$ and $\tilde{X} = 110$, respectively. It is observed from Figs. 10 and 11 that the designed H_∞ time-delay controller can also robustly stabilize the system no matter of the changes on elastic stiffness of each story unit and the time delay in control. Extensive simulation results indicate that

Figure 8. Time histories of the system response for the robustness of the no-delay controller against the elastic stiffness of the building structure ($\tilde{\mathbf{K}} = \mathbf{K} \times 80\%$): (a) the first story unit; (b) the sixth story unit; and (c) the active control force.

the H_∞ time-delay controller is applicable when $\tilde{X} \geq 52$ and control performance will deteriorate if $\tilde{X}$ is smaller than 52.

4.3. Robustness against the Time Delay Uncertainties

In this last section, the robustness of the H_∞ controller against the structural parameter uncertainties is investigated. It prompts the following question: “will designed time-delay controller be robust for time delay?” Also in this section, the robustness of the H_∞ controller against the time delay uncertainties of the building structure is studied. The time delay τ

is used for the time-delay controller design and the real time delay of active control system is expressed as $\bar{\tau} = \tau \times \tilde{Y}\%$. When using the time-delay controller designed by $\tau = 0.1$ s to control the building structure, Figs. 12 and 13 show the simulation results for the cases $\tilde{Y} = 50$ (namely $\bar{\tau} = 0.05$ s) and $\tilde{Y} = 200$ (namely $\bar{\tau} = 0.2$ s), respectively. It is observed from Figs. 12 and 13 that the designed H_∞ time-delay controller can robustly stabilize the system and remarkably suppress the responses of building structure in spite of the changes on the time delay in control. Extensive simulation results indicate that the H_∞ time-delay controller is always applicable when

Figure 9. Time histories of the system response for the robustness of the no-delay controller against the elastic stiffness of the building structure ($\tilde{\mathbf{K}} = \mathbf{K} \times 120\%$): (a) the first story unit; (b) the sixth story unit; and (c) the active control force.

$0 \leq \tilde{\gamma} \leq 240$ (namely $0 \leq \tilde{\tau} \leq 0.24$ s), and is not good when $\tilde{\gamma}$ is larger than 240.

5. CONCLUSIONS

By using the H_∞ control method, this paper studies the aseismic robust H_∞ control of a building structure with a time delay. An H_∞ controller with a time delay is presented in this paper. The simulation results indicate that when no time delay exists in the control system, the no-delay controller can evidently reduce the responses of building structure. When a time delay exists, the control performance becomes worse

if the time delay is not compensated in control design. The time-delay controller proposed in this paper can effectively deal with the time delay in control system. Simulation results also show that the proposed time-delay controller has good robustness for both structural parameter and the time delay uncertainties.

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Figure 10. Time histories of the system response for the robustness of the time-delay controller against the elastic stiffness of the building structure ($\tau = 0.05$ s, $\bar{\mathbf{K}} = \mathbf{K} \times 70\%$): (a) the first story unit; (b) the sixth story unit; and (c) the active control force.

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Figure 11. Time histories of the system response for the robustness of the time-delay controller against the elastic stiffness of the building structure ($\tau = 0.05$ s, $\bar{\mathbf{K}} = \mathbf{K} \times 110\%$): (a) the first story unit; (b) the sixth story unit; and (c) the active control force.

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Figure 12. Time histories of the system response for the robustness of the time-delay controller against the time delay ($\tau = 0.1$ s, $\tilde{\tau} = \tau \times 50\% = 0.05$ s): (a) the first story unit; (b) the sixth story unit; and (c) the active control force.

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Figure 13. Time histories of the system response for the robustness of the time-delay controller against the time delay ($\tau = 0.1$ s, $\tilde{\tau} = \tau \times 200\% = 0.2$ s): (a) the first story unit; (b) the sixth story unit; and (c) the active control force.

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