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Tri-Section Method for Solving Fuzzy Non-Linear Equations

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Abstract: *In this paper, an efficient numerical method to solve fuzzy non-linear equations is proposed. It is an extension of the bi-section method, however, total number of iterations of the new method is less than the bi-section method. Numerical experiment supports its fast convergence.*

Keywords: *Fuzzy non-linear equation, Linear fuzzy real number, Tri-section method, Iterative method.*

1. Introduction

Since many real world root-finding problems are too complicated to be defined in precise terms, uncertainty is often needed. So, the variables or parameters may be expressed in terms of fuzzy numbers initially introduced by Zadeh [1].

There are many methods for solving fuzzy non-linear equations. Most of those methods are based on well-known methods of crisp non-linear equations, such as Newton's method [2], fixed point iteration [3], harmonic mean method [4], Broyden's and Newton's methods [5]. In 2013, Saha and Shirin [6] proposed an iterative method of fuzzy non-linear equation using the classical bi-section method over linear fuzzy real numbers. Performance of their method is good and the method is conceptually clear, however, it is relatively slow to converge [7]. In this paper, we provide an algorithm to improve the convergence using tri-section.

2. Preliminaries

In this section, we provide some definitions of linear fuzzy real numbers [8] used in this research.

Definition 2.1 [8] (Linear fuzzy real number) Let R be the set of all real numbers. For some real numbers $a, b, c,$ let $\mu: R \to [0,1]$ be a function defined by

$$\mu(x) = \begin{cases} 0, & if \ \ x < a \text{ or } \ x > c, \\ \dfrac{x-a}{b-a}, & if \ \ a \le x < b, \\ 1, & if \ \ x = b, \\ \dfrac{c-x}{c-b}, & if \ \ b < x \le c. \end{cases}$$

Then a notation $\mu(a,b,c)$ is called a *linear fuzzy real number* with associated triple of real numbers (a,b,c), where $a \le b \le c$, shown in Figure 1.

Let LFR be the set of all linear fuzzy real numbers. Since any real numbers $t \in R$ can be written as a linear fuzzy real number $r(t) \in LFR$, where $r(t) = \mu(t,t,t)$, we can see $R \subseteq LFR$. Fruthermore, the LFR-valued interval $[r(s), r(t)]$ can be considered as the real-valued interval $[s,t]$. Thus, a fuzzy interval $[\mu_1, \mu_2]$ over LFR is the extension of an interval over R.

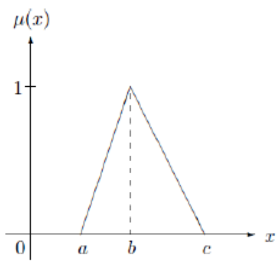

Figure1. *Linear fuzzy real number* $\mu(a,b,c)$

Definition 2.2 [8] (Fuzzy arithmetic) Let $\mu_1 = \mu(a_1, b_1, c_1)$ and $\mu_2 = \mu(a_2, b_2, c_2)$ be two linear fuzzy real numbers. Then addition, subtraction, multiplication, and division of μ_1 and $\mu_2 \in LFR$ are defined by

(1) $\mu_1 + \mu_2 = \mu(a_1 + a_2, b_1 + b_2, c_1 + c_2)$

(2) $\mu_1 - \mu_2 = \mu(a_1 - c_2, b_1 - b_2, c_1 - a_2)$

(3) $\mu_1 \cdot \mu_2 = \mu(\min\{a_1 a_2, a_1 c_2, a_2 c_1, c_1 c_2\}, b_1 b_2, \max\{a_1 a_2, a_1 c_2, a_2 c_1, c_1 c_2\})$

(4) $\frac{\mu_1}{\mu_2} = \mu_1 \cdot \frac{1}{\mu_2}$, where $\frac{1}{\mu_2} = \mu(\min\left\{\frac{1}{a_2}, \frac{1}{b_2}, \frac{1}{c_2}\right\}, \text{median}\left\{\frac{1}{a_2}, \frac{1}{b_2}, \frac{1}{c_2}\right\}, \max\left\{\frac{1}{a_2}, \frac{1}{b_2}, \frac{1}{c_2}\right\})$

We can see that $t \cdot \mu(a, b, c) = \mu(t \cdot a, t \cdot b, t \cdot c)$ for $t > 0$, because a real number t can be considered as a linear fuzzy real number $r(t) = \mu(t, t, t)$.

Definition 2.3 [8] (Fuzzy function) Let $f: R \to R$ be a real-valued function and $\mu(a, b, c) \in LFR$. Let $\bar{a} = \min\{f(a), f(b), f(c)\}, \bar{b} = \text{median}\{f(a), f(b), f(c)\}, \bar{c} = \max\{f(a), f(b), f(c)\}$. Then the function $\bar{f}: LFR \to LFR$ defined by

$$\bar{f}(\mu(a, b, c)) = \mu(\bar{a}, \bar{b}, \bar{c})$$

is called the LFR-*valued function* associated with f. If the associated function f is non-linear, then the LFR-valued function $\bar{f}$ is said to be *non-linear*. The sign $\bar{f}(\mu(a, b, c))$ is said to be *positive*, if the sign of $f(b)$ is positive and the sign $\bar{f}(\mu(a, b, c))$ is said to be *negative*, if the sign of $f(b)$ is negative.

Definition 2.4 [8] (Fuzzy non-linear equation) Let $\bar{f}: LFR \to LFR$ be a non-linear LFR-valued function. Then $\bar{f}(\mu_x) = 0$ is called a *non-linear equation* in LFR with the unknown μ_x. For example, $\bar{f}(\mu_x) = \mu_x^2 - 2 = 0$ is a fuzzy non-linear equation in LFR.

Definition 2.5 [8] (Fuzzy sequence) Let $\{\mu^{(k)}\}_{k=0}^{\infty}$ be a sequence of LFR where $\mu^{(k)} = \mu(a^{(k)}, b^{(k)}, c^{(k)})$. The LFR sequence $\{\mu^{(k)}\}$ has the limit $\mu^* = \mu(a^*, b^*, c^*)$ and we write $\lim_{k\to\infty} \mu^{(k)} = \mu^*$, if the sequences $\{a^{(k)}\}$, $\{b^{(k)}\}$, $\{c^{(k)}\}$ and have the limit a^*, b^*, and c^*, respectively. If $\lim_{k\to\infty} \mu^{(k)}$ exists, we say the LFR sequence $\{\mu^{(k)}\}$ is *convergent*. Otherwise, we say the sequence is *divergent*.

3. Tri-Section Method

In this section, we provide an algorithm using tri-section to solve fuzzy non-linear equation. Consider fuzzy non-linear equation $\bar{f}(\mu_x) = 0$ on the fuzzy interval $[\mu_1, \mu_2] = [\mu(a_1, b_1, c_1), \mu(a_2, b_2, c_2)]$ over LFR. Let $[\mu_1^{(n)}, \mu_2^{(n)}]$ be the fuzzy interval in the n-th iteration, where $\mu_1^{(1)} = \mu(a_1, b_1, c_1)$ and $\mu_2^{(1)} = \mu(a_2, b_2, c_2)$. If $\bar{f}(\mu_1^{(n)})\bar{f}(\mu_2^{(n)}) < 0$, then there exists a root μ_x in $[\mu_1^{(n)}, \mu_2^{(n)}]$ such that $\bar{f}(\mu_x) = 0$ by Intermediate Value Theorem [7]. We divide the interval $[\mu_1^{(n)}, \mu_2^{(n)}]$ into three sub-intervals:

$$[\mu_1^{(n)}, \alpha_1], [\alpha_1, \alpha_2], [\alpha_2, \mu_2^{(n)}]$$

where $\alpha_1 = \frac{2\mu_1^{(n)} + \mu_2^{(n)}}{3}$ and $\alpha_2 = \frac{\mu_1^{(n)} + 2\mu_2^{(n)}}{3}$. Let $\mu_x^{(n)} = \frac{\mu_1^{(n)} + \mu_2^{(n)}}{2}$ be nth approximating solution of a zero μ_x. Once one of the three sub-intervals satisfies the assumption of Intermediate Value Theorem, we proceed an iteration with the sub-interval to the next iteration. We now provide the new algorithm, referred to as the LFR Tri-section method, to solve fuzzy non-linear equation $\bar{f}(\mu_x) = 0$.

Algorithm 3.1 (LFR Tri-section method)

INPUT: Fuzzy equation $\bar{f}(\mu_x) = 0$, fuzzy interval $[\mu_1, \mu_2]$, integer N

OUTPUT: Approximate LFR solution $\mu_x^{(n)}$

Step 1: For $n = 1, 2, \cdots, N$, do steps 2~3.

Step 2: Set $\alpha_1 = \frac{2\mu_1 + \mu_2}{3}$, $\alpha_2 = \frac{\mu_1 + 2\mu_2}{3}$, and $\mu_x^{(n)} = \frac{\mu_1 + \mu_2}{2}$.

Step 3: If $\bar{f}(\mu_1)\bar{f}(\alpha_1) < 0$, then $\mu_2 = \alpha_1$

elseif $\bar{f}(\alpha_1)\bar{f}(\alpha_2) < 0$, then $\mu_1 = \alpha_1, \mu_2 = \alpha_2$

else set $\mu_1 = \alpha_2$.

Step 4: Output(all $\mu_x^{(n)}$) and STOP.

Example 3.2 Solve the fuzzy non-linear equation $\mu_x^2 - 2 = 0$ on the fuzzy interval $[\mu(-0.5, 0, 0.5), \mu(1.5, 2, 2.5)]$ over LFR.

Solution. Let $\bar{f}(\mu_x) = \mu_x^2 - 2 = 0$. Since $\bar{f}(\mu(-0.5, 0, 0.5)) < 0$ and $\bar{f}(\mu(1.5, 2, 2.5)) > 0$, $\mu_x^2 - 2 = 0$ has a root in $[\mu_1, \mu_2] = [\mu(-0.5, 0, 0.5), \mu(1.5, 2, 2.5)]$. For the first iteration of the LFR Tri-section method, we use the fact that $\bar{f}(\alpha_1) < 0$ and $\bar{f}(\alpha_2) < 0$, where $\alpha_1 = \mu(\frac{1}{6}, \frac{2}{3}, \frac{7}{6})$, $\alpha_2 = \mu(\frac{5}{6}, \frac{4}{3}, \frac{11}{6})$, and $\mu_x^{(1)} = \mu(\frac{1}{2}, 1, \frac{3}{2})$. So, we should select the interval $[\alpha_2, \mu_2]$ for our second iteration. Continuing in this manner gives the values in Table 1. The actual root of the crisp equation $x^2 - 2 = 0$ over R is 1.414213562373095. Note that b_{13} in the approximate solution $\mu_x^{(13)} = \mu(a_{12}, b_{12}, c_{12})$ from LFR Tri-section is accurate to six decimal places. We can see in Table 1, the LFR Tri-section method required 13 iterations, whereas the bi-section method needed 20 iterations. Therefore, the LFR Tri-section method converges faster than the bi-section method.

Table1. *Approximate solutions for* $\mu_x^2 - 2 = 0$

n	$\mu_x^{(n)}$ from Bi-section	$\mu_x^{(n)}$ from LFR Tri-section
1	$\mu_x^{(1)} = \mu(0.500000, 1.000000, 1.500000)$	$\mu_x^{(1)} = \mu(0.500000, 1.000000, 1.500000)$
2	$\mu_x^{(2)} = \mu(1.000000, 1.500000, 2.000000)$	$\mu_x^{(2)} = \mu(1.166667, 1.666667, 2.166667)$
3	$\mu_x^{(3)} = \mu(0.750000, 1.250000, 1.750000)$	$\mu_x^{(3)} = \mu(0.944444, 1.444444, 1.944444)$
4	$\mu_x^{(4)} = \mu(0.875000, 1.375000, 1.875000)$	$\mu_x^{(4)} = \mu(0.944444, 1.444444, 1.944444)$
5	$\mu_x^{(5)} = \mu(0.937500, 1.437500, 1.937500)$	$\mu_x^{(5)} = \mu(0.919753, 1.419753, 1.919753)$
6	$\mu_x^{(6)} = \mu(0.906250, 1.406250, 1.906250)$	$\mu_x^{(6)} = \mu(0.911523, 1.411523, 1.911523)$
7	$\mu_x^{(7)} = \mu(0.921875, 1.421875, 1.921875)$	$\mu_x^{(7)} = \mu(0.914266, 1.414266, 1.914266)$
8	$\mu_x^{(8)} = \mu(0.914063, 1.414063, 1.914063)$	$\mu_x^{(8)} = \mu(0.914266, 1.414266, 1.914266)$
9	$\mu_x^{(9)} = \mu(0.917969, 1.417969, 1.917969)$	$\mu_x^{(9)} = \mu(0.914266, 1.414266, 1.914266)$
10	$\mu_x^{(10)} = \mu(0.916016, 1.416016, 1.916016)$	$\mu_x^{(10)} = \mu(0.914165, 1.414165, 1.914165)$
11	$\mu_x^{(11)} = \mu(0.915039, 1.415039, 1.915039)$	$\mu_x^{(11)} = \mu(0.914198, 1.414198, 1.914198)$
12	$\mu_x^{(12)} = \mu(0.914551, 1.414551, 1.914551)$	$\mu_x^{(12)} = \mu(0.914210, 1.414210, 1.914210)$
13	$\mu_x^{(13)} = \mu(0.914307, 1.414307, 1.914307)$	$\mu_x^{(13)} = \mu(0.914213, 1.414213, 1.914213)$
14	$\mu_x^{(14)} = \mu(0.914185, 1.414185, 1.914185)$	
15	$\mu_x^{(15)} = \mu(0.914246, 1.414246, 1.914246)$	
16	$\mu_x^{(16)} = \mu(0.914215, 1.414215, 1.914215)$	
17	$\mu_x^{(17)} = \mu(0.914200, 1.414200, 1.914200)$	
18	$\mu_x^{(18)} = \mu(0.914207, 1.414207, 1.914207)$	
19	$\mu_x^{(19)} = \mu(0.914211, 1.414211, 1.914211)$	
20	$\mu_x^{(20)} = \mu(0.914213, 1.414213, 1.914213)$	

4. Conclusion

In this paper, we presented the LFR Tri-section method to solve fuzzy non-linear equations over linear fuzzy real numbers. The method was obtained by modification of the bi-section method. Numerical experiments show that the LFR Tri-section method converges faster than the bi-section method.

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