ORIGINAL ARTICLE

Hypsometric relationship in *Tectona grandis* L. F. stands using quantile regression

Relação hipsométrica em povoamentos de Tectona grandis L. F. utilizando regressão quantílica

Luciano Rodrigo Lanssanova¹ , Franciele Alba da Silva² , Sebastião do Amaral Machado² , Allan Libanio Pelissari² , Afonso Figueiredo Filho³ , Luan Fiorentin² , Clebson Lima Cerqueira²

¹Instituto Federal de Mato Grosso – IFMT, Juína, MT, Brasil

²Universidade Federal do Paraná – UFPR, Curitiba, PR, Brasil

³Universidade do Centro-Oeste – UNICENTRO, Irati, PR, Brasil

How to cite: Lanssanova, L. R., Silva, F. A., Machado, S. A., Pelissari, A. L., Figueiredo Filho, A., Fiorentin, L., & Cerqueira, C. L. (2021). Hypsometric relationship in *Tectona grandis* L. F. stands using quantile regression. *Scientia Forestalis*, 49(131), e3559. <https://doi.org/10.18671/scifor.v49n131.07>

Abstract

Quantile regression is an alternative to the traditional methods of model fitting for problems caused by the presence of outliers or asymmetry in the data distribution. Our aim was to evaluate and propose hypsometric relationship models fitting *Tectona grandis* L. f. stands by means of the quantile regression and least squares method. The diameter was measured at 1.3 meters from the ground and the total height of 4,068 sampled trees were divided into two bases, one for adjustment and the other for validation. Later the non-linear logistic model was adjusted via quantile regression for which the first (0.25), second (0.50) and third quartiles (0.75) were tested. The least squares method (OLS) was also used to compare it with the quantile regression. To evaluate the precision and the adjustment of the evaluated methods, the statistics Akaike's Information Criterion (AIC), square root of the mean error (RMSE), mean absolute error (MAE) and coefficient of determination (R^2) were used. In sequence, the analysis of variance was performed to verify the existence of significant difference between the estimate from the evaluated methods and the observed heights. The adjusted logistic model was accurate to estimate the total heights of *Tectona grandis* stands. Quantile regression allowed a more complete view of the relationship between response height and diameter at breast height since it was possible to build a model for each quantile of interest. Quantile regression showed good results and can be recommended for calibration of hypsometric models in *Tectona grandis* stands due to the ease of adjustment and implementation in forest inventories.

Keywords: Conditional quantiles; Diameter/height ratio; Simplex; Discrepant data.

Resumo

O uso da regressão quantílica é uma alternativa aos métodos tradicionais de ajuste de modelos para problemas causados pela presença de *outliers* ou assimetrias na distribuição de dados. Com isto, o objetivo deste trabalho foi avaliar e propor modelos de relação hipsométrica para *Tectona grandis* L. f. ajustados por meio do método de regressão quantílica e mínimos quadrados. O diâmetro foi medido a 1,3 metros do solo e a altura total de 4.068 árvores amostradas foi dividida em duas bases, uma para ajuste e a outra para validação. Posteriormente, o modelo logístico não linear foi ajustado através de regressão quantílica, para o qual foram testados o primeiro (0,25), segundo (0,50) e terceiro quartis (0,75). O método dos mínimos quadrados (MMQ) também foi utilizado para compará-lo com a regressão quantílica. Para avaliar a precisão e o ajuste dos métodos avaliados, foram utilizados os critérios de informação estatística Akaike (AIC), raiz quadrada do erro médio (RMSE), erro médio absoluto (MAE) e coeficiente de determinação (R^2). Em sequência, a análise de variância foi realizada para verificar a existência de diferença significativa entre a estimativa dos métodos avaliados e as alturas observadas. O modelo logístico ajustado foi preciso para

Financial support: Instituto Federal de Mato Grosso, PROPES – Pró-reitoria de pesquisa.

Conflict of interest: Nothing to declare.

Corresponding author: luciano.lanssanova@jna.ifmt.edu.br

Received: 15 June 2020.

Accepted: 25 November 2020.

Editor: Paulo Henrique Müller Silva.

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estimar as alturas totais dos estandes de *Tectona grandis*. A regressão quantílica permitiu uma visão mais completa da relação entre altura de resposta e diâmetro à altura do peito, uma vez que foi possível construir um modelo para cada quantil de interesse. Essa metodologia de ajuste mostrou bons resultados e pode ser recomendada para a calibração de modelos hipsométricos em povoamentos de *Tectona grandis* devido à facilidade de ajuste e implementação nos inventários florestais.

Palavras-chave: Quantis condicionais; Razão diâmetro/altura; Simplex; Dados discrepantes.

INTRODUCTION

The height estimation in a stand has a high relevance, considering the impossibility of measuring all individuals in the forest inventory, due to the time and costs related to the measurement of this variable (Floriano et al., 2006; Özçelik et al., 2018). In this sense, several researches were carried out with the purpose of minimizing costs with height measurement, such as the evaluation of simple input models for volume estimation (Lansanova et al., 2018) and comparison of different models and methodologies to estimate the height in forest stands (Zang et al., 2016; Loureiro et al., 2016).

Models fitting to estimate height is quite usual, and the ordinary least squares estimator method is the most used in the forestry area to obtain the regression coefficients (Mendonça et al., 2011; David et al., 2016; Nicoletti et al., 2016; Cerqueira et al., 2017). This method uses the conditional means to explain the relationship between the variables of interest; however, it is influenced by extreme points and limitations for data with asymmetric and non-normally distributed (Barroso et al., 2015). In the presence of discrepant values (outliers), the use of this method may result in inadequate estimates with low accuracy and biases (Koenker & Bassett, 1978).

The use of quantile regression is an alternative to problems caused by the presence of outliers or asymmetry in the data distribution. In addition, unlike traditional model fitting approaches, regression coefficients can be obtained through mathematical programming, such as linear programming (Koenker & Bassett, 1978), using conditional quantiles, which makes it flexible to select the quantiles that best represents the relationship between variables (Özçelik et al., 2018).

Since the introduction of quantile regression by Koenker & Bassett (1978), the method has been used in forestry research for self-thinning boundary lines (Zhang et al., 2005), diameter growth (Bohora & Cao, 2014), tree taper (Cao & Wang, 2015), and in crown modeling (Sun et al., 2017). In addition, Zang et al. (2016), adjusted height-diameter equations for Larch plantations in China. Özçelik et al. (2018) used the method in estimating tree heights from diameter for two species from Turkey: *Pinus brutia* Ten and *Cedrus libani* A. Rich. However, in Brazil there are few studies on the adjustments of diameter height equations with the quantile regression method, with the exception for the research of Pontes Neto (2012).

The aim of this work was to evaluate hypsometric relationship models fitting *Tectona grandis* L. f. stands by means of quantile regression. For this purpose, the hypothesis that it allows to obtain accurate estimates of the total height was considered, being an option for data with occurrence of outliers and asymmetric distribution.

MATERIAL AND METHODS

This study was carried out in seedling plantation of *Tectona grandis*, aged from 5 to 25 years, located in Brasnorte, Mato Grosso state, Brazil. For this purpose, the diameter outside bark at 1.3 m (d) and the total height (h) were measured from 4,068 sample trees. Subsequently, the descriptive statistical analysis of the data was applied, as well as the construction of a boxplot for the diameter and total height dendrometric variables in order to verify the existence of outliers.

The logistic non-linear model (1) was adjusted, in which the estimates of the regression coefficients (β_i) via quantile regression were obtained through Simplex, in which the minimization of the sum of the absolute errors $\sum |y_i - x_i' \beta|$ was considered as the objective function. The model was adjusted by the inner points algorithm proposed by Koenker & Park (1996), which has the purpose

of computing quantum regression estimates for cases in which the response function is non-linear in the parameters. For this, the *quantreg* package (Koenker, 2013) of the R program (R Development Core Team, 2017) was used with three quantile variations in the adjustments, evaluating the first (0.25), the second (0.50) and the third quartile (0.75).

The adjustment from the least squares, on the other hand, was carried out through an iterative process with the *nls* function, which is indicated to adjust non-linear models (Ferreira, 2013). The program uses the Gauss-Newton algorithm as default and a maximum number of 500 iterations was specified using the *maxiter* function.

$$h = \frac{\beta_0}{1 + e^{\left[\frac{(\beta_1 - d)}{\beta_2} \right]}} + \varepsilon \quad (1)$$

Where: β_i = regression coefficients; d = diameter at 1.3 m (cm); h = total height (m); e = base of Napierian logarithms; ε = stochastic error.

To evaluate the accuracy and adjustment of the evaluated methods, the statistics Akaike's Information Criterion (AIC) (2), square root of mean error (RMSE) (3), mean absolute error (MAE) (4) and coefficient of determination (R^2) (5) were used. All parameters estimated by the methods tested had their significance evaluated by the t test, at % significance level. Besides the adjustment and precision statistics, the residual graphs (6) as well as the estimated curves from each evaluated method were made.

$$AIC = 2k - 2\ln(\hat{L}) \quad (2)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (3)$$

$$MAE = \frac{1}{n} |y_i - \hat{y}_i| \quad (4)$$

$$R^2 = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y}_i)^2} \quad (5)$$

$$Residuals(\%) = \frac{y_i - \hat{y}_i}{y_i} . 100 \quad (6)$$

where $\hat{L}$ is the likelihood function for the model; y_i and $\hat{y}_i$ are the observed and predicted values of tree height, respectively; and $\bar{y}_i$ is the average value of $\hat{y}_i$.

Of the 4,068 trees used in the research, 60 individuals were randomly separated for method validation, in which they were evaluated by the same statistics used in the adjustment. From these data, a graph was also made showing the linear correlations between the different methods evaluated and the values observed. Afterwards, analysis of variance was performed to verify the existence of significant difference between the estimate from the evaluated methods and the observed heights. Then, the multiple comparison procedure was applied through Tukey's test to the 95% probability level.

RESULTS AND DISCUSSION

Table 1 shows the minimum, mean, maximum, quantiles values of the distribution and the respective coefficients of variation of total height (h) and diameter at 1.3 m (d). Although

the variable diameter has a high range between maximum and minimum of 48.38 cm, it has a low interquartile range of 5.29 cm, as well as symmetrical distribution. From the 3rd quartile of total height it is possible to infer that 75% of the individuals measured are below 19.8 m, the maximum value of the set is 28.8 m.

Table 1. Descriptive statistics of the variables diameter at 1.3 m (d) and total height (h) for *Tectona grandis*

Variable	Descriptive statistics						
	x_{min}	$\bar{x}$	Q1	Q2	Q3	x_{max}	cv%
Diameter at 1.3 m (cm)	4.45	18.67	15.59	18.14	20.88	52.83	19.92%
Total height (m)	4.4	17.09	14.3	16.9	19.8	28.8	21.76%

x_{min} = minimum value; $\bar{x}$ = arithmetic mean; Q1 = first quartile; Q2 = second quartile; Q3 = third quartile; x_{max} = maximum value; e cv% = coefficient of variation.

Discrepant values were observed in the data set (Figure 1), for both diameter and total height, the curve that shows the relationship between these two variables was quite steep, possibly due to factors related to age and site quality (Machado et al., 2008). Considering a stand of the same age, the individuals that develop in sites of better quality will have bigger diameter and height in relation to the trees in bad sites. Thus, the slope of the hypsometric curve tends to reduce in the most productive sites to the less productive ones (David et al., 2016).

Figure 1. Scatter plot and boxplot of the variables diameter at 1.3 m and total height for *Tectona grandis*.

The adjustment and precision statistics showed similar results for the different quantile positions in the adjustment as well as for the least squares method (Table 2). It is worth noting that although the coefficient of determination is not an adequate measure to evaluate the adjustment of non-linear models (Juliano & Williams, 1987; Spiess & Neumeyer, 2010), it is a commonly used measure and easy to interpret when evaluating models with forest variables (Schröder et al., 2014). All coefficients by the four methods evaluated were significant at $p < 0.01$ by t-test.

Table 2. Regression coefficients and goodness of fit of the hypsometric relationship models in different quantiles for *Tectona grandis*.

Method	Coefficients			Fit				Validation		
	B_0	B_1	B_2	R^2	AIC	RMSE	MAE	R^2	RMSE	MAE
MQO	21.49**	11.169**	4.692**	0.51	19104.06	2.57	2.03	0.80	2.62	2.02
Quantile (0.25)	18.84**	10.77**	4.47**	0.51	19613.11	3.12	2.43	0.79	3.36	2.38
Quantile (0.50)	21.94**	11.30**	5.15**	0.51	19390.69	2.57	2.02	0.81	2.56	1.98
Quantile (0.75)	24.77**	11.68**	5.20**	0.51	19979.68	3.16	2.51	0.81	2.95	2.52

B's = Regression coefficients; AIC = Akaike information criterion (AIC); RMSE = root mean squared error (RMSE), R^2 = coefficient of determination (R^2) and MAE = mean absolute error (MAE).

The quantile regression, using Q2 (0.50), which coincides with the mean of the data distribution, resulted in higher fitting statistics than the other quantiles evaluated. Possibly, this is because the mean and the median are the most appropriate position measurements to represent normally distributed datasets.

The quantile regression gives a more complete view of the relationship between the studied variables, since it allows to observe the functional relationship in different levels of the response variable, since they are able to incorporate a possible heteroscedasticity, which would be detected from the variation of the parameter estimates in the different quantiles evaluated (Santos, 2016). In this way, it is possible to outline the relation in central regions of the distribution, through the median and in the tails of the conditional distribution, as done in this work, using quantiles 0.25 and 0.75.

Thus, it is possible to evaluate the behavior of the asymptote (parameter β_0) and the inflection point (parameter β_1) in the Logistic model for the three quantiles tested (Table 2). The inflection point, which represents the value of the independent variable (diameter) where the dependent variable (height) reaches the half of β_0 (Mendonça et al., 2011), was similar for the three quantiles evaluated. This is because the central tendency measures (average and median) are close (Table 1), since the data distribution does not present a strong asymmetry, with an asymmetry coefficient equal to 0.15. For the upper horizontal asymptote, which corresponds to the maximum value of the variable (height) in the different quantiles tested, it was verified that the maximum height was 24.84 m for Q3, and 18.88 m for Q1.

We present the graphical residual analysis for the different methods evaluated for height estimation in Figure 2. The result obtained by the evaluated methods was not robust to the presence of outliers, indicating that the presence of discrepant data is not influencing the estimation of the curve adjusted by quantum regression (Barroso et al., 2015). In general, the residual dispersion (Figure 2) for the heights estimated by the different methods evaluated showed homogeneity of variance in all estimated height classes, with few points presenting a variation above 100% amplitude. In addition, the residuals showed no bias, with proportionality between estimates for the response variable. The main difference between the estimates of the quantile regression and the ordinary least squares method is the distance of the points observed by the estimated curve, which is measured by minimizing the weighted average of the sum of the vertical distances in the quantile regression. Thus, for points below the line, weight equal to 1 subtracted from the established quantile is attributed, as in the present research, used for 0.25, 0.50 and 0.75, whereas for points above the line, the weight is related to the quantile value (Hao & Naiman, 2007).

Figure 2. Residual plots and observed versus adjusted values for the different methods evaluated for the height estimate.

Figure 3 shows the curves adjusted by the method of least squares and quantile regression considering the three quantiles evaluated (0.25, 0.50, 0.75). Thus, it was possible to evaluate the impact of the diameter at breast height on the entire distribution of total height, and not only on its mean. Different diameter results in quantiles are related to differences in height response to changes in coefficients at various points in the conditional distribution of the variable depends (Silva & Porto Júnior, 2006).

Figure 3. Curves estimated for total height by the different evaluated methods.

The ages that compose the sample for the adjustment of the model varied from 5 to 16 years (Figure 4). The first quantile evaluated (0.25) yielded the lowest error (4.45%) for the estimation of heights with ages below 8 years. The third quantile (0.75) and the least squares method presented the largest errors for this same age range, with 14.99% and 5.26%, respectively.

Figure 4. Interaction between different methods for estimating total height and age for *Tectona grandis*.

The general non-linear regression model, based on the mean regression technique, was not able to fully describe the height-diameter relationship (Figure 4). Thus, it is possible to observe the flexibility of the quantum regression when describing different patterns of height-diameter relation (Fu et al., 2018; Özçelik et al., 2018; Zhang et al., 2020).

The results of quantile regression using the median were similar to regression based on modeling the mean of the data. Thus, the model adjusted with the quantile at 0.50 covered most of the observations predicted by the method of least squares. Cade & Noon (2003) and Muggeo et al. (2013) found similar results.

Heights above 22 meters were not estimated by the traditional method of regression. By using quantum regression with the quantile third (0.75), it was possible to predict the variable of interest in this range of distribution (Figure 4). Thus, it is verified that the quantile regression estimates several ranges of changes, from minimum to maximum, providing a more complete picture of response when compared to the method of least squares.

In this context, quantile regression estimates can be used to construct forecast and tolerance intervals without assuming any parametric error distribution and without specifying how heterogeneity of variance is linked to changes in means (Puiatti et al., 2018). In a fully

parametric model where the error distribution takes some specified form, the various quantiles of the response distribution are estimated by a specified multiple of the estimated standard deviation of the parametric error distribution, which is then added to the estimated mean function (Cade & Noon, 2003).

In the data used for validation, all the methods evaluated performed well, with R^2 ranging from 0.79 to 0.81, and with low RMSE and MAE ranging from 2.56 to 3.36 and 1.98 to 2.52, respectively. All linear correlations between the different methods and the heights observed in the field were all greater than 0.89 (Figure 5).

Figure 5. Pearson's linear correlation between the different evaluated methods and the observed heights

The comparative analysis of variance between the different methods showed that there was a significant difference between the methods, at the level of 5% significance (Tables 3 and 4). Thus, when proceeding with the test of means, it was verified that the methods evaluated are more distant from the observed values, referring to the amounts 0.25 and 0.75, respectively. The other evaluated methods do not differ statistically from the observed heights.

Table 3. Analysis of variance for the different methods to estimate the total height.

FV	GL	SQ	QM	<i>F</i> calculate	<i>P</i> value
Métodos	4	347.2	86.79	4.88*	0.0007
Resíduos	295	5243.4	17.77		

Table 4. Averages test for the different methods evaluated to estimate the height of *Tectona grandis*.

Methods	Averages
Quantile 3 (0.75)	17.92 a
least squares	16.16 ab
Quantile 2 (0.50)	16.16 ab
Observed values	16.14 ab
Quantile 1 (0.25)	14.52 b

The quantile regression is based on the minimization of the absolute errors, allowing to evaluate the impact of the independent variables in different positions along the distribution of the dependent variable (Koenker & Bassett, 1978), unlike the traditional regression through

the mean as a measure of position. Thus, it is possible to evaluate the response of each quantile, as in the present research, using the quantiles 0.25, 0.50 and 0.75. It is important to emphasize that the choice of the quantile can also be considered as a solution to a given absolute minimization problem (Hao & Naiman, 2007).

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The fitting of hypsometric models in researches in Brazil is almost predominantly performed by the method of ordinary least squares (Caldeira et al., 2003; Bartoszeck et al., 2004; Machado et al., 2008; Oliveira et al., 2015; Nicoletti et al., 2016; Cerqueira et al., 2017; Souza et al., 2017; Schmitt, 2017). However, quantile regression may be an alternative, due to the ease of fit and implementation in forest inventories. In addition, it can be applied to asymmetric data sets, whose presence of outliers can influence the fit of the model, resulting in a more robust method than traditional fittings (Barroso et al., 2015).

CONCLUSIONS

The adjusted logistical model was accurate to estimate the total heights of *Tectona grandis* stands. The median quantile regression was efficient to adjust the non-linear regression model from the measured diameters, being an alternative for the adjustment of hypsometric models, due to the ease of adjustment and implementation in forest inventories.

The quantile regression allows a complete view of the relationship between the response variable and the observed covariates, since it is possible to build a model for each quantile of interest. Thus, it becomes an option that makes it possible to adjust several hypsometric curves referring to different quantiles of the distribution.

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Authors' contributions: LRL: Conceptualization, data curation, formal analysis, funding acquisition, methodology, writing original draft preparation, writing; FAS: writing-review and editing; SAM: writing-review and editing; ALP: writing-review and editing; AFF: writing-review; LF: writing review; CLC: writing-review.