IDENTIFICATION OF CLASSES OF BILINEAR TIME SERIES MODELS

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Abstract

In this paper, two classes of bilinear time series model are obtained under certain conditions from the general bilinear autoregressive moving average model. Bilinear autoregressive (BAR) and bilinear moving average (BMA) models have been identified. From the general bilinear model, BAR and BMA models have been proved to exist for $q = Q = 0, = > j = 0$, and $p = P = 0, = > i = 0$, respectively. These models are found useful in modelling most of the economic and financial data.

1. Introduction

Bilinear time series model is a class of time series model, which assume both linear and non-linear components of autoregressive moving average processes. The linear part of the bilinear model is the aggregation of autoregressive and moving average processes, while the non-linear part is the product of the two processes. The general bilinear

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time series model as defined by Kendell and Ord [5]; Bibi and Oyet [1], and Iwueze [4] is

$$X_t = \sum_{i=1}^p \phi_i X_{t-i} + \sum_{j=1}^q \theta_j \epsilon_{t-j} + \sum_{i=1}^P \sum_{j=1}^Q \beta_{ij} X_{t-i} \epsilon_{t-j} + \epsilon_t, \quad (1)$$

where X_t is the bilinear process, X_{t-i} and ϵ_{t-j} are time varying autoregressive and moving average processes, respectively, with ϕ_i and θ_j as their respective parameters; β_{ij} are the non-linear parameters of $X_{t-i} \epsilon_{t-j}$, $1 \leq i \leq p$, $1 \leq j \leq q$ and $1 \leq i \leq P$, $1 \leq j \leq Q$; ϵ_t is the error term, $\epsilon_t \sim N(0, \sigma_{\epsilon_t}^2)$. Model (1) is a difference equation for a time varying bilinear process of order (p, q, P, Q) , also known as $BL(p, q, P, Q)$.

Granger and Anderson [3] identified a complete non-linear part of bilinear model with $p = q = 0$. That is,

$$X_t = \sum_{i=1}^P \sum_{j=1}^Q \beta_{ij} X_{t-i} \epsilon_{t-j} + \epsilon_t. \quad (2)$$

It is expressed in the matrix form as

$$X_t = (X_{t-1}, X_{t-2}, \dots, X_{t-P})^T (\epsilon_{t-1}, \epsilon_{t-2}, \dots, \epsilon_{t-Q}) \underline{\beta} + \epsilon_t, \quad (3)$$

where $\underline{\beta}$ is the matrix of parameters of order P and Q .

$$\underline{\beta} = (\beta_{ij}), i = 1, 2, \dots, P; j = 1, 2, \dots, Q.$$

Usoro and Omekara [7] used the non-linear part of bilinear time series model to express pure diagonal bilinear model. From model (2), if $i = j$, the coefficient matrix $\underline{\beta} = (\beta_{ij})$, $i = 1, 2, \dots, P$; $j = 1, 2, \dots, Q$ of the non-linear part of the bilinear model exists with elements in the principal diagonal of the matrix. Maravell [6] and Franses [2] ascertained the reliability of bilinear model and its performance over alternative linear models when modelling economic and financial data.

2. Model Identification

This section deals with identification of classes of bilinear models from model (1) under certain conditions.

2.1. Bilinear autoregressive (BAR) model

Given the difference equation

$$X_t = \sum_{i=1}^p \phi_i X_{t-i} + \sum_{j=1}^q \theta_j \epsilon_{t-j} + \sum_{i=1}^P \sum_{j=1}^Q \beta_{ij} X_{t-i} \epsilon_{t-j} + \epsilon_t.$$

If $q = Q = 0$, $\Rightarrow j = 0$, the model becomes

$$X_t = \sum_{i=1}^p \phi_i X_{t-i} + \sum_{i=1}^P \beta_{i0} X_{t-i} \epsilon_t + \epsilon_t, \quad (4)$$

where β_{i0} are the coefficients of the non-linear autoregressive part of the bilinear model. ϕ_i , X_{t-i} , ϵ_t are as described earlier.

Proof. Let $X_t = Y_t + Z_t + \epsilon_t$, where,

$$Y_t = \sum_{i=1}^p \phi_i X_{t-i} + \sum_{j=1}^q \theta_j \epsilon_{t-j}, \quad (5)$$

and

$$Z_t = \sum_{i=1}^P \sum_{j=1}^Q \beta_{ij} X_{t-i} \epsilon_{t-j}. \quad (6)$$

From Equation (5), if $q = 0$, $\Rightarrow j = 0$, the model becomes

$$Y_t = \sum_{i=1}^p \phi_i X_{t-i}. \quad (7)$$

Equation (6) is expressed in matrix form as

$$Z_t = (X_{t-1}, X_{t-2}, \dots, X_{t-P})^T (\epsilon_{t-1}, \epsilon_{t-2}, \dots, \epsilon_{t-Q}) \beta.$$

The matrix expansion is

$$Z_t = \begin{pmatrix} X_{t-1}^{\epsilon_{t-1}} & X_{t-2}^{\epsilon_{t-1}} & \cdot & \cdot & \cdot & X_{t-P}^{\epsilon_{t-Q}} \\ X_{t-1}^{\epsilon_{t-2}} & X_{t-2}^{\epsilon_{t-2}} & \cdot & \cdot & \cdot & X_{t-P}^{\epsilon_{t-Q}} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{t-1}^{\epsilon_{t-Q}} & X_{t-2}^{\epsilon_{t-Q}} & \cdot & \cdot & \cdot & X_{t-P}^{\epsilon_{t-Q}} \end{pmatrix} \begin{pmatrix} \beta_{11} & \beta_{12} & \cdot & \cdot & \cdot & \beta_{1Q} \\ \beta_{21} & \beta_{22} & \cdot & \cdot & \cdot & \beta_{2Q} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \beta_{P1} & \beta_{P2} & \cdot & \cdot & \cdot & \beta_{PQ} \end{pmatrix}. \quad (8)$$

From (8), if $Q = 0$, $\Rightarrow j = 0$, then

$$Z_t = \begin{pmatrix} X_{t-1}^{\epsilon_{t-0}} & X_{t-2}^{\epsilon_{t-0}} & \cdot & \cdot & \cdot & X_{t-P}^{\epsilon_{t-0}} \\ X_{t-1}^{\epsilon_{t-0}} & X_{t-2}^{\epsilon_{t-0}} & \cdot & \cdot & \cdot & X_{t-P}^{\epsilon_{t-0}} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{t-1}^{\epsilon_{t-0}} & X_{t-2}^{\epsilon_{t-0}} & \cdot & \cdot & \cdot & X_{t-P}^{\epsilon_{t-0}} \end{pmatrix} \begin{pmatrix} \beta_{10} & \beta_{10} & \cdot & \cdot & \cdot & \beta_{10} \\ \beta_{20} & \beta_{20} & \cdot & \cdot & \cdot & \beta_{20} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \beta_{P0} & \beta_{P0} & \cdot & \cdot & \cdot & \beta_{P0} \end{pmatrix}, \quad (9)$$

$$Z_t = (X_{t-1}^{\epsilon_{t-0}} \ X_{t-2}^{\epsilon_{t-0}} \ \cdots \ X_{t-P}^{\epsilon_{t-0}}) \begin{pmatrix} \beta_{10} \\ \beta_{20} \\ \cdot \\ \cdot \\ \cdot \\ \beta_{P0} \end{pmatrix} = \beta_{10}X_{t-1}^{\epsilon_t} + \beta_{20}X_{t-2}^{\epsilon_t} \\ + \cdots + \beta_{P0}X_{t-P}^{\epsilon_t},$$

$$Z_t = \sum_{i=1}^P \beta_{i0} X_{t-i} \epsilon_t. \quad (10)$$

Combining (7) and (10), X_t becomes,

$$X_t = \sum_{i=1}^p \phi_i X_{t-i} + \sum_{i=1}^P \beta_{i0} X_{t-i} \epsilon_t + \epsilon_t.$$

This completes the proof. Hence, Equation (4) is a bilinear autoregressive (BAR) model of order p, P . That is $BL(p, P)$.

2.2. Bilinear moving average (BMA) model

Corollary. *Given the difference equation*

$$X_t = \sum_{i=1}^p \phi_i X_{t-i} + \sum_{j=1}^q \theta_j \epsilon_{t-j} + \sum_{i=1}^P \sum_{j=1}^Q \beta_{ij} X_{t-i} \epsilon_{t-j} + \epsilon_t.$$

If $p = P = 0$, $\Rightarrow i = 0$, the model becomes

$$X_t = \sum_{j=1}^q \theta_j \epsilon_{t-j} + \sum_{j=1}^Q \beta_{0j} X_t \epsilon_{t-j} + \epsilon_t, \quad (11)$$

where β_{0j} are the coefficients of the non-linear moving average part of the bilinear model. $\theta_j, X_t, \epsilon_{t-j}$ are as described earlier.

Proof. Let $X_t = R_t + S_t + \epsilon_t$, where

$$R_t = \sum_{i=1}^p \phi_i X_{t-i} + \sum_{j=1}^q \theta_j \epsilon_{t-j}, \quad (12)$$

and

$$S_t = \sum_{i=1}^P \sum_{j=1}^Q \beta_{ij} X_{t-i} \epsilon_{t-j}. \quad (13)$$

From Equation (12), if $p = 0$, $\Rightarrow i = 0$, the model becomes

$$R_t = \sum_{j=1}^q \theta_j \epsilon_{t-j}. \quad (14)$$

Equation (13) is expressed in matrix form as

$$S_t = (X_{t-1}, X_{t-2}, \dots, X_{t-P})' (\epsilon_{t-1}, \epsilon_{t-2}, \dots, \epsilon_{t-Q}) \underline{\beta}.$$

The matrix expansion is

$$S_t = \begin{pmatrix} X_{t-1} \epsilon_{t-1} & X_{t-2} \epsilon_{t-1} & \cdot & \cdot & \cdot & X_{t-P} \epsilon_{t-Q} \\ X_{t-1} \epsilon_{t-2} & X_{t-2} \epsilon_{t-2} & \cdot & \cdot & \cdot & X_{t-P} \epsilon_{t-Q} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{t-1} \epsilon_{t-Q} & X_{t-2} \epsilon_{t-Q} & \cdot & \cdot & \cdot & X_{t-P} \epsilon_{t-Q} \end{pmatrix} \begin{pmatrix} \beta_{11} & \beta_{12} & \cdot & \cdot & \cdot & \beta_{1Q} \\ \beta_{21} & \beta_{22} & \cdot & \cdot & \cdot & \beta_{2Q} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \beta_{P1} & \beta_{P2} & \cdot & \cdot & \cdot & \beta_{PQ} \end{pmatrix}, \quad (15)$$

From (8), if $p = 0$, $\Rightarrow i = 0$, then

$$S_t = \begin{pmatrix} X_{t-0} \epsilon_{t-1} & X_{t-0} \epsilon_{t-2} & \cdot & \cdot & \cdot & X_{t-0} \epsilon_{t-Q} \\ X_{t-0} \epsilon_{t-1} & X_{t-0} \epsilon_{t-2} & \cdot & \cdot & \cdot & X_{t-0} \epsilon_{t-Q} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{t-0} \epsilon_{t-1} & X_{t-0} \epsilon_{t-2} & \cdot & \cdot & \cdot & X_{t-0} \epsilon_{t-Q} \end{pmatrix} \begin{pmatrix} \beta_{01} & \beta_{01} & \cdot & \cdot & \cdot & \beta_{01} \\ \beta_{02} & \beta_{02} & \cdot & \cdot & \cdot & \beta_{02} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \beta_{0Q} & \beta_{0Q} & \cdot & \cdot & \cdot & \beta_{0Q} \end{pmatrix}, \quad (16)$$

$$\begin{aligned}
S_t = (X_{t-0}^{\epsilon_{t-1}} \ X_{t-0}^{\epsilon_{t-2}} \ \cdots \ X_{t-0}^{\epsilon_{t-Q}}) & \begin{pmatrix} \beta_{01} \\ \beta_{02} \\ \cdot \\ \cdot \\ \cdot \\ \beta_{0Q} \end{pmatrix} = \beta_{01}X_{t-0}^{\epsilon_{t-1}} + \beta_{02}X_{t-0}^{\epsilon_{t-2}} \\
& + \cdots + \beta_{0Q}X_{t-0}^{\epsilon_{t-Q}}, \\
S_t = \sum_{j=1}^Q \beta_{0j}X_{t-0}^{\epsilon_{t-j}}. & \tag{17}
\end{aligned}$$

Combining (14) and (17), X_t becomes,

$$X_t = \sum_{j=1}^q \theta_j \epsilon_{t-j} + \sum_{j=1}^Q \beta_{0j} X_{t-0}^{\epsilon_{t-j}} + \epsilon_t$$

This completes the proof. Hence, Equation (11) is a bilinear moving average (BMA) model of order q, Q . That is $BL(q, Q)$.

3. Conclusion

Bilinear time series model is a class of time series model that has found applications in various areas of time series modelling, especially when dealing with data series that assume non-linearity behaviour. Bilinear time series model, like ordinary autoregressive moving average model, is characterised by individual autoregressive and moving average processes. The introduction of the non-linear part is what distinguished a bilinear process from the linear process. Here, bilinear autoregressive and bilinear moving average models have been established and proved.

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