NEW RESULTS ON RESISTANCE-HARARY INDICES

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Abstract

Resistance-Harary index of a connected graph G is defined as

$$RH(G) = \sum_{\{u,v\} \subseteq V(G)} \frac{1}{r_G(u,v)},$$

where $r_G(u, v)$ is the resistance distance between vertices u and v in G . It originates from many physical problems. In this paper, we completely determine Resistance-Harary indices of two classes graphs.

Keywords: resistance-Harary index, path, cycle.

1. Introduction

Topological indices of molecular graphs are very important invariants of a graph, which can be used to explore properties of a molecular

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structured graph. It has been studied by a lot of scientists both in mathematics, physics and in chemistry for many years.

Let G be a simple and connected graph with $|V(G)| = n$ and $|E(G)| = m$. For any $v \in V(G)$, $d(v)$ is the degree of vertex v , the distance between vertices u and v , denoted by $d(u, v)$, is the length of a shortest path between them. The resistance distance $r(u, v)$ (if more than one graphs are considered, we write $r_G(u, v)$ in order to avoid confusion) between vertices u and v in G is defined as the effective resistance between the two nodes of the electronic network obtained so that its nodes correspond the vertices of G and each edge of G is replaced by a resistor of unit resistance, which is computed by the methods of the theory of resistive electrical networks based on Ohm's and Kirchhoff's laws.

In 1993, Klein and Randić [3] defined a new distance function named resistance on the basis of electrical network theory. The *Kirchhoff index* $Kf(G)$ of a graph G is defined as

$$Kf(G) = \sum_{\{u,v\} \subseteq V(G)} r_G(u, v).$$

In 2017, Chen et al. [2] introduced a new graph invariant reciprocal to Kirchhoff index, named *Resistance-Harary index*, as

$$RH(G) = \sum_{\{u,v\} \subseteq V(G)} \frac{1}{r_G(u, v)},$$

where $r_G(u, v)$ is the resistance distance between vertices u and v in G . It originates from many physical problems. Let P_n and C_n denote Path and Cycle with n vertices, respectively. All other notations and terminologies are referred to [1].

2. Main Results

In this section, we begin to determine *Resistance-Harary indices* of Path P_n and Star S_n with n vertices.

Theorem 1. For any positive integer $n \geq 2$, $RH(P_n) = n \sum_{i=2}^n \frac{1}{i}$.

Proof. Let $V(P_n) = \{v_1, v_2, \dots, v_n\}$. According to the structures of Path, any circuit connecting any two vertices is a series circuit. There are in all $(n-1)$ values $1, 2, \dots, n-1$ of $r(v_i, v_j)$ in P_n , and the total numbers are $n-1, n-2, \dots, 1$, respectively. Then

$$RH(P_n) = \frac{n-1}{1} + \frac{n-2}{2} + \dots + \frac{1}{n-1} = n \left(\frac{1}{2} + \dots + \frac{1}{n} \right) = n \sum_{i=2}^n \frac{1}{i}.$$

□

Theorem 2. For any positive integer $n \geq 2$, $RH(S_n) = \frac{(n-1)(n+2)}{4}$.

Proof. Let Star $S_n = \{v_1, v_2, \dots, v_n\}$, where v_1 is the center of Star. According to the structures of Star, we have consider two cases as follows:

Case 1. $x, y \in \{v_2, \dots, v_n\}$. $r(x, y) = 2$ and the total number of $r(x, y)$ is $\frac{(n-1)(n-2)}{2}$.

Case 2. $x = v_1$ and $y \in \{v_2, \dots, v_n\}$. $r(x, y) = 1$ and the total number of $r(x, y)$ is $(n-1)$. Then

$$RH(S_n) = \frac{(n-1)(n-2)}{2} \cdot \frac{1}{2} + (n-1) \cdot \frac{1}{1} = \frac{(n-1)(n+2)}{4}.$$

□

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