это преобразование эквивалентно инвариантности всех функций и других параметров относительно переобозначений $A \rightleftharpoons B$. Новая модель микрокристаллизации 50% двухкомпонентных металлических расплавов позволяет изучить процессы перехода от возникающей при определенных условиях термодинамики неупорядоченной кристаллической фазы к полностью упорядоченной фазе, а также выявить времена релаксации, способствующие этому процессу.

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Unified Cluster Model of Nuclei and Subnuclear Particles Formation

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Abstract

This work presents an attempt to look at intranuclear phenomena from the unified point of view, treating them as processes of formation and growth of compact clusters with distinct collective quantum properties.

Key Words: unified point of view, intranuclear phenomena, formation and growth, compact clusters, nuclei, subnuclear particles

Introduction

Different analytical and numerical methods are used to study the mechanisms of reaching the equilibrium in nuclear processes, such as formation of stable nuclei, exotic processes, cluster radioactivity, spontaneous nuclear fission and synthesis of superheavy elements in stars, deep inelastic scattering of relativistic particles on nuclei (e.g. [1-15]). Such investigations are based on the model concepts of the nature of nuclear matter and the dynamics of the processes under consideration.

Works [16-18] describe an attempt to look at intranuclear phenomena from the unified point of view, treating them as processes of formation and growth of compact objects (clusters) with distinct collective quantum properties (strong interaction) in a closed stochastic system of nucleons oscillating around local positions of equilibrium. The closed system in question can be either a parent nucleus or two nuclei in contact after their colliding (a double system). At the initial moment of time, this system contains seeds of nuclear matter: alpha-particles and (or) their fragments — tritons and deuterons. The random interaction of these seeds results in their coalescence, and compact nuclear matter clusters are formed in the system. It is assumed that, analogously to the growth processes of nanocrystals with strong interatomic bonds, oscillations of nucleons on the surface of contacting nuclear clusters with temporarily (virtually) broken external bonds can lead to the mutual compensation of these unused bonds. As a result, consolidation of adjacent clusters takes place and larger objects of nuclear medium with different masses are formed. Due to Coulomb repulsion, these objects can escape from the area of interaction.

The proposed model was applied to the following problems: 1) calculations of the most probable mass numbers A_n of clusters within the entire range of the existing nuclide systematization; 2) computed estimation of a discrete spectrum of the most probable numbers A , typical for cluster radioactivity; 3) discovering the most probable and average mass numbers $\langle A \rangle$ of clusters formed in processes of spontaneous nuclear fission; 4) determination of the most probable values A and evaluation of the average number $\langle A \rangle$ of superheavy clusters formed in processes of nucleosynthesis in stars; 5) description of relativistic deep inelastic processes in nuclear medium. Intranuclear processes (r-processes) that go during lifetime of excited states are considered. These states are given by the energy of oscillatory and rotational levels determined in shell model of nucleus.

The basis for analytical description of the processes under consideration is a phenomenological concept of the distribution density wave $\varphi(a, t)$ of the objects in their sizes a space.

1. Methodology

Let us consider the formation and growth of compact clusters in conservative stochastic systems defined as total-mass-limited assemblies of randomly interacting quantum objects. To describe quantitatively objects aggregation we use a conception [16-18] of distribution density wave $\varphi(a, t)$ in the space of cluster sizes a . The wave travels with the time t toward an increase in the cluster size. Such one-dimensional approach makes it possible to ignore deviation of a geometrical shape of an object from an ideal one. On the assumption of the universal relation $\Delta a \cdot \Delta k \geq 1/4\pi$ (k is wave number) for a wave packet half-width and a spectral line half-width for coherent processes one can write down the following uncertainty relation for a coordinate and a momentum in space of cluster sizes:

$$\Delta a \cdot \Delta p \cong \frac{\hbar}{2}. \quad (1.1)$$

Here, $\Delta p \sim p = m \Delta a / \Delta t$ is uncertainty of momentum, m is cluster mass, $\hbar$ is the reduced Planck constant. Uncertainty of momentum is equal to the momentum itself in the order of magnitude, i.e. interaction of objects either occurs or does not occur. Physical meaning of relation (1.1) is in the fact that during the time interval Δt of elementary (single) act of interaction between objects the exact cluster size cannot be determined until this interaction is finished either through capture of one object by another or their partial or complete disruption, or elastic scattering. It is connected with the fact that unless the elementary act is finished, it is impossible to determine to which object each of the interacting surface elements (nucleons in

case of nuclear material) is assigned. The sign of approximate equality reflects quasi-coherent behavior of considered systems specified by possible fluctuations connected with the specific character of high intensive processes (high-velocity collisions breaking the initial structure of objects).

Since in relation (1.1) we are dealing with object sizes that vary continuously with time, this relation can be considered as differential. Formally it follows from (2.1) that $\Delta a \propto \sqrt{\Delta t}$, i.e. instant rate of object size growth is infinitely large:

$$da/dt = \lim_{\Delta t \rightarrow 0} \Delta a / \Delta t \propto \lim_{\Delta t \rightarrow 0} 1/\sqrt{\Delta t} = \infty.$$

Because of random character of aggregation process of investigated objects the analytical derivative da/dt cannot be determined, and the factor of a fractional derivative is equal to 1/2. To describe such a stochastic process using any analytical function $F(a)$ one should use a differential of the type $dF = F'da + (1/2)F''(da)^2$ (the Ito differential).

In the closed system of clusters the distribution density function $\varphi(a, t)$ satisfies the condition of total mass conservation:

$$M_s(t) + M_{cl}(t) = M_s(t) + \int_0^{a_{\max}} \varphi(a, t) m(a, t) da = \text{const}, \quad (1.2)$$

where $M_s(t)$, $M_{cl}(t)$ are current total masses of seeds and clusters, respectively, $a_{\max}$ is the maximum cluster size in the system; and the mass of volumetric close-packed cluster can be defined either as $m = m_0(a/a_0)^3$, where a_0 , m_0 are size and mass of a cubic shape seed, or as $m = \alpha \rho a^3$, where ρ is cluster matter density, α is geometric factor (for a cube $\alpha = 1$, for a sphere $\alpha = \pi/6$). At the end of aggregation process the total mass of seeds is infinitely small, and hence, the first term in conservation equation (2.2) can be neglected. Total cluster mass can be considered as integral of motion, i.e. one can consider that the mass increment tends to zero: $\Delta M_{cl}(t) \rightarrow 0$ at $t \rightarrow t_f$ (t_f is the time instant of aggregation completion). Revealing increment of a certain integral in expression (1.2), we obtain

$$\begin{aligned} \lim_{t \rightarrow t_f} \Delta M_{cl} &= \lim_{t \rightarrow t_f} \Delta \int_0^{a_{\max}} \varphi(a, t) m(a, t) da = \\ &= \alpha \rho \int_0^{a_{\max}} \left\{ \lim_{t \rightarrow t_f} \Delta [a^3 \varphi(a, t)] \right\} da = 0. \end{aligned} \quad (1.3)$$

Since at the end of the process cluster sizes and quantities are large comparing to those of seeds, the “simultaneous” failure of a significant number of clusters at their interaction with seeds is highly unlikely. Frozen distribution of clusters in sizes is established in the system, and an integrand in expression (1.3) does not change its sign, at least, within the time interval ($t_f - \Delta t, t_f$). Hence from (1.3) it follows that

$$\lim_{t \rightarrow t_f} \Delta [a^3 \varphi(a, t)] = 0.$$

Introducing the dimensionless variable a/a_0 and revealing the last expression according to rules [2], for the above differential dF we obtain:

$$d \ln \varphi = -3 \{ d \ln(a/a_0) + \frac{3}{2} [d(a/a_0)/(a/a_0)]^2 \}, \quad t \rightarrow t_f.$$

With high $a \gg a_0$ the second term in braces can be neglected. It is finally found that the density function of size distribution of large clusters at the end of the aggregation process φ_{fin} can be approximated by the following relationship

$$\varphi_{fin} \propto a^{-3} \propto m^{-1}, a \gg a_0, t \rightarrow t_f. \quad (1.4)$$

This asymptotic kind of distribution obtained using uncertainty relation (1.1) is not connected with the initial distribution φ_0 , and does not depend on the type of kinetic equation for φ . Distribution (1.4) can be regarded as “invariant” of any kinetic equation describing behavior of clusters with quantum properties.

In accordance with the above kind of analytic function differential, evolution of distribution density function $\varphi(a, t)$ during a stochastic process of aggregation can be described in diffusion approximation using Fokker-Planck equation written for the space of cluster sizes:

$$\frac{\partial \varphi(a, t)}{\partial t} + \frac{\partial}{\partial a} [v \varphi(a, t)] - \frac{1}{2} \frac{\partial^2}{\partial a^2} [\eta \varphi(a, t)] = 0. \quad (1.5)$$

Here, $v = \langle da \rangle / dt$, $\eta = \langle (da)^2 \rangle / dt$ is the mean rate of kinematic transport φ and diffusion coefficient in the space a , respectively. To linearize this equation and obtain approximate solutions we will accept that the average rate of kinematic transfer φ matches in order of magnitude the growth rate of mean size of cluster:

$$v = \langle da \rangle / dt \cong d \langle a \rangle / dt.$$

In accordance with the ratio (2.1) the diffusion coefficient can be written as

$$\eta \cong \hbar / 2m = \hbar / 2m_0 (a/a_0)^3.$$

We introduce dimensionless variables $\xi = a/a_0$, $\tau = t/t_i$, where t_i is a typical time scale of object interactions determined by their nature and aggregation mechanism. Then, taking into account the accepted assumptions, equation (1.5) takes the following form

$$\frac{\partial \varphi(\xi, \tau)}{\partial \tau} + \Psi(\tau) \frac{\partial \varphi(\xi, \tau)}{\partial \xi} - \frac{1}{2} \beta \frac{\partial^2}{\partial \xi^2} [\varphi(\xi, \tau) \cdot \xi^{-3}] = 0. \quad (1.6)$$

Here, $\Psi(\tau) = d \langle \xi \rangle / d\tau$ is dimensionless growth rate of mean size of clusters and $\beta \cong \hbar t_i / 2m_0 a_0^2$. To estimate the value β we will draw on ratio (1.1) and the mass conservation law in an elementary process of interaction between a cluster and a seed in the assumption that seed mass gets equally “smeared” on a cluster surface:

$$\rho a^3 + \rho a_0^3 = \rho (a + \Delta a)^3,$$

where ρ is matter density. From conditions $a_0 < a$, $\Delta a \ll a$ we will find that $\Delta a \cong a_0^3 / 3a^2$. Then from ratio (1.1) we obtain that $\beta \cong (1/81)(a_0/a) \ll 1$. Hence β is a small parameter.

Revealing the derivative in the third term of equation (1.6), we obtain the following equation

$$\frac{\partial \varphi}{\partial \tau} + \Psi \frac{\partial \varphi}{\partial \xi} - \frac{\beta}{2\xi^3} \frac{\partial^2 \varphi}{\partial \xi^2} + \frac{3\beta}{\xi^4} \frac{\partial \varphi}{\partial \xi} - \frac{6\beta}{\xi^5} \varphi = 0. \quad (1.7)$$

The study into asymptotic properties of equation (1.7) has shown the following. With high $\xi \gg 1$, the law of average size growth becomes:

$$\langle a \rangle \cong a_0 K \left(\frac{t}{t_i} \right)^Z. \quad (1.8)$$

The constants K, Z are determined by an expression coming from uncertainty relation (1.1) transcribed for mean size of clusters

$$\langle m \rangle \frac{(\Delta \langle a \rangle)^2}{\Delta t} \cong \frac{\hbar}{2}. \quad (1.1')$$

With $\Delta t = t_i$, we obtain the following expression:

$$Z^2 \cdot K^{2/Z} \cong \frac{\hbar t_i}{2 m_0 a_0^2} \left(\frac{\langle a \rangle}{a_0} \right)^{\frac{2}{Z}-5}. \quad (1.9)$$

Value Z is given from the condition $K \equiv \text{const}$, i.e. a right-hand portion of expression (2.9) should not depend on $\langle a \rangle$. Specific values of constants in law (1.8) are related with the value t_i , i.e. they are determined by a mode and mechanism of object interactions in a closed system.

Partial solution of equation (2.7) at $\xi \gg 1$ looks like:

$$\varphi_\lambda = C(\lambda) \exp[-\lambda^2(\xi - \langle \xi \rangle)], \quad \langle \xi \rangle \cong K \tau^Z. \quad (1.10)$$

Here $C(\lambda)$ is a function of the arbitrary real number λ . General solution fitting the law of mass conservation and asymptotic distribution (1.4) ($C(\lambda) = B\lambda^5, B \equiv \text{const}$), can be written as:

$$\begin{aligned} \varphi(\xi, \tau) &= \int_{-\infty}^{\infty} C(\lambda) \exp[-\lambda^2(\xi - \langle \xi \rangle)] d\lambda = \\ &= B 2! / (\xi - \langle \xi \rangle)^3, \quad \xi \gg 1. \end{aligned} \quad (1.11)$$

Comparing this expression with distribution (1.4) one can conclude that at $t \rightarrow t_f$ the condition $\langle a \rangle_{fin} \ll a_{\max}$ is to be satisfied. At the end of the process of object irreversible aggregation as a result of interaction between large clusters and small seeds in a stochastic conservative system the distribution density of large clusters is in inverse proportion to their masses, and average size of clusters is significantly less than the final maximum size.

Approximate view of function φ close to maximum of the size distribution density curve can be determined by omitting in equation (1.7) terms with the first variable derivative ξ , since in small neighborhood of local maximum the analytical function changes poorly: $\partial \varphi / \partial \xi \cong 0$. Having written the function φ as a product of space and time factors and having solved the equation for the spatial factor P by Ventzel-Kramers-Brillouin method, we obtain the following expression:

$$P = \frac{(\beta/2)^{1/4} (\xi/\lambda)^{1/2}}{(c^2 + \xi^5)^{1/4}} \times \left| \sin \left\{ \frac{2\lambda}{5} \left(\frac{2}{\beta} \right)^{1/2} \left[\sqrt{c^2 + \xi_n^5} - \frac{c}{2} \ln \frac{\sqrt{c^2 + \xi_n^5} + c}{\sqrt{c^2 + \xi_n^5} - c} \right] + \right\} \right|, \quad (1.12)$$

where $c^2 \equiv 6\beta/\lambda^2$. The presence of the oscillating factor $\sin\{\dots\}$ in the expression (1.12) means that function P , in principle, can have variety of local maximums with ξ_n coordinates. What this means is existence of variety of the most probable (stable) sizes of clusters. The

constant in the expression (1.12) is found from the condition that the first maximum ($n = 0$) corresponds to seeds ($\xi_0 = 1$). Since $\beta \ll 1, \lambda > 1$, the parameter $c^2 \ll 1$. Then the condition of maximum looks like

$$\frac{2\sqrt{2}}{5} \frac{\lambda}{\sqrt{\beta}} \left(\xi_n^{5/2} - 1 \right) \cong n\pi, \quad n = 1, 2, 3, \dots \quad (1.13)$$

The condition of P-function “zeros” with consideration for small c^2 and c values looks as:

$$\frac{2\sqrt{2}}{5} \frac{\lambda}{\sqrt{\beta}} \xi_k^{5/2} + \text{const}' \cong \pi k, \quad k = 1, 2, 3, \dots \quad (1.14)$$

We consider size distribution of “small” clusters for which kinematic transport of distribution density is expressed weaker than diffusion. Having omitted in equation (1.7) the second term responsible for transport and having solved the obtained equation, we will get the following expression for a space factor:

$$P = \xi^3 - \frac{\lambda^2}{20} \xi^8 \quad (1.15)$$

Function maximum condition (1.15) takes the form:

$$P' = 3\xi^2 - \frac{8\lambda^2}{20} \xi^7 = 0, \quad P'' = 6\xi - \frac{56\lambda^2}{20} \xi^6 < 0. \quad (1.16)$$

“Joining” of maximum condition (1.16) with the first maximum condition in expression (1.12) $n = 0, \xi_0 = 1$ allows one to determine the arbitrary constant λ :

$$3 - \frac{8}{20} \lambda^2 = 0 \Rightarrow \lambda^2 = \frac{15}{2}, \quad P'' = -21 < 0. \quad (1.17)$$

With the knowledge of λ , it is possible to determine the coordinate of P-function first “zero”:

$$P = 0 \Rightarrow \xi = \left(\frac{20}{\lambda^2} \right)^{1/5} = 1.216728684. \quad (1.18)$$

“Joining” of expressions (1.14) and (1.18) with $k = 0$ makes it possible to find a constant value in expression (1.14) and write the following condition for coordinates of function P “zeros”:

$$\frac{2\sqrt{2}}{5} \frac{\lambda}{\sqrt{\beta}} \left(\xi_k^{5/2} - \frac{\sqrt{20}}{\lambda} \right) \cong \pi k, \quad k = 1, 3, 5, \dots \quad (1.19)$$

What this means is existence of unstable clusters with corresponding to expression (1.19) sizes, the first “unstable” size in this case is equal to $\xi = 1.216728684$.

The linearization of Fokker-Planck kinetic equation (1.5) allows one to establish basic properties of cluster size distribution density function. Expressions (1.8), (1.9) can be used for finding laws of time-dependent average size growth of large clusters ($\langle a \rangle \gg a_0$). We consider two limiting growth modes: 1) seed small flux, when each of the seeds has time to occupy its energy advantageous place on cluster surface before interaction with the following seed begins; 2) seed large flux, when seeds affect the cluster almost simultaneously which is equivalent to cluster collision. In the first case parameter t_i does not depend on a cluster size and the law of growth can be written as:

$$\langle a \rangle \cong \left(5/2\sqrt{2} \right)^{2/5} a_0 (t/t_1)^{2/5}, \quad t_1 = a_0 \sqrt{m_0 t_i / \hbar}. \quad (1.20)$$

Here, t_1 is the time unit in the mode of seed small flux. In case of seed large flux we accept the t_i parameter equal to the perturbation back-and-forth travel time in a cluster: $t_i = 2\langle a \rangle / c_0$, where c_0 is velocity of perturbation distribution in cluster substance velocity of (“sound”). Then the law of growth can be written as

$$\langle a \rangle \cong (3/2)^{1/3} a_0 (t/t_2)^{1/3}, \quad t_2 = a_0 \sqrt{a_0 m_0 / \hbar c_0}. \quad (1.21)$$

Here, t_2 is the time unit in the seed large flux mode (cluster collision).

It should be noted that laws of growth for average size objects can be deduced directly from uncertainty relation (1.1'), if to substitute $\Delta t = t_i$ in it, determine $\Delta \langle a \rangle$, and then to solve the differential equation $d\langle a \rangle / dt \approx \Delta \langle a \rangle / \Delta t = \Delta \langle a \rangle / t_i$ with zero initial condition. With the seed small flux we obtain the following chain:

$$\begin{aligned} \Delta \langle a \rangle &\cong \sqrt{\frac{\hbar t_i}{2m_0}} \frac{a_0^{3/2}}{\langle a \rangle^{3/2}} \rightarrow \\ \frac{\Delta \langle a \rangle}{\Delta t} &\cong \sqrt{\frac{\hbar}{2m_0 t_i}} \frac{a_0^{3/2}}{\langle a \rangle^{3/2}} \cong \frac{d\langle a \rangle}{dt} \rightarrow \\ \langle a \rangle^{3/2} d\langle a \rangle &\cong \left(\frac{\hbar a_0^{3/2}}{2m_0 t_i} \right)^{1/2} dt. \end{aligned}$$

Solving the last differential relation in quadratures, we will obtain the following expression:

$$\langle a \rangle \cong \left(\frac{25\hbar a_0^3}{8m_0 t_i} \right)^{1/5} t^{2/5}. \quad (1.22)$$

This exactly agrees with law (1.20), if to reveal the parameter t_1 there. Similarly, with $\Delta t = 2\langle a \rangle / c_0$ we obtain:

$$\langle a \rangle \cong \left(\frac{3}{2} \right)^{1/3} \left(\frac{\hbar c_0 a_0^3}{m_0} \right)^{1/6} t^{1/3}. \quad (1.23)$$

This agrees with law (1.21). In the first approximation one can assume that time-dependent laws of growth (1.22), (1.23) for average size of objects are invariants of any kinetic equation describing aggregation of clusters with quantum properties.

In number of problems connected with investigations of high-intensive processes with high energy (for example, at high temperatures of crystal objects) it is reasonable to determine the value of Δt time interval of elementary interaction act based on the Heisenberg “energy-time” uncertainty relation:

$$\Delta t \cong \hbar / \Delta E. \quad (1.24)$$

Here, ΔE is energy level width of an insulated excited state of a quantum mechanical system. Substituting expression (1.24) in relation (1.1'), we will describe the following chain of differential relations:

$$\frac{\Delta\langle a \rangle}{\Delta t} \cong \left(\frac{a_0^3 \Delta E}{2m_0} \right)^{1/2} \frac{1}{\langle a \rangle^{3/2}} \cong \frac{d\langle a \rangle}{dt} \rightarrow$$

$$\langle a \rangle^{3/2} d\langle a \rangle \cong \left(\frac{a_0^3 \Delta E}{2m_0} \right)^{1/2} dt. \quad (1.25)$$

The ΔE value is determined by nature of objects and a process mode. Having solved differential relation (1.25) in quadratures one can find approximate laws of average size $\langle a \rangle$ growth with time in high-intensive processes of object aggregation.

We consider an issue of upper limits of object sizes. From uncertainty relation (1.1) and mass conservation condition in elementary interaction between a large cluster and a seed, when $\Delta a \cong a_0^3/3a^2$, we obtain the following expression for the maximum object size

$$a_{\max} \cong \frac{2}{9} \alpha \frac{\rho}{\hbar} \frac{a_0^6}{\Delta t_{\min}} \cong \frac{2}{9} \frac{m_0}{\hbar} \frac{a_0^3}{\Delta t_{\min}}. \quad (1.26)$$

Here, $\Delta t_{\min}$ is a minimum time interval of an elementary act of object interaction determined by physical nature of the process.

2. Results and discussion

To determine the most probable mass numbers we will use expression (1.13) of section 2 with an arbitrary real number $\lambda = \sqrt{15/2}$ found out in Chapter 1 and known expression [7, 9] connecting the nucleus radius $R = a/2$ and the mass number:

$$R \cong r_0 A^{1/3}, \quad (2.1)$$

where $r_0 = 1.3$ Fm is a typical parameter of strong interaction. If one accepts that this expression is approximately fulfilled for seeds as well, then from equation (1.13) of chapter 1 it is possible to obtain the following expression for the most probable mass numbers A_n :

$$A_n \cong A_0 \left(\frac{5}{2\sqrt{2}} \frac{\sqrt{\beta}}{\lambda} \pi n + 1 \right)^{6/5}, \quad n = 1, 2, 3, \dots \quad (2.2)$$

Here, A_0 is seed mass number, and the parameter $\beta \cong \hbar t_i / 2m_0 a_0^2$ is determined by a seed mass and size and the typical time scale t_i of interaction between objects. It is reasonable to accept as such scale a period T_{hf} of high-frequency nucleon oscillations in a nucleus. This parameter can be determined as $t_i = T_{hf} = 2r_0/c_0 = 5 \cdot 10^{-23}$ s, where $c_0 = 5 \cdot 10^7$ m·s⁻¹ is an average thermal nucleon velocity in degenerate Fermi gas determined by the known value of average kinetic energy of ideal Fermi gas per a nucleon, equal to $E_F = 22$ MeV [9].

To find characteristic mean sizes of cluster nuclides we will use expressions (1.22) and (1.23) of Chapter 1 and transcribe them as follows

$$\langle a \rangle_1 \cong \left(5/2\sqrt{2} \right)^{2/5} (6\hbar/\pi\rho\alpha_i)^{1/5} t^{2/5}, \quad (2.3)$$

$$\langle a \rangle_2 \cong (3/2)^{1/3} (6\hbar c_0/\pi\rho)^{1/6} t^{1/3}. \quad (2.4)$$

Here, ρ is density of nuclear matter. In order to eliminate the parameter a_0 from equations, it was accepted that $m_0 \cong (\pi/6)\rho a_0^3$. The absence of a seed size in (2.3) and (2.4) reflects an asymptotic stage of the process, when the system “forgets” about initial conditions

and its evolution flows according to nucleus internal dynamics. This dynamics is connected with excitation of oscillatory and rotational energy levels. General time of the process is determined by shell energy level Γ using Heisenberg rule:

$$\tau = \hbar/\Gamma. \quad (2.5)$$

This is valid for insulated excitations of quantum systems.

Verification of the proposed formalism can be implemented by the example of “rigid” (deep inelastic) processes involving hadrons described in [19, 20]. We consider these processes as the following chain of events: head-on collision of “nucleon-nucleon” or “lepton-nucleon” $\rightarrow$ quark knocking-out from gluon cloud $\rightarrow$ random interactions in intra-hadron medium $\rightarrow$ new particle formation (hadron jets). The new particles are considered as clusters consisting of intra-hadron medium components (partons): quarks, gluons, quark-antiquark couples. New particle generation conditions can be written as:

$$(M_{elem} + M_{cl})c^2 = M_0 c^2.$$

Here, $M_{elem}c^2$ is total energy of partons, $M_{cl}c^2$ is total energy of new particles, M_0c^2 is total energy of two original colliding particles. At the end of this process all the partons transformed into bound states, $M_{elem} = 0$, therefore relation (1.3) is fulfilled, and the proposed in section 2 stochastic approach is applicable. Then law (1.21) of Chapter 1 for describing the evolution of clusters in mean size provides the following relation between length a_{unit} , time t_{unit} and mass m_{unit} units with deep inelastic interaction of fundamental particles:

$$m_{unit} a_{unit}^3 / t_{unit}^2 = \hbar c. \quad (2.6)$$

Here c is the maximum velocity of interaction propagation in hadron medium: light speed in vacuum. On the basis of relation (2.6) and the data from [7, 14] for “conventional” quark masses (u- and d- quarks) deep inside hadrons and for characteristic space scales, one can obtain the following evaluation of the time unit t_{unit}^{af} for current quarks (cq) in the state of asymptotic freedom (af):

$$a_{unit}^{af} \sim 10^{-16} \text{ m}, m_{unit} \sim m_u^{cq} \approx 5 \text{ MeV}, m_{unit} \sim m_d^{cq} \approx 7 \text{ MeV}; \Rightarrow t_{unit}^{af} \sim 10^{-26} \text{ s}.$$

The quark transition time t_{trans} from asymptotic freedom to a bound state (confinement) inside a hadron is evaluated according to formula (1.21) of chapter 1 as $t_{trans} \sim t_{unit}^{af} (a_{unit}^{conf} / a_{unit}^{af})^3 \sim 10^{-23} \text{ s}$ ($a_{unit}^{conf} \sim 10^{-15} \text{ m}$). The obtained value corresponds to the time scale of strong interaction.

In processes considered in [19] the minimum value of space unity is $a_{unit}^{min} \sim 10^{-18} \text{ m}$. It can be considered as the upper estimation of current quark size. Then it goes from the relation (2.6) that $t_{unit}^{min} \sim 10^{-29} \text{ s}$, and the time t_{hadr}^{form} for formation of hadrons having mean size of $\langle a \rangle \sim 10^{-15} \text{ m}$, in accordance with formula (1.21) of chapter 1, is $t_{hadr}^{form} \approx t_{unit}^{min} (\langle a \rangle / a_{unit}^{min})^3 \sim 10^{-20} \text{ s}$. This value is significantly higher than the life time of unstable hadrons (“resonances”) $t_{life}^{res} \leq 10^{-21} \text{ s}$: $t_{hadr}^{form} \gg t_{life}^{res}$. This reflects formation of stable hadron jets in described processes of catastrophic collisions.

If in accordance with the results of [21], we accept that perturbation propagation rate in quark-gluon medium is equal to speeds of sound $c_s = 0.3c$ and $c_s = 0.57c$, then in the above estimates of the time units (the time of quark transformation into a bound state and the time of hadron formation) factors 1.8 and 1.3 will appear. These corrections do not change the estimate

of strong interaction time scale and increase the inequality $t_{hadr}^{form} \gg t_{life}^{res}$ reflecting the formation of stable hadron jets.

The asymptotic distribution $\varphi \propto m^{-1}$ means that: 1) probability of nucleon fragmentation in deep inelastic scattering is higher than probability of their conservation; 2) the number of pions in hadron jet is 1.5 times higher than the number of nucleons. These conclusions are in compliance with concepts of work [19] about nature of rigid processes. In particular, at proton frontal collision the following reaction takes place: $pp \rightarrow \pi^0 \pi^+ \pi^+ nn$. One can see that there are 3 pions and 2 neutrons in the formed hadron jet.

Consequently, the proposed formalism allows adequate description of an asymptotic form of well-known accidents in the world of elementary particles. This gives grounds to try to apply it in solving the above described problems of intranuclear processes description.

Values A_n calculated in [17, 18] with the help of formula (2.2) with three types of seeds correspond to the mass numbers of the nuclides and their isotopes over the currently known range [7, 12], including the transfermium elements. Substituting into (1.26) the travel time of the electromagnetic wave over the cluster a/c (where c is the speed of light in a vacuum) as the minimum possible Δt_{min} and considering dependence (2.1), one can obtain the following expression for the mass number A_{end} of the final nuclide [16]:

$$A_{end} = \left(\frac{\pi}{27} \right)^{3/2} \left(\frac{\rho c}{\hbar} \right)^{3/2} (4r_0^2)^3 A_0^3.$$

Taking the α particle ($A_0 = 4$) for the seed, we obtain that $A_{end} \approx 470$. This value is about 15% greater than the final value 410 corresponding to the systematics of nuclei [10]. From formula (1.12) of Chapter 1 with $\beta \ll 1$, one can obtain the following expression for relative density of objects distribution in sizes:

$$P \approx \left(\frac{a_0}{a} \right)^{3/4} \left| \cos \frac{2\sqrt{2}}{5} \frac{\lambda}{\beta^{1/2}} \left[\left(\frac{a}{a_0} \right)^{5/2} - 1 \right] \right|. \quad (2.7)$$

Figure 1 provides the calculated, according to formulae (2.1) and (2.7), relationship between relative density of distribution of nuclide nuclei and their mass numbers, when seeds are alpha-particles.

Figure 2.1. Relative density of nuclei distribution (α -particles are seeds).

Oscillating character of this relation reflects the existence of the most probable mass numbers A_{mp} relatively stable nuclides (elements and isotopes) corresponding to local maximums of the curve $P(A)$. It should be noted that the nature of this curve qualitatively corresponds to the nature of the pulsed relation [10] between the nucleon binding energy in the nucleus and the mass number on the stability line $Z(N)$ (Z is proton number, N is neutron

number). From the value A_{mp} , using the dependence formula between the number of protons Z_{stab} in stable nuclei and the mass number in the liquid drop model approximation

$$Z_{stab} \approx A / \left(0.015A^{2/3} + 2 \right)$$

(Weizsacker formula, see [7], p. 277), it is possible to calculate a number of elements and isotopes [7, 12] from ${}^4\text{He}_2$, to ${}^{256}\text{Cf}_{98}$. Calculations have been as well performed using the method [10] based on the shell model of nucleus: ${}^{88}\text{Sr}_{38}$, ${}^{91}\text{Zr}_{40}$, ${}^{96}\text{Mo}_{42}$, ${}^{98}\text{Tc}_{43}$, ${}^{101}\text{Ru}_{44}$, ${}^{144}\text{Nd}_{60}$, ${}^{173}\text{Yb}_{70}$, ${}^{195}\text{Pt}_{78}$, ${}^{197}\text{Au}_{79}$, ${}^{201}\text{Hg}_{80}$, ${}^{204}\text{Tl}_{81}$, ${}^{210}\text{At}_{85}$, ${}^{222}\text{Rn}_{86}$, ${}^{251}\text{Cf}_{98}$, ${}^{252}\text{Es}_{99}$, ${}^{258}\text{Md}_{101}$, ${}^{259}\text{No}_{102}$, ${}^{262}\text{Lr}_{103}$.

In case of a mass number of the above determined finite nucleus $A_{end} \approx 470$, in accordance with systematization [10], we get that $Z = A + 46/3 = 172$ and $N = 298$ (hypothetical nucleus ${}^{470}\text{X}_{172}$). In addition, the pulsed curve in Fig. 1 provides mass numbers of finite nuclides of the seventh period of a modified periodic system of elements [10]: $A_{last} = 285$ and 307. The corresponding proton numbers $Z = A + 46/3$ are equal to $Z = 110$ and 118. The obtained agreement allows prediction of the most probable mass numbers within unstudied at present the range: $A_{mp} = 311, 314, 317, 320, \dots, 408, 412, \dots, 466, 469$. Numbers $A_{mp} = 408$ and 412 are close to mass number 410 of the hypothetical finite nuclide in the systematization [10]. Thus the proposed cluster model predicts the existence of higher stable nuclides.

General diagram of stable neutron-rich nuclei is given at Figure 2.

Figure 2. Diagram of neutron-rich nuclei:

1 – parabolic line of 2β -stability [10] regarding for experimental data with prolongation up to hypothetical final nuclide $Z = 152$, $N = 258$; 2 – the boundary of well-known neutron-rich nuclei [7]; 3 – supposed boundary of the existence of neutron-rich nuclei (tritium medium); 4 – the boundary of neutron stability [22] (1n NSB); 5 – final nuclides $Z = 172$, $N = 299$ u $Z = 179$, $N = 311$ [17, 18]; 6 – extrapolation of line 1; 7 – elements calculated by formula (2.2) jointly with Weizsacker formula of β -stability [7]; 8 – stable isotopes calculated by formula (2.2) jointly with above Weizsacker formula; 9 – elements calculated by formula (2.2) jointly with nuclei systematics [10]; 10 – isotopes calculated by formula (2.2) jointly with systematics [46]; 11 – “island” of stability $Z = 164$, $N = 318$ mentioned in [10].

One can see that the calculations performed by cluster model of present work correspond to parabolic line of 2β -stability [10], and notions of monograph [7]. Point of intersection of lines 3 and 4 correspond to hypothetical stable exotic nucleus ${}^{281}\text{X}_{93}$. In addition, the boundary 3 lies not far from the “island” of stability $Z = 164$, $N = 318$ mentioned in [10].

It should be borne in mind that at the proposed approach the calculated clusters are formed deep in the sea of nuclear matter and not in particular reactions. In escaping from either the parent nucleus or the double system, clusters can emit or capture individual nucleons and seeds of nuclear matter. The established correspondence allows the characteristic values for various processes of equilibrium approach (cluster radioactivity, spontaneous fission, synthesis of heavy and superheavy elements) to be extracted from the above values.

Calculated mass numbers of clusters formed inside heavy nuclei in processes of cluster radioactivity (cr) make up the following discrete spectrum: $A_{cr} = 6, 7, 9, 11-16, 18, 20, 24, 26, 28, 30, 32, 34, 37-40$. The spectrum was terminated at the mass number at which the arithmetic mean over the spectrum approximately (to an accuracy of unity) coincided with the value $\langle A \rangle = (a/2r_0)^3 \cong 21$ calculated by the formula (2.3) for the increase over time in the average size of clusters due to the adding of seeds. Here, the total running time of the process, $t = 6.287 \cdot 10^{-22}$ s, corresponds to the excitation of the vibrational level with energy $\Gamma = 1$ MeV. Nuclear weight spectra was calculated in [16] for four cluster formation channels in which the initial protoclusters ($n = 1$) were ^6Li , ^7Li , ^8Be , ^9Be and in each channel the seeds ($n = 0$) under consideration were deuterons ($A_0 = 2$), tritons ($A_0 = 3$), and α - particles ($A_0 = 4$). This allowed the arbitrary number λ and the parameter β to be excluded from further calculations. The following discrete spectrum was calculated: $A_{cr} = 10, 12, 14, 16, 20, 24, 26, 28, 30, 32, 37, 40$ (Figure 3). In experiments [4, 7] on cluster radioactivity the values $A = 14, 24, 28, 32$ were recorded. Under discussion is the possibility of recording the outgoing clusters with mass numbers close to stated predicted spectrum.

Figure 3. Calculated spectrum of the most probable mass numbers at cluster radioactivity: N is the absolute frequency of cluster formation, A is the mass number.

In the field of spontaneous fission it is possible to single out two sets of the most probable cluster mass numbers, each being repeated for two or three seed types: $A_1 = 72, 79, 84, 88, 103, 108$ и $A_2 = 126, 134, 139, 142, 147, 155, 158$. The arithmetic means for the two sets of mass numbers are equal to $\bar{A}_1 \cong 90$ и $\bar{A}_2 \cong 140$. The obtained values correspond to the coordinates of the maxima of the total nuclear fission yields [7, 13]. It is possible to describe spontaneous nuclear fission as a result of excitation of the first rotational level with the energy $\Gamma_{rot} = 100$ keV using the formulas (2.1) and (2.4). The total time of an intranuclear process corresponding to the given value Γ_{rot} , is equal to $t_{rot} = 6.287 \cdot 10^{-21}$ s, and the mentioned formulas determine an average mass number of light fragments as $\langle A \rangle_{light} = 100$. An average mass number of heavy fragments $\langle A \rangle_{heavy}$ is determined from the mass conservation $\langle A \rangle_{mat} = \langle A \rangle_{light} + \langle A \rangle_{heavy}$, $\langle A \rangle_{mat}$ is mass number of parent nucleus. In case of light actinides (Th, U) we get $\langle A \rangle_{heavy} = 135-138$. In diagrams [7, 13] of fragment mass distribution in spontaneous heavy nuclear fission ($A = 235, 238$) an average mass number of lighter fragments is 90–100, an average mass number of heavier fragments is about 140.

If one assumes that fast nucleosynthesis in stars occurs as a result of transition of nucleon system from high-frequency vibration level with the period $t_i = 5 \cdot 10^{-23}$ s to the first rotational level with the typical energy 100 keV during the lifetime of this level $t_{rot} = 6.287 \cdot 10^{-21}$, then formulas (2.1) and (2.3) provide the following estimate of an average mass number of

superheavy (*sh*) elements: $\langle A \rangle_{sh} = 330$. The obtained value agrees with the prediction from [8] about possibility of nucleus formations with $A > 300$ in neutron stars. As for the induced fission as the process competing with nucleosynthesis, in this case one can accept as a typical scale t_i the sonic wave travel time t_0 in a nucleus formed as a result of nucleosynthesis: $t_0 = a/c_0$. With $A = 330$, $c_0 = 5 \times 10^7 \text{ m}\cdot\text{s}^{-1}$, we obtain $t_i = 3.6 \times 10^{-22} \text{ s}$. Then from formulas (2.1) and (2.3) we obtain that with the above lifetime of the first rotational level $6.287 \cdot 10^{-21} \text{ s}$ the average mass number of light fragments of superheavy elements is $\langle A \rangle_{light} = 100$. In compliance with the mass conservation law the average mass number of heavy fragments is equal to $\langle A \rangle_{heavy} = 230$. The obtained values correspond approximately to the first and the last peaks of final abundance of elements in Galaxy [3]. This compliance gives grounds to expect that the above estimate of the average mass number of superheavy elements that can be formed as a result of nucleosynthesis in supernovas and neutron stars, is reliable.

One can try to use the proposed model to consider effects in nuclear medium with deep inelastic scattering of neutrino and antineutrino across a target nucleus [5]. The process is described by the equation $\nu_l(k) + N(p) \rightarrow l^-(k') + X(p')$, $l = e, \mu$, where $N(p)$ is relativistically exited nucleon. From equation (1.25) one can obtain the following relation for an average mass of particles, i.e., products of intensive relativistic process:

$$\Delta E = mc^2 \Rightarrow \langle a \rangle = \frac{c}{\sqrt{2}} t = \frac{c}{\sqrt{2}} \frac{a_i}{c} = \frac{a_i}{\sqrt{2}} \Rightarrow \langle m \rangle = \frac{m_i}{2\sqrt{2}}. \quad (2.8)$$

Here, a_i, m_i are initial particle size and mass reacting with a neutrino. If it is a nucleon N having the mass $m_i = 1440 \text{ MeV}$ [14], the average mass of particles is equal to $\langle m \rangle \cong 510 \text{ MeV}$ that with an accuracy of up to 3% corresponds to K-meson (kaon) having a mass of 494 MeV and basic modes of disintegration $\nu\mu^+, \pi^0\pi^+$ [7, 14]. Thus, the proposed here approximate model in combination with the all-known data [7, 14] is in qualitative compliance with the ideas of work [5] where formation of a single kaon and a set of elementary particles is observed. If we except that a typical time scale t_i of interaction between objects is equal to travelttime of a relativistic particle at a space scale of a seed - proton with the charge size $a_p = 1.72 \text{ Fm}$ [7], then, from formula (2.7) with $\beta = \hbar/2m_p a_p c = 0.061$ we obtain a pulsed curve: the relative probability for formation of products of deeply inelastic object interaction versus their relative sizes $\xi = a/a_p$ (Figure 4).

Figure 4. Spectrum of particle relative sizes

From here we obtain a spectrum of the known particles [7, 14] that in the given reaction can be formed as a result of relativistic effects in the sea of partons leading to relativistic mass increase (rest mass of a produced particle is calculated as $m_{pr} = m_p \xi^3$): nucleon $N(1520)$,

lambda-particle $\Lambda(2110)$, $\eta_c(2980)$, $\chi(3510)$, $\psi(4160)$, B -meson (5271). The given in brackets values of rest masses (in MeV) fit into a width of the corresponding pulsation in ξ . The spectrum is cut at the seventh pulsation with maximum at $\xi = 1.828$ that approximately corresponds to the maximum possible value $\xi_{\max} = 1.824$ obtained according to formula (1.26) of chapter 1 for a seed – proton. This value is in compliance with the rest mass 5690 MeV. Masses of known baryons reach 5600 MeV [7]. The availability of a lambda-particle in the calculated spectrum means the possibility for formation of hyper-nuclei described in [7].

If we accept that the time for product formation is equal to the traveltime of relativistic particles inside the maternal nucleus $t_{form}^{ur} = a_m/c$, then from formulae (1.25) and (2.1) we obtain that the average mass number of deeply inelastic process products is related to the mass number A_m of a maternal nucleus through a simple dependence similar to formula (2.8):

$$\langle A \rangle_{pr} = A_m / 2\sqrt{2} \quad (2.9)$$

If we use ${}^9\text{Be}$ as a maternal nucleus, then from formula (2.9) we get $\langle A \rangle_{pr} = 3$, i.e., tritium nucleus. In case of targets [5] from carbon ${}^{12}\text{C}$, calcium ${}^{40}\text{Ca}$, iron ${}^{56}\text{Fe}$ we obtain $\langle A \rangle_{pr} = 4, 14, 20$, i.e. nuclei of helium, nitrogen, and neon are formed, respectively. If as maternal objects we use light actinides with $A_m = 238 - 244$, then $\langle A \rangle_{pr} = 84 - 87$, i.e. when interacting with relativistic particles, actinide nuclei will turn into nuclei corresponding to fission products [7, 13]. A question arises whether neutrino is a catalyst for spontaneous fission?

The developed asymptotic model of cluster formation in the nuclear matter complements the method for predicting nuclear mass with the help of radial basis function [6], which makes it possible to find mass numbers in the range from 20 to 260. In particular, the proposed method allows one to estimate the time of the process of approaching equilibrium based on characteristic mass numbers and the corresponding nuclide sizes. Such task was solved in [23] to fit the study into kinetics of deep inelastic interaction between a beam of heavy copper ions and a target of gold with the collision energy 365 MeV [9]. It has been demonstrated that times of formation of interaction products, having average mass numbers $\langle A \rangle = 60$ and 100 typical for the process under consideration, are in the order of magnitude 10^{-20} s. The resultant values are significantly lower than the lifetime $10^{-16} - 10^{-14}$ s of an intermediate compound nucleus. This allows us to speak about relatively fast (explosive) process progress. It should be noted that presented in [9] product distribution behavior in the reaction corresponds qualitatively to asymptotic properties of a distribution density function established in section 1. Thus, for large mass numbers the law (1.4) is approximately fulfilled, and against the background of a smooth curve local maximums take place at $A = 56$ and 63. Calculations based on formula (2.2) show that the most probable mass numbers of cluster nuclides in the interregion between cluster radioactivity and spontaneous fission are equal to 51, 58, 65, 67.

Conclusion

Presented results determine limits of application of cluster concepts both on nuclear medium dynamics in stars and formation of fundamental and elementary particles with regard for uncertainty principle. The proposed cluster model corresponds to the known concepts about stability of nuclides. This makes it possible to predict the most probable sizes and the corresponding mass numbers of nuclei within the unstudied range up to the hypothetical finite nucleus. The model was used also to describe deep inelastic relativistic processes in nuclear medium and to calculate average masses of the generated nuclei up to actinide fission products

and a spectrum of sizes and masses of generated elementary particles up to the heaviest known baryon.

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