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The prospects of statistical modeling of turbulent flows

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doi 10.18411/gq-31-07-2019-18
idsp sciencerussia-31-07-2019-18

Abstract

The problems and prospects of statistical modeling of energy containing structure of developed turbulent flows are shown. It is shown that the method of RANS can not provide high quality of such models due to unconditional averaging of Navier-Stokes equations. Also, it is shown that high quality of mathematical models can be achieved using the method of autonomous statistical modeling of turbulence ASMTurb, distinctive feature of which is conditional averaging of Navier-Stokes equations over a turbulent and non-turbulent fluid of a turbulent flow.

Keywords: turbulent flows, statistical modeling, intermittency.

1. Introduction

Development of an effective physical-mathematical model which would provide high accuracy and speed of calculation of averaged hydrodynamic values of a developed turbulent flow is still a key problem of statistical hydrodynamics. To solve this problem, it is first of all necessary to determine the type of statistical characteristics most suitable for mathematical modeling. In the classical RANS method, such characteristics are unconditional statistical momenta (“average” and “pulsation” values of hydrodynamic quantities); in the method of autonomous statistical modeling of turbulent flows ASMTurb [1] which takes into account the effect of hydrodynamic intermittency [2] they are conditional statistical momenta for each of the intermittent media with a turbulent and non-turbulent flow. [3, 4, 5, 6].

At present, the method of RANS is being intensively used for mathematical modeling of all possible processes of turbulence (for example, processes in case of turbulent combustion). Such stable attention to the RANS method is explained by a high speed of predicting the structure of a turbulent flow in comparison with the methods DNS and LES, i.e. a high calculation speed of hydrodynamic characteristics of the turbulent flow under study. At the same time, the practice showed that for more than a century-long history of using the method of RANS the attempts to construct a mathematical model of high quality had failed.

The aim of this work is to reveal the problems of the RANS method and determine the prospects of increasing the quality of statistical modeling of developed turbulent flows. For this, it is necessary to answer the following questions: for what reason the method of RANS does not allow building physical-mathematical models of high quality; which of statistical characteristics of a turbulent flow are most suitable for modeling; what approach in statistical modeling of turbulence processes is the most effective?

We will start solving the set up questions with discussion of the problems and peculiarities of statistical modeling of turbulent flows, to be more exact, large scale (energy containing) structure of developed turbulent flows with transverse shear.

2. Problems and peculiarities of statistical modeling of turbulent flows

The problems of the RANS method are well-known – mathematical models built according to this method can not provide a high accuracy of calculation of average and pulsation characteristics of turbulent flows. This especially refers to correlations of pulsation hydrodynamic quantities. At the same time, a recently developed method of autonomous modeling of turbulent flows ASMTurb, the peculiarity of which is consideration of intermittence effect, has shown that models built according to this method give a “surprisingly” good agreement of calculations with the known experimental data [3,4].

To reveal the problems of the RANS method and show the features of statistical modeling of turbulent flows, let us consider the possible variants of statistical averaging of differential Navier-Stokes equations. In case of an incompressible fluid flow, Navier-Stokes equations can be written in the form:

$$\begin{cases} \frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_k} \left(u_i u_k + \frac{p \delta_{ik}}{\rho} - \sigma_{ik} \right) = 0, & i = 1, 2, 3 \\ \frac{\partial u_k}{\partial x_k} = 0 \end{cases} \quad (1)$$

with summation in respect to $k = 1, 2, 3$ at fixed values $i = 1, 2, 3$; here, the tensor of viscous stresses. $\sigma_{ik} = \nu(\partial u_i / \partial x_k + \partial u_k / \partial x_i)$.

According to [3-6], unconditional statistical averaging of equations (1) gives equations of RANS in the form:

$$\begin{cases} \frac{\partial \langle u_i \rangle_R}{\partial t} + \frac{\partial \langle u_i \rangle_R \langle u_k \rangle_R}{\partial x_k} + \frac{\partial \langle u'_{iR} u'_{kR} \rangle_R}{\partial x_k} + \frac{\partial \langle (p_R \delta_{ik} - \sigma_{Rik}) / \rho \rangle_R}{\partial x_k} = 0 \\ \frac{\partial \langle u_k \rangle_R}{\partial x_k} = 0, \end{cases} \quad (2)$$

where $\langle u_i \rangle_R$ – unconditional means of the velocity vector components, $\langle u_i \rangle_R \equiv \langle u_{iR} \rangle_R$, $u'_{iR} = u_{iR} - \langle u_i \rangle_R$. At the same time, conditional statistical equations (1) give equations of ASMTurb namely: for the flow of turbulent fluid in the form:

$$\begin{cases} \frac{\partial \langle u_i \rangle_t}{\partial t} + \frac{\partial \langle u_i \rangle_t \langle u_k \rangle_t}{\partial x_k} + \frac{\partial \langle u'_{it} u'_{kt} \rangle_t}{\partial x_k} + \frac{\partial \langle (p_t \delta_{ik} - \sigma_{tik}) / \rho \rangle_t}{\partial x_k} = 0 \\ \frac{\partial \langle u_k \rangle_t}{\partial x_k} = 0, \end{cases} \quad (3)$$

where $\langle u_i \rangle_t$ – conditional means of the velocity vector component within the turbulent medium, $\langle u_i \rangle_t \equiv \langle u_{it} \rangle_t$, $u'_{it} = u_{it} - \langle u_i \rangle_t$; for the flow of non-turbulent fluid in the form

$$\begin{cases} \frac{\partial \langle u_i \rangle_n}{\partial t} + \frac{\partial \langle u_i \rangle_n \langle u_k \rangle_n}{\partial x_k} + \frac{\partial \langle u'_{in} u'_{kn} \rangle_n}{\partial x_k} + \frac{\partial \langle (p_n \delta_{ik} - \sigma_{nik}) / \rho \rangle_n}{\partial x_k} = 0 \\ \frac{\partial \langle u_k \rangle_n}{\partial x_k} = 0 \end{cases} \quad (4)$$

where $\langle u_i \rangle_n$ – conditional means of the velocity vector components inside the non-turbulent medium. It is worth to note that $\langle u_i \rangle_n \equiv \langle u_{in} \rangle_n$, $u'_{in} = u_{in} - \langle u_i \rangle_n$.

As each of the systems (3)-(4) is statistically independent, this allows modeling conditional means of each of the intermittent media with turbulent and non-turbulent fluid autonomously, i.e. independently of each other. This independence is determined by the fact that there are no one-point correlations of hydrodynamic quantities of turbulent and non-turbulent fluids, for example $\langle u'_{it} u'_{kn} \rangle_t = 0$. It is important to emphasize that autonomy of each system (3)-(4) allows to accept it's own hypotheses of closures for correlations $\langle u'_{it} u'_{kt} \rangle_t$ and $\langle u'_{in} u'_{kn} \rangle_n$

After solving equations (3)-(4) and determining conditional means $\langle u_i \rangle_t$ and $\langle u_i \rangle_n$ we have the possibility to find total means by the formula

$$\langle u_i \rangle = \gamma \langle u_i \rangle_t + (1 - \gamma) \langle u_i \rangle_n \quad (5)$$

which are equivalent to the unconditional mean, i.e. $\langle u_i \rangle \cong \langle u_i \rangle_R$. To do this, however, it is necessary to know the distribution of values of the external intermittency coefficient of turbulent fluid γ . As we see, equations of RANS and ASMTurb, i.e. differential equations (2) as well as (3) and (4) are identical in appearance. It is not surprising as hydrodynamic quantities in RANS and ASMTurb methods are characteristics of the continuous medium flow and from the point of view of A.N. Kolmogorov's probability theory they are dense and continuous sets; mathematically rigorous derivation of equations (2)-(4) is explained in detail in [4], where we give the proof of the expression:

$$\left\langle \frac{\partial f}{\partial x} \right\rangle_r = \frac{\partial \langle f \rangle_r}{\partial x}, \quad r = t, n, R \quad (6)$$

It is appropriate to note that in ASMTurb method, the process of production of a turbulent and "disappearance" of a non-turbulent fluid takes place only in a thin "superlayer" which separates these fluids. It is therefore that equations (3) and (4) do not contain "sources".

So, it follows that equations of RANS were derived correctly, however, the "double structure" experimentally found in [7] was lost due to unconditional averaging of Navier-Stokes equations. In other words, the problems in the method of statistical modeling RANS are related to the absence of intermittency effects, so that it is not possible to set different closure hypotheses in each of the intermittent media. The calculations presented in [3-4] convincingly confirm a low efficiency of physical and mathematical models of ASMTurb (see Figure 1 and Figure 2).

As far as the problems in statistical modeling of turbulent flows, which considers the intermittency effects are concerned, we should say the following.

Building of physical and mathematical models by the ASMTurb method comes to define successive stages namely: to construct a physical and mathematical model for conditional statistical characteristics of a non-turbulent fluid flow; to construct a mathematical model for calculation of the intermittency coefficient; to carry out calculation of statistical characteristics which are related to conditional statistical characteristics algebraically via the intermittency coefficient. The calculations presented in [3-4] convincingly confirm a high efficiency of physical and mathematical models of ASMTurb (see Figure 3 and Figure 4). It is clear that the main difficulties in building ASMTurb models are related to modeling of the intermittency coefficient in expressions for a total statistical mean (5).

At the same time, in many practically important problems, modeling of total means of a turbulent flow is not required.

3. The prospects of statistical modeling according to ASMTurb method

As is known, all turbulence processes proceed only in a turbulent fluid of a turbulent flow. At the same time, one of the advantages of ASMTurb method is the possibility to construct physical and mathematical models only for a turbulent fluid. This circumstance allows to significantly simplify statistical modeling of complex processes such as, turbulent mixing and turbulent diffusion combustion [3, 8, 9].

For example, a mathematical model for describing turbulent mixing of two wakes of the same incompressible fluid contains a system of equations only for conditional means of a turbulent fluid. Such a model is described in detail in [3-4]; at the level of the second central momenta it has the form:

$$\left\{ \begin{array}{l} \frac{\partial \langle u_i \rangle_t}{\partial t} + \frac{\partial \langle u_i \rangle_t \langle u_k \rangle_t}{\partial x_k} + \frac{\partial \langle u'_{it} u'_{kt} \rangle_t}{\partial x_k} + \frac{\partial \langle (p_t \delta_{ik} - \sigma_{tik}) / \rho \rangle_t}{\partial x_k} = 0 \\ \frac{\partial \langle u_k \rangle_t}{\partial x_k} = 0 \\ \frac{\partial \langle E_t \rangle_t}{\partial t} + \langle u_k \rangle_t \frac{\partial \langle E_t \rangle_t}{\partial x_k} + \frac{\partial \langle (E_t + p'_t / \rho) v'_t \rangle_t}{\partial x_k} + \langle u'_{it} u'_{kt} \rangle_t \frac{\partial \langle u_i \rangle_t}{\partial x_k} + \langle \varepsilon_t \rangle_t = 0 \end{array} \right. \quad (7)$$

The procedure for solving the system of equations (7), written in the approximation of the free boundary layer for the self-similar flow domain, is presented in detail in [3-4]. At the same time, after invoking the well-known (and relatively “simple”) closure hypotheses of gradient type, the calculated profiles of the conditional mean longitudinal velocity $\langle u \rangle_t$ and turbulent energy $\langle E_t \rangle_t$ almost completely coincided with the available experimental data.

4. Some results on testing of statistical modeling methods

The results of testing RANS and ASMTurb methods on the example of classical turbulent flows are presented in [3]. Here we will only present some calculation results for statistically stationary automodel mode of a turbulent flow in the mixing zone obtained in [3-4].

The results on calculation of unconditional means of RANS are presented in Figure 1 and Figure 2. The results on calculation of total and conditional means of ASMTurb are presented in Figure 3 and Figure 4. In these figures, the curves represented calculation, the signs – experimental data [10]. To calculate the total mean of pulsation intensity, the following expression was used:

$$\langle u'^2 \rangle = \gamma \langle u_t'^2 \rangle_t + (1 - \gamma) \langle u_n'^2 \rangle_n + \langle u'^2 \rangle_{dop} \quad (8)$$

where $\langle u_t'^2 \rangle_t = \langle E_t \rangle_t$ and $\langle u'^2 \rangle_{dop} = \gamma(1 - \gamma)((\langle u \rangle_t - \langle u \rangle_n)^2)$.

Figure 1. The unconditional mean profile of longitudinal velocity $U_R = \langle u \rangle_R / u_1$.

Figure 2. The unconditional mean profile of turbulent energy $\langle E \rangle_{*R} = \langle E \rangle_R / \Delta U^2$.

Figure 3. The profiles of a conditional and total mean of longitudinal velocity $\langle U \rangle_r = \langle u \rangle_r / u_1$: 1- Δ - $\langle U \rangle_t$; 2- $\square$ - $\langle U \rangle_n$; 3- $\circ$ - $\langle U \rangle$.

Figure 4. The profiles of a conditional and total mean of pulsation intensity of longitudinal velocity $\langle u' \rangle_r = \langle u'^2 \rangle_r / \Delta U^2$: 1- Δ - $\langle u' \rangle_t$; 2- $\square$ - $\langle u' \rangle_n$; 3- $\circ$ - $\langle u' \rangle$; 4- $\langle u'^2 \rangle_{dop}$.

As can be seen, curves 1 in Figure 3 and Figure 4, which were calculated only on the basis of the system of equations (7), almost completely coincide with the experimental data. Thus, the built by us mathematical model of a turbulent flow in the wake mixing zone allows to calculate both conditional (for flows of turbulent and non-turbulent fluids) and total statistical means of such flow characteristics at the level of second central momenta (see Figure 3 and Figure 4). It is worth to note that this model contains only two semiempirical parameters, this being a turbulent and non- turbulent fluid.

5. Conclusion

The results of the carried out by us investigation research allow to set up questions. The following points have been cleared up. The method of RANS can not provide building as unconditional averaging of Navier-Stokes equations results in the loss of hydrodynamic intermittency effects and, consequently, the loss of accuracy when calculating the statistical characteristics of a turbulent flow. In other words, the structure of flows of a turbulent and non-turbulent fluid in a specifically taken "turbulent" flow differs principally. This fact does not allow us to set up different closure hypotheses for RANS models, although the need for such a task is obvious due to this difference in structures. It has become clear that the search for "good" closure hypotheses for RANS models in principle can not principally give good results. This is confirmed by our calculations of unconditional statistical means.

The importance of the RANS method for building mathematical models of hydrodynamic processes of turbulence can not be overestimated - its fruitful use has been realized during many years in different fields of science and engineering. However, the urgent necessity in increasing the quality of RANS models forces us to look for new ways in the statistical modeling of turbulent flows.

Consideration of the intermittency phenomenon in statistical modeling of turbulent flows has become promising for building deeper, more exact and effective (in terms of minimum time expenditure for calculation of statistical characteristics) physical and mathematical models of such flows. ASMTurb method, is just this method.

One of the characteristic peculiarities of ASMTurb method is performance of statistical averaging of instantaneous differential equations of hydrodynamics with the help of formal use of conditional PDF. The obtained questions allow modeling conditional statistical averages characteristics of each of the intermittent media with a turbulent and nonturbulent fluid of a concrete turbulent and non-turbulent fluid independently of each other while, for modeling of total averages, simple algebraic relations of statistical hydrodynamics theory are used, i.e. here attraction of differential equations for total averages is not required. The possibility to construct mathematical models only for the flow of a turbulent fluid is one of the main advantages of ASMTurb method. As all main processes of turbulent mixing take place exactly in a turbulent fluid, this approach simplifies much the process of statistical modeling.

It is worth noting that the method of building mathematical models ASMTurb models is, in fact, the use RANS method for each of the intermittent media of a turbulent flow separately. This is related to the fact that equations for unconditional averages in RANS method formally agree with the equations for conditional averages of each of the intermittent media with a turbulent and non-turbulent fluid. However, the possibility to set up different boundary conditions and different closure hypotheses for each of the intermittent media of a turbulent flow separately makes the ASMTurb method principally new and high effective.

It should be noted that the possibility of building mathematical models only for the flow of a turbulent fluid without attracting of non-turbulent fluid opens new prospects for statistical modeling of complex processes such as, for example, turbulent mixing of different fluids (gases) and turbulent diffusion combustion.

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