Extraction of Zernike Coefficients Modes using Holographic Wavefront Sensing: Modeling and Simulation

S. Kara Mohammed, D. Bahloul, S. Cherif, and M. Rahmani

Abstract—We propose a holographic wavefront sensor that uses a binary computer-generated hologram (BCGH). This CGH is coded with a single particular Zernike aberration mode with different coefficients and then all the holograms are multiplexed and binarized to get the final hologram. This multiplexed hologram is used to produce a spots (white pixel on the detector) according to the presence of particular aberration. The Numerical results by Matlab program corresponding to the CGHs are presented.

Index Terms—Zernike aberration modes, wavefront sensing, Fourier transform, computer-generated hologram.

I. INTRODUCTION

As an important part of the adaptive optics system, wavefront sensor can detect the unknown wavefront aberrations coming through various sources e.g., optical component, atmosphere, etc. In modal wavefront sensors the aberration can be decomposed into a set of orthogonal polynomials (e.g. Zernike polynomials [1], or Zernike aberration mode). If only the coefficient of each Zernike aberration mode is fixed, we will retrieve the aberration in formation and fully reconstruct the distorted wavefront. Various techniques exist in the literature for wavefront sensing [2]-[4], each with their own benefits and drawbacks. Recently, a holographic wavefront sensor has been proposed [5], [6], in which a multiplexed hologram acts as a wavefront sensor. It completely removes the need of any computation. However, optical recording of multiplexed hologram is tedious process.

The computer-generated hologram is the better option to implement this type of sensor [7], [8].

Our work is about the development of a holographic wavefront sensor to be used for opthalmic applications such as in vivo retinal imaging and human eye's aberrations measurements.

Manuscript received May 20, 2015; revised July 27, 2015.

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II. COMPUTER GENERATED HOLOGRAM

We record each CGH by interfering a reference wave $\psi_R^k = \psi_p \exp\left(i \frac{2\pi}{\lambda} a_k Z_k\right)$ with an object wave ψ_O^k . ψ_R^k is the reference wave distorted by a Zernike aberration mode $\varphi_k = a_k Z_k$ with k the single index. ψ_O^k is the object wave passing through an image with white pixel and an FT lens. The record process is represented in Fig. 1.

Fig. 1. Recording process for a single hologram.

According to the principle of holography [9], the intensity at the recording plane of the single hologram is calculated by:

$$I_k = [\psi_a^k + FT(d_p)] [\psi_a^k + FT(d_p)]^* \quad (1)$$

$$I_k = |\psi_a^k|^2 + |FT(d_p)|^2 + \psi_a^k [FT(d_p)]^* + [\psi_a^k]^* FT(d_p) \quad (2)$$

$$I_k = \left| \psi_p \exp\left(i \frac{2\pi}{\lambda} a_k Z_k\right) \right|^2 + |FT(d_p)|^2 + \psi_p \exp\left(i \frac{2\pi}{\lambda} a_k Z_k\right) [FT(d_p)]^* + \left[\psi_p \exp\left(i \frac{2\pi}{\lambda} a_k Z_k\right) \right]^* FT(d_p) \quad (3)$$

For the amplitude hologram, its transmission τ can be viewed as proportional to the interferogram intensity distribution I_k . We choose the aberrated wave ψ_a^k as the replay wave to reconstruct the hologram, and multiply the four terms on right hand side of (3) equation using ψ_a^k . In the result, first two terms are zero diffraction order, third term reconstructs the conjugate object beam, the fourth term gives the original object beam, and if we take its inverse Fourier transform, we retrieve the image with white pixel. Fig. 2 gives the reconstruction process.

Fig. 2. Reconstruction process for a single hologram.

The corresponding spot in the detector plane represent the existence of the aberration $a_k Z_k$; therefore, we can achieve the goal to detect the unknown aberration quickly and effectively.

III. NUMERICAL SIMULATION AND RESULTS

In order to show the interest of holographic wavefront sensor, CGH for the coma1 aberration with different coefficients has been simulated by applying equation (3). The associated parameters in numerical simulations are set as follows: wavelength 632.8nm, pixel number 256×256, coefficients values (from -2λ to 2λ) without the zero value, which is not required to design the hologram. The obtained interference pattern between a plane wavefront and the aberrated wavefront is as shown in Fig. 1.

Fig. 3. Interference fringes between a plane wavefront and the aberrated wavefront with the coma1 coefficients from -2λ to 2λ .

As we can see on Fig. 3, it is difficult to visually distinguish the difference between positive and negative value of the same coefficient (e.g., -2λ and 2λ) in interferometric estimation, whereas our holographic wavefront sensor can easily distinguish between the two, as is shown below Fig. 4.

Fig. 4. Interference fringes between the aberrated wavefront and Fourier transform of dot image with the coma1 coefficients from -2λ to 2λ .

These holograms are added to get the multiplexed hologram, which is then binarized to get the CGH mask. This is shown in Fig. 5.

Fig. 5. Binary mask for coma1, coded with amplitudes of Fig. 4.

To check the wavefront sensing for coma1 with this CGH, we use the same aberrated wavefronts one by one as reference and compute the spot images at Fourier plane. The results are shown in Fig. 6. The dots corresponding to the amplitudes are visible clearly.

Fig. 6. Sensor output for detection of coma1 with amplitude -2λ and -1.0λ , which is coded in the design of CGH mask of Fig. 5. (a, c) original dot images, (b, d) reconstructed dot images.

Fig. 7. Sensor output for detection of coma1 with amplitude 0.5λ and 1.5λ , which is coded in the design of CGH mask of Fig. 5. (a, b) original dot images, (c, d) reconstructed dot images.

If we illuminate this hologram with a reference wave Ψ_R^k we get the corresponding dot image dp in the Fourier plane, provided that the hologram of reference wavefront Ψ_R^k and FT (dp) is already coded in H .

The position of dot image is $d1(15, 15)$. Here, $d1(15, 15)$ means that there is a white pixel at $(15, 15)$, whereas the rest of the pixels are black.

We take $x=15$ for $coma1$, so that signal on the line $x=15$ will represent the presence of this mode only.

As shown in Fig. 6, the position of dot image reconstruction from multiplexed hologram is $d3(15, 48)$. It represents the existence of the aberration coma 1 with amplitude -1.0λ .

The corresponding sensor outputs, with 0.5 and 1.5 amplitudes (in λ), are shown in Fig. 7.

Fig. 8. Plots of sensor outputs, which are coded in the design of CGH mask of Fig. 5. a) detection of coma1 amplitude 0.5λ , b) detection of coma1 amplitude 1.5λ .

As shown in Fig. 8, for the amplitude value, which is coded in the hologram, the sensor gives the signal at the coded value.

For the amplitude value, which is not coded in the hologram, the sensor gives the signal at the nearby-coded value. The results with the amplitude 0.6 and 1.75 are shown in Fig. 9 and Fig. 10 respectively.

Fig. 9. Plots of sensor output at $x=15$ for detection of coma1 with amplitude 0.6λ , which is not coded in the design of CGH mask of Fig. 5.

Fig. 9 shows that for 0.6 (coma1 in λ) amplitude in the input beam, the sensor gives the spot at its nearest-coded value, i.e., 0.5 .

Fig. 10 shows that for 1.25 and 1.75 (coma1 in λ) amplitudes which are not coded in the design of CGH mask of Fig. 5, the sensor gives two feeble spots at equidistant nearest-coded values, i.e., at 1.0 and 1.5 for 1.25λ and 1.5 and 2.0 , for 1.75λ .

Fig. 10. Plots of sensor outputs, which are not coded in the design of CGH mask of Fig. 5. a) detection of coma1 amplitude 1.25λ , b) detection of coma1 amplitude 1.75λ .

IV. CONCLUSION

In this paper, we simply introduced the basic principle of CGH, which acts as the wavefront sensor. Interference between the aberrated wavefront with a single mode-coefficient combination and the Fourier transform of a single dot is numerically calculated for one hologram. Different coefficients for a single mode (coma1) and corresponding different positions of spots are taken to compute various holograms and then all the holograms are multiplexed to get the CGH mask. There is a clear visual distinction between negative and positive value of the same amplitude (e.g., -0.5 and 0.5) in holographic sensor, whereas in the interferometric estimation Fig. 3, the visual result is same for both signs of same value.

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