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MODELING OF CORN GRAIN DRYING BY RUNGE- KUTTA METHOD

Uziel Francisco Grajeda-González, Héctor Flores-Breceda, Juana Aranda-Ruiz, Juan Antonio Vidales-Contreras, Humberto Rodríguez-Fuentes and Alejandro Isabel Luna-Maldonado*

Department of Agricultural and Food Engineering, Faculty of Agriculture, Autonomous University of Nuevo Leon, México

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KEYWORDS

Drying

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ABSTRACT

Corn grain drying modelling is used to estimate moisture loss along time, and with this information improve the drying process, maximizing energy resources. The numerical method of Runge-Kutta is an alternative for the solution of ordinary differential equations, since obtaining the coefficients of the method is possible to simulate a very real approximation of the actual behaviour of drying process. In this research, the constants of the fourth order Runge-Kutta numerical method were calculated, a determination coefficient with $R^2=1$ between analytical solution and a numerical method was found. The mean error between the two solutions was 3.13×10^{-5} .

* Corresponding author

E-mail: alejandrolunaml@uanl.edu.mx (Alejandro Isabel Luna-Maldonado)

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1 Introduction

Many problems in engineering and science can be predicted from differential equations. Numerical methods are indeed a crucial part to solve differential equations that cannot be neglected (Kanevsky et al., 2007). Nowadays, computers are used extensively to solve various differential equations. Among the most significant numerical methods, Runge- Kutta method is most prominent one and it was developed by Carl Runge and Martin Wilhelm Kutta in 1901 (Roberts, 2011). Carl Runge also developed numerical methods for solving differential equations arose from his studies of atomic spectra. These numerical methods are frequently used in these days and help in the iterations which are performed faster with modern computers (O'Connor & Robertson, 2015a). Moreover, the thesis of Martin Wilhelm Kutta contains the Runge-Kutta method for solving ordinary differential equations (O'Connor & Robertson, 2015b). In many cases such as model of fatigue crack growth, it is not possible solve an ordinary differential equation by analytical methods. This type of models could be solved using numeric integration schemes such as Euler, Runge-Kutta 2nd, 4th or higher order algorithms (Amann & Kadau, 2016). Runge-Kutta method is one of most important method to solve models of continuous stages and simulate the long-time dynamical behavior of systems with special structures (Tang et al., 2016). According to Wen & Yu (2016) Runge-Kutta has proven to be a numerical method with mathematical stability, since their results converge under given initial conditions.

High moisture content is the most important problem in the mechanized harvested corn grain. Most grain is physiologically ripe when its moisture content is around 30%. In this state, it should be harvested rapidly to avoid attack in the field from predators and, in the case of rice to diminish grain loss from shattering and cracking. Therefore, farmers are required to dry their crops to solve this problem and avoid penalties by the administration of grain storage companies, normally sun drying method is used by the farmers but it is time and energy consuming. According to Proctor (1994), sun drying of grain remains the most common drying method in tropical developing countries. It is first employed when the crop is

standing in the field prior to harvest; maize cobs may be left on the standing plant for several weeks after attaining maturity. Although it is not required labor or other inputs field drying may render the grain subject to insect infestation and mould growth, prevent the land being prepared for the next crop and is vulnerable to theft and damage from animals. Drying in the field may also be carried out after harvest with the harvested plants laid in stacks with the grain, maize cobs or panicles raised above the ground and exposed directly to the sun. The objectives of this research were find the solution to modeling of drying corn grain with a fourth order Runge-Kutta numerical method and compare the accuracy of the numerical solution obtained by the fourth order Runge-Kutta numerical method versus the analytical solution.

2 Materials and Methods

Equation 1 shows the fourth order Runge Kutta method with its parameters k_1 , k_2 , k_3 y k_4 .

$$Y_{t+\Delta} = Yt + \Delta t \left(\frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} \right) \quad (1)$$

Parameters k_1 , k_2 , k_3 & k_4 of fourth order Runge Kutta method can be obtained through equations 2 to 5 (Zill, 2012).

$$k_1 = f'(t, y) \quad \dots (2)$$

$$k_2 = f' \left(t + \frac{\Delta t}{2}, y + k_1 \frac{\Delta t}{2} \right) \quad \dots (3)$$

$$k_3 = f' \left(t + \frac{\Delta t}{2}, y + k_2 \frac{\Delta t}{2} \right) \quad \dots (4)$$

$$k_4 = f' \left(t + \Delta t, y + k_3 \frac{\Delta t}{2} \right) \quad \dots (5)$$

The steps which were involved in the Excel algorithm program of Runge-Kutta methods are (1) evaluate the slope in a value of the state variable (Figure 1), (2) to calculate y_1 in $t + \Delta t/2$ using k_1 and evaluate the slope in y_1 (Figure 2), (3) to calculate y_2 in $t + \Delta t/2$ using k_2 and evaluate the slope in y_2 (Figure 3), (4) to calculate y_3 in $t + \Delta t/2$ using k_3 and evaluate the slope in y_3 (Figure 4) (5) to calculate weighted slope and use it to estimate y in $t + \Delta t$ (Figure 5).

Figure 1 Slope 1 in Runge-Kutta method.

Figure 2 Slope 2 in Runge-Kutta method.

Figure 3 Slope 3 in Runge-Kutta method.

Figure 4 Slope 4 in Runge-Kutta method.

Figure 5 Weighted slope and real and estimated values in Runge-Kutta method.

Figure 6 Prediction model of moisture percentages on a dry basis differential equation.

To dry maize grain, a type-oven dryer was used at an initial temperature of 24°C to a temperature of 45°C and a corn grain layer of 20 cm depth (30 kg of mass) was placed in the oven tray and then drying process was started. After that, sampling of corn grain was made from three points of corn grain layer each hour. Corn grain masses were measured using a balance and then the moisture contents were determined by the equation of dry basis moisture (Equation 6).

$$\% \text{Moisture}_{\text{dry basis}} = \frac{\text{Water}}{\text{Dried corn grain}} \times 100(6)$$

3 Results and Discussion

During drying test, corn grain mass and initial moisture were 40kg and 30.36%, respectively. The corn grain mass and final moisture were reduced 35.8 kg and 10.45%, respectively after 8 hours of drying test (Table 1), the kinetics of drying process of corn grain was studied by Krokida et al. (2003). For modeling in Excel Worksheet the following equations on the initial moisture conditions were programmed :

$h = 0.5;$

Constant $k = -0.13315497;$

$K1 = \$C\$3 * (\$E\$3 * C7);$

$K2 = \$C\$3 * (\$E\$3 * (C7 + (0.5 * D7)));$

$K3 = \$C\$3 * (\$E\$3 * (C7 + (0.5 * E7)));$

$K4 = \$C\$3 * (\$E\$3 * (C7 + (0.5 * F7)));$

$\%H_{\text{Runge-Kutta}} = \$C\$3 * (\$E\$3 * (C7 + F7))$

Where: C= time increment; E= intrinsic rate of grain corn drying K_1 =slope 1; K_2 = slope 4

Table 1 Percentages of dry basis moisture obtained from samples of corn grain from dryer.

Time (hours)	Moisture dry basis (%)
0	30.36
1	28.00
2	26.00
3	24.00
4	22.00
5	20.98
6	17.00
7	13.42
8	10.45

Table 2 Percentage of corn grain dry basis moisture by differential equation, parameters k_1 , k_2 , k_3 , k_4 and % of corn grain dry basis moisture by Runge-Kutta method *versus* time.

Time (hours)	%Moisture _{Ec.Dif}	k_1	k_2	k_3	k_4	%Moisture _{Runge-Kutta}
0	30.36000	-2.02373	-1.95628	-1.95853	-1.89318	30.36
0.5	28.40225	-1.89323	-1.83013	-1.83223	-1.77110	28.4022454
1	26.57074	-1.77115	-1.71212	-1.71408	-1.65689	26.5707359
1.5	24.85733	-1.65693	-1.60171	-1.60355	-1.55004	24.8573307
2	23.25441	-1.55009	-1.49842	-1.50015	-1.45009	23.2544139
2.5	21.75486	-1.45013	-1.40180	-1.40341	-1.35658	21.7548606
3	20.35201	-1.35662	-1.31140	-1.31291	-1.26910	20.3520056
3.5	19.03961	-1.26914	-1.22684	-1.22825	-1.18727	19.0396132
4	17.81185	-1.18730	-1.14773	-1.14905	-1.11071	17.81185
4.5	16.66326	-1.11074	-1.07372	-1.07495	-1.03908	16.6632587
5	15.58874	-1.03911	-1.00448	-1.00563	-0.97208	15.5887339
5.5	14.58350	-0.97210	-0.93970	-0.94078	-0.90939	14.5834995
6	13.64309	-0.90942	-0.87911	-0.88012	-0.85075	13.6430873
6.5	12.76332	-0.85077	-0.82242	-0.82336	-0.79589	12.7633173
7	11.94028	-0.79591	-0.76939	-0.77027	-0.74457	11.940279
7.5	11.17032	-0.74459	-0.71977	-0.72060	-0.69655	11.170314
8	10.45000	-0.69657	-0.67336	-0.67413	-0.65164	10.45
8.5	9.77614	-0.65166	-0.62994	-0.63066	-0.60962	9.77613516
9	9.14573	-0.60963	-0.58932	-0.58999	-0.57031	9.14572429
9.5	8.55597	-0.57032	-0.55131	-0.55195	-0.53353	8.55596526
10	8.00424	-0.53354	-0.51576	-0.51635	-0.49913	8.00423665

$h = 0.5$; Constant $k = -0.13315497$

% $H_{n+1} = y_n + [K_1/6 + K_2/3 + K_3/3 + K_4/6]$, moisture content in corn grain.

Exact or analytical solution of the differential equation of corn drying of maize grain was %Moisture_{Dif.Eq} = $30.36e^{(-0.133t)}$, the solving to the mathematical model is observed with Martinello et al. (2013) with a determination coefficient equals to 1 (Figure 6).

Figure 7 Prediction model of moisture percentages on a dry basis with the Runge –Kutta numerical method.

Table 2 shows the percentages of corn grain moisture on dry basis obtained by differential equation, parameters k_1 , k_2 , k_3 and k_4 and corn grain moisture on dry basis obtained by

Runge- Kutta method. The average error between the percentages of corn grain moisture obtained with the differential equation and the Runge-Kutta method was 3.138×10^{-5} . The solution for drying grain corn by the Runge - Kutta method was %Moisture_(Runge-Kutta) = $30.36e^{(-0.133t)}$ with a determination coefficient equals to 1 (Figure 7). It was observed that error between methods to determine corn grain moisture contents is relatively low.

Conclusions

The moisture content was predicted according to an exponential model, which was “%Moisture_(Runge-Kutta) = $30.36e^{(-0.133t)}$ ” with a determination coefficient of $R^2 = 1$. Further, the error between the percentages of moisture obtained with the differential equation and the Runge-Kutta numerical method was 3.138×10^{-5} .

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Conflict of interest

Authors would hereby like to declare that there is no conflict of interests that could possibly arise.

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