SIMULATION OF ELECTROMAGNETIC WAVE PROPAGATION IN THE MEDIUM USING FINITE DIFFERENCE METHOD ON THE DISPLACED GRIDS

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Abstract. The second-order of approximation numerical method for solution of Maxwell Equations in nondispersive medium, called FDTD (Finite-Difference Time-Domain), also known as Yee algorithm, was used to create the software package for modelling of electromagnetic waves propagation in medium (with the Courant stability condition). The Total Field/Scattered Field (TFSF) source is considered in detail. Inhomogeneous medium is also included into the model and there are some examples, which show, how the program works with it. The different boundary conditions (periodic, reflecting and absorbing) were looked into. Perfectly Matched Layer (PML) absorbing boundary conditions are used in the program and the corresponding results are discussed. Different types of sources are realized in program, the results of the waves propagation are shown.

Keywords: Maxwell equations, Yee algorithm, electromagnetic waves, TM-, TE waves, the algorithm of perfectly matched layers, total field/scattered field equipment

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1. INTRODUCTION

It is well known that Maxwell equations are the basis of all the electromagnetic phenomena. That's why the solution of these equations is used in wide range of problems, including the propagation of electromagnetic waves in medium, which is the subject of the article. A plenty of different numerical methods have been

developed in order to solve this task. One of the most popular is the finite-difference method on offset grids, Finite Difference Time Domain (FDTD) [1], also known as Yee algorithm [2]. There are many advantages of this method. The first advantage is simple data decomposition: the data is stored in simple way; it is easy to work with it. Secondly, on each time step the values of both electric and magnetic fields are known in each grid cell. Thirdly, this method possesses a Divergence-Free Nature. Despite the great number of applications, the problem of creation a universal solver for this method on arbitrary grids has not yet been resolved. The program for modelling of electromagnetic waves has been created on the base of this method and has been applied to various

tasks. The results of modelling are shown in the article. For the optimization of this program, the algorithm can be combined with the method FDFD, based on transfer to frequency domain [3].

2. NUMERICAL ALGORITHM

2.1. FINITE DIFFERENCE ANALOGUES OF MAXWELL EQUATIONS

Maxwell's equations:

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \vec{E} - \vec{M},$$

$$\frac{\partial \vec{D}}{\partial t} = \vec{\nabla} \times \vec{H} - \vec{J},$$

$$\vec{\nabla} \cdot \vec{D} = 0,$$

$$\vec{\nabla} \cdot \vec{B} = 0.$$

where E – electric field strength, D – electrical induction, H – magnetic field strength, B – magnetic induction, J – electric current density, M – magnetic current density. For linear, isotropic, nondispersing materials:

$$\vec{D} = \varepsilon \varepsilon_0 \vec{E},$$

$$\vec{B} = \mu \mu_0 \vec{H},$$

where ε – dielectric permeability, ε_0 – vacuum dielectric permeability, μ – magnetic permeability, μ_0 – vacuum magnetic permeability.

$$\vec{J} = \sigma \vec{E},$$

$$\vec{M} = \sigma^* \vec{H},$$

where σ – electrical conductivity, σ^* – magnetic conductivity. In this way,

$$\frac{\partial \vec{H}}{\partial t} = -\frac{1}{\mu} \vec{\nabla} \times \vec{E} - \frac{1}{\mu} \sigma^* \vec{H},$$

$$\frac{\partial \vec{E}}{\partial t} = -\frac{1}{\varepsilon} \vec{\nabla} \times \vec{H} - \frac{1}{\varepsilon} \sigma \vec{E}.$$

In the FDTD (Finite-Difference Time-Domain) method the components of electric and magnetic fields are placed in

Fig. 1. The grid cell FDTD algorithm.

such a way, that each component of one field is surrounded by the components of another (Fig. 1).

For example, in one-dimensional case we have the values of E field, emitted by some source (Gaussian pulse, plane wave, harmonic source, etc.). On the next time step the H field is calculated in all grid cells. Notice, that each component of H field is surrounded by the two components of E field (as if is 1d case). On the third time step we calculate the E field, possessing the values of H field on the previous step, and so on (Fig. 2).

Fig. 2. Scheme of integration over time.

The numerical equations [2] we will acquire, using the approximation of space and time derivatives by their final differences:

$$\frac{\partial u}{\partial x}(i\delta x, j\delta y, k\delta z, n\delta t) = \frac{u_{i+1/2,j,k}^n - u_{i-1/2,j,k}^n}{\delta x} + O[(\Delta x)^2],$$

$$\frac{\partial u}{\partial t}(i\delta x, j\delta y, k\delta z, n\delta t) = \frac{u_{i,j,k}^{n+1/2} - u_{i,j,k}^{n-1/2}}{\delta t} + O[(\Delta t)^2].$$

There is a problem that on the left side of equations there is a field in the time moment n , while on the right side-in the moments $n - 1/2$ and $n + 1/2$. That's why let us consider E field as:

$$E_x|_{i,j,k}^n = \frac{E_x|_{i,j,k}^{n+1/2} + E_x|_{i,j,k}^{n-1/2}}{2}.$$

After all calculations, we have for all field components:

$$E_x|_{i,j+1/2,k+1/2}^{n+1/2} = C_\alpha|_{i,j+1/2,k+1/2} E_x|_{i,j+1/2,k+1/2}^{n-1/2} + C_b|_{i,j+1/2,k+1/2} (H_z|_{i,j+1,k+1/2}^n - H_z|_{i,j,k+1/2}^n - H_y|_{i,j+1/2,k+1}^n + H_y|_{i,j+1/2,k}^n),$$

$$E_y|_{i-1/2,j+1,k+1/2}^{n+1/2} = C_\alpha|_{i-1/2,j+1,k+1/2} E_y|_{i-1/2,j+1,k+1/2}^{n-1/2} + C_b|_{i-1/2,j+1,k+1/2} (H_x|_{i-1/2,j+1,k+1}^n - H_x|_{i-1/2,j+1,k}^n - H_z|_{i,j+1,k+1/2}^n + H_z|_{i-1/2,j+1,k+1/2}^n),$$

$$E_z|_{i-1/2,j+1/2,k+1}^{n+1/2} = C_\alpha|_{i-1/2,j+1/2,k+1} E_z|_{i-1/2,j+1/2,k+1}^{n-1/2} + C_b|_{i-1/2,j+1/2,k+1} (H_y|_{i,j+1/2,k+1}^n - H_y|_{i-1,j+1/2,k+1}^n - H_x|_{i-1/2,j+1,k+1}^n + H_x|_{i-1/2,j,k+1}^n),$$

$$H_x|_{i-1/2,j+1,k+1}^{n+1} = D_\alpha|_{i-1/2,j+1,k+1} H_x|_{i-1/2,j+1,k+1}^n + D_b|_{i-1/2,j+1,k+1} (E_y|_{i-1/2,j+1,k+3/2}^{n+1/2} - E_y|_{i-1/2,j+1,k+1/2}^{n+1/2} - E_z|_{i-1/2,j+3/2,k+1}^{n+1/2} + E_z|_{i-1/2,j+1/2,k+1}^{n+1/2}),$$

$$H_y|_{i,j+1/2,k+1}^{n+1} = D_\alpha|_{i,j+1/2,k+1} H_y|_{i,j+1/2,k+1}^n + D_b|_{i,j+1/2,k+1} (E_z|_{i,j+3/2}^{n+1/2} - E_z|_{i,j+1/2}^{n+1/2} - E_x|_{i,j+1/2,k+3/2}^{n+1/2} + E_x|_{i,j+1/2,k+1/2}^{n+1/2}),$$

where (the size of the cell is Δ):

$$C_\alpha|_{i,j,k} = \left(1 - \frac{\sigma_{i,j,k}\Delta t}{2\varepsilon_{i,j,k}}\right) / \left(1 + \frac{\sigma_{i,j,k}\Delta t}{2\varepsilon_{i,j,k}}\right),$$

$$C_b|_{i,j,k} = \left(\frac{\Delta t}{\varepsilon_{i,j,k}\Delta}\right) / \left(1 + \frac{\sigma_{i,j,k}\Delta t}{2\varepsilon_{i,j,k}}\right),$$

$$D_\alpha|_{i,j,k} = \left(1 - \frac{\sigma_{i,j,k}^*\Delta t}{2\mu_{i,j,k}}\right) / \left(1 + \frac{\sigma_{i,j,k}^*\Delta t}{2\varepsilon_{i,j,k}}\right),$$

$$D_b|_{i,j,k} = \left(\frac{\Delta t}{\mu_{i,j,k}\Delta}\right) / \left(1 + \frac{\sigma_{i,j,k}^*\Delta t}{2\varepsilon_{i,j,k}}\right).$$

The cases of TE and TM waves are very important. For them the numerical equations are:

TM-wave:

$$E_z|_{i-1/2,j+1/2}^{n+1/2} = C_\alpha|_{i-1/2,j+1/2} E_z|_{i-1/2,j+1/2}^{n-1/2} + C_b|_{i-1/2,j+1/2} (H_y|_{i,j+1/2}^n - H_y|_{i-1,j+1/2}^n + H_x|_{i-1/2,j}^n - H_x|_{i-1/2,j+1}^n),$$

$$H_x|_{i-1/2,j+1}^{n+1} = D_\alpha|_{i-1/2,j+1} H_x|_{i-1/2,j+1}^n + D_b|_{i-1/2,j+1} (E_z|_{i-1/2,j+1/2}^{n+1/2} - E_z|_{i-1/2,j+3/2}^{n+1/2} + n+1/2),$$

$$H_y|_{i,j+1/2}^{n+1} = D_\alpha|_{i,j+1/2} H_y|_{i,j+1/2}^n + D_b|_{i,j+1/2} (E_z|_{i+1/2,j+1/2}^{n+1/2} - E_z|_{i-1/2,j+1/2}^{n+1/2} + n+1/2).$$

TE-wave:

$$E_x|_{i,j+1/2}^{n+1/2} = C_\alpha|_{i,j+1/2} E_x|_{i,j+1/2}^{n-1/2} + C_b|_{i,j+1/2} (H_z|_{i,j+1}^n - H_z|_{i,j}^n),$$

$$E_y|_{i-1/2,j+1/2}^{n+1/2} = C_\alpha|_{i-1/2,j+1/2} E_y|_{i-1/2,j+1/2}^{n-1/2} + C_b|_{i-1/2,j+1/2} (H_z|_{i-1,j+1}^n - H_z|_{i,j+1}^n),$$

$$H_z|_{i,j+1/2}^{n+1} = D_\alpha|_{i,j+1/2} H_z|_{i,j+1/2}^n + D_b|_{i,j+1/2} (E_x|_{i,j+3/2}^{n+1/2} - E_x|_{i,j+1/2}^{n+1/2} + n+1/2 + E_z|_{i-1/2,j+1}^{n+1/2} - E_z|_{i+1/2,j+1}^{n+1/2} + n+1/2).$$

It is known that numerical methods do not converge if the wrong parameters are chosen. Moreover, there is a numerical

dispersion: the numerical solution may appear to be non-physical: a spherical wave may propagate not in the same way in different directions. It may even have a form, resembling a cube. To avoid this misbehavior, the parameters must be chosen, according to the following rules.

Courant stability condition:

$$c\tau \sqrt{\frac{1}{h_x^2} + \frac{1}{h_y^2} + \frac{1}{h_z^2}} < 1,$$

where $c = \frac{1}{\sqrt{\mu\epsilon}}$ – is the speed of light in medium.

The general numerical dispersion relation:

$$\left(\frac{1}{c\Delta t} \sin\left(\frac{\omega\Delta t}{2}\right) \right)^2 = \left(\frac{1}{\Delta x} \sin\left(\frac{k_x\Delta x}{2}\right) \right)^2 + \left(\frac{1}{\Delta y} \sin\left(\frac{k_y\Delta y}{2}\right) \right)^2 + \left(\frac{1}{\Delta z} \sin\left(\frac{k_z\Delta z}{2}\right) \right)^2.$$

2.2. Boundary conditions

In the developed program 3 types of boundary conditions are realized: reflective, periodic and absorbing.

The most simple are reflective conditions. The most natural way to use them is “perfect electric conductor”: we just make the field in particular equal to zero. Another way to implement these conditions is colliding with the same wave, but propagating in another direction: we insert into the boundary cells the values of field, equal to those of the field near boundary. We can also multiply the field in particular cells by the really small value. However, it is non-physical. That’s why, it can be used only in test programs. The last way to model reflection conditions – cells with high conductivity.

Periodic boundary conditions are used in general for modelling periodic structures in the direction of one of the axis (which helps to make the grid much smaller and to accelerate the calculations). These conditions are implemented through setting field on one boundary equal to field on the opposite boundary. There are no problems in 1d case. In other cases it is not always so easy to implement these conditions. For example, there are three cases: the wave, falling at an angle, pulsing source and periodic boundary conditions. If among these situations two are realized (not all three) then the method is working correctly (**Fig. 3**).

The last type of boundary conditions is absorbing. In this article the particular type of absorbing boundary conditions, called Perfectly Matched Layer [5], is used. The main idea of this method is creation of the conductive layer on the boundaries with the size more than one cell. The main advantage of this type of absorbing boundary conditions is that they work with the waves of arbitrary angle of propagation and emitted by any source. This algorithm is based on splitting each component of the field on two orthogonal components

Fig. 3. PML boundary conditions.

(thus, in 3d case we get a system of 12 numerical equations).

There are some features in implementation of PML: the conductivity must increase from center of the domain to the boundaries, otherwise boundary conditions do not work. Moreover, the minimization of the reflected wave must be looked into in considerable detail.

The system of numerical equations in 3d will now be:

$$\left(\varepsilon \frac{\partial}{\partial t} + \sigma_y \right) E_{xy} = \frac{\partial}{\partial y} (H_{zx} + H_{zy}),$$

$$\left(\varepsilon \frac{\partial}{\partial t} + \sigma_z \right) E_{xz} = -\frac{\partial}{\partial z} (H_{yx} + H_{yz}),$$

$$\left(\varepsilon \frac{\partial}{\partial t} + \sigma_z \right) E_{yz} = \frac{\partial}{\partial z} (H_{xy} + H_{xz}),$$

$$\left(\varepsilon \frac{\partial}{\partial t} + \sigma_x \right) E_{yx} = -\frac{\partial}{\partial x} (H_{zy} + H_{zx}),$$

$$\left(\varepsilon \frac{\partial}{\partial t} + \sigma_x \right) E_{zx} = \frac{\partial}{\partial x} (H_{yx} + H_{yz}),$$

$$\left(\varepsilon \frac{\partial}{\partial t} + \sigma_y \right) E_{zy} = -\frac{\partial}{\partial y} (H_{xy} + H_{xz}),$$

$$\left(\mu \frac{\partial}{\partial t} + \sigma_y^* \right) H_{xy} = -\frac{\partial}{\partial y} (E_{zx} + E_{zy}),$$

$$\left(\mu \frac{\partial}{\partial t} + \sigma_z^* \right) H_{xz} = \frac{\partial}{\partial z} (E_{yx} + E_{yz}),$$

$$\left(\mu \frac{\partial}{\partial t} + \sigma_z^* \right) H_{yz} = -\frac{\partial}{\partial z} (E_{xy} + E_{xz}),$$

$$\left(\mu \frac{\partial}{\partial t} + \sigma_x^* \right) H_{yx} = \frac{\partial}{\partial x} (E_{zx} + E_{zy}),$$

$$\left(\mu \frac{\partial}{\partial t} + \sigma_x^* \right) H_{zx} = -\frac{\partial}{\partial x} (E_{yx} + E_{yz}),$$

$$\left(\mu \frac{\partial}{\partial t} + \sigma_y^* \right) H_{zy} = \frac{\partial}{\partial y} (E_{xy} + E_{xz}).$$

For TM and TE wave cases we can also easily get the numerical system [1].

2.3. WAVE SOURCES

Harmonic source, gaussian pulse and plane wave are used as sources in the program. It is not so hard to implement the first two sources (just enter the values of field in some cells). However, the modelling of plane wave is a little bit more difficult. It can be modelled with the help of Total Field-Scattered Field technic (TFSF; **Fig. 4**). The main difficulty is creation of a source, which emit field in one direction, not in all directions symmetrical (in the comparison with many other sources, which are modelled through inserting the field values in particular cells).

It is implemented in the following way. The computational domain is split into the "Total field" region, in which there are both fields, emitted by a source and reflected from the object, and "Scattered field", in which there is only reflected field. The border between these two areas is a source of a plane wave. In order to

Fig. 4. Total field and scattered field areas.

implement this idea, the E field is set in all the cells of the border, while in the border cells of the scattered field region it is subtracted from the values of the field in those cells. The H field in the “Scattered field” region, if E -field is a gaussian pulse $g(t)$, is (considering the time and space offset between fields):

$$H = -\sqrt{\frac{\varepsilon}{\mu}} g\left(t + \frac{n\Delta y}{2c} + \frac{\Delta t}{2}\right).$$

2.4. COMPUTATIONAL RESULTS

In this section there are some results of work of the created program.

In the 2d-case the following parameters are used: frequency 5 GHz; the size of the computational area: 5x5 m (100x100 grid

Fig. 5. Modelling of horn antenna.

cells; 5 cm size of the cell). In Fig. 5 the results of modelling of horn antenna are shown.

In Fig. 6 we can see the propagation of waves, emitted by two harmonic sources.

The following numerical experiments are conducted on the grid 200x200 with the size of the cell 5 mm. In the Fig. 7 the wave is propagating through conductor.

Fig. 6. Modelling of electromagnetic field, emitted by 2 harmonic sources.

Fig. 7. Modelling of a plane wave, propagating through conductor.

Fig. 8. Modelling of a plane wave, propagating through dielectric.

Fig. 9. Fall of a plane wave on conductor with the size of 1 cell, conductivity 0.05, 2, 10, 100.

In the Fig. 8 – through dielectric. The amplitude of the wave is 1V/m. The exact shape of impulse can be seen in 1d profile.

The modelling of a plane wave, propagating through conductor with the size of 1 cell and conductivity 0.05, 2, 10, 100 is shown in Fig. 9.

3. CONCLUSION

To sum up, in this program the propagation of electromagnetic waves was modelled in 1d, 2d and 3d cases, different types of sources and boundary conditions were implemented, the propagation of waves in inhomogeneous medium was investigated, antennas were

modelled. This program can be used in wide range of applied problems. It has already been used in radiolocation and it is planned to use it to model photonic crystals.

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