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A GENERALIZED EQUIVALENT TEMPERATURE MODEL IN A TIME-VARYING ENVIRONMENT

UOGÓLNIONY MODEL TEMPERATURY RÓWNOWAŻNEJ W ZMIENNYM W CZASIE OTOCZENIU

Accelerated degradation test at high temperature level is a common method to accelerate the degradation of products by elevating temperature, and the obtained degradation data are then used to obtain the estimate of the performance at normal temperature after extrapolating the degradation through accelerating model. However, the normal temperature is ever-changing rather than constant. Therefore, a generalized equivalent temperature model based on power law degradation path is proposed to establish a connection between accelerated degradation data and degradation data at normal temperature. The model takes the equal degradation measure as a principle and the conclusion is demonstrated that the increments of the degradation under the same magnitude, same time and different orders of temperature stresses are same. The result shows that the empirical equivalent temperature model is a special case of the proposed model. The accuracy of the proposed model is finally demonstrated by a case study of nitrile rubber O-rings.

Keywords: *accelerated degradation test, equivalent temperature, nature storage, degradation path, rubber O-rings.*

Przyspieszone badania degradacji (badania starzeniowe) prowadzone w warunkach wysokiej temperatury stanowią powszechnie stosowaną metodę przyspieszania starzenia produktów poprzez podwyższanie temperatury. Otrzymane w takich badaniach dane degradacyjne wykorzystuje się do szacowania wydajności produktu w temperaturze normalnej na zasadzie ekstrapolacji. Głównym ograniczeniem tej metody jest fakt, że normalna temperatura nie jest stała lecz zmienia się w czasie. Dlatego też, aby skorelować dane z przyspieszonej degradacji z danymi dotyczącymi starzenia w normalnej temperaturze, zaproponowaliśmy uogólniony model temperatury równoważnej oparty na krzywej degradacji opisanej prawem potęgowym. W modelu przyjęto zasadę równego stopnia degradacji i wykazano, że przyrosty degradacji przy tej samej wartości i czasie działania naprężeń termicznych różnego rzędu są takie same. Wyniki pokazują, że empiryczny model temperatury równoważnej jest szczególnym przypadkiem proponowanego przez nas modelu. Trafność opisanego w pracy modelu wykazano na podstawie studium przypadku dotyczącego uszczelnień nitylowych, tzw. oringów.

Słowa kluczowe: *przyspieszone badania starzenia, temperatura równoważna, przechowywanie w warunkach naturalnych, krzywa degradacji, oringi nitylowe.*

1. Introduction

Accelerated test is an efficient method to collect information of products by measuring performance data directly over time from the test at high stress while the collected data are used to extrapolate the information through a physically reasonable statistical model to obtain the estimate of life or long-term performance at lower stress, normal use or storage condition [12]. The frequently used stresses include use rate, voltage, humidity, pressure, especially temperature [13, 16, 23]. Long-term reliability of gold (Au) and copper (Cu) ball bonds in fineline ball grid array package under storage condition 30°C was estimated by high temperature storage bake test at elevated temperatures of 150°C, 175°C and 200°C [8]. Wang predicted the storage life of aerospace electromagnetic relay under storage temperature 25°C -32°C based on auto-regressive and moving average model and wavelet transform model [21]. Anisotropic magnetoresistive read sensors were exposed to elevated temperatures to estimate end-of-life conditions under normal operating temperatures [5]. Huang predicted the life of tantalum capacitors under working temperature which was

specified as 35°C [4]. Vakulov studied the properties changing in storage condition by accelerating ageing test on the example of rubbers K-14-1 and K-14-2 [18].

From the examples above, a common phenomenon is observed that the temperature in storage condition or normal operating is often assumed as a constant temperature [7](e.g. 25°C, 30°C), or a temperature interval. However, the temperature in real storage or operating condition often varies with the season and region. For some long-life products, a minor temperature difference may lead to a major difference in the result of life assessment. As a result, it is important to consider the impact of the changing temperature appropriately. Particularly, it is of interest to wonder whether there is a temperature under which the life or performance of the product is equal to the life or performance at real storage or operating temperature. This issue consists of the primary goal of this paper, proposing an equivalent temperature model.

In literature, a substantial number of degradation models have been developed to model the accelerated degradation data [10]. In general, these existing models can be divided into linear and nonlinear

models. In these models, power law degradation path model can describe both linear degradation path and nonlinear degradation path by different powers, and several empirical studies show that the expected degradation at time t is often proportional to a power law [2, 17, 19]. Moreover, this kind of model is widely used in engineering fields like electrical connectors, rubber materials [9], green laser diodes [11] and film resistances [1]. In addition, the power law degradation path has also been used to optimize the accelerated degradation experiment. These observations indicate the wide application of the power law model [24, 25]. Accordingly, the proposed equivalent temperature model is based on power law degradation path due to its flexibility and good performance.

In the inference process of the model, the conclusion is demonstrated to simplify the model that the increments of the degradation under the same magnitude, the same time and different orders of temperature stresses are same. Meanwhile, the influences to equivalent temperature of the two key parameters in the model, the time index α and accelerated parameter β , are analyzed by taking the first partial derivative of the equivalent temperature model. The result shows that equivalent temperature would decrease with the increasing of β and α . Besides, the model is compared with the average temperature model and the empirical equivalent temperature model which is a special case of the proposed model in the linear case while $\alpha=1$. Thus the proposed model is also known as generalized equivalent temperature model. Moreover, the validity of the proposed model is proved by rubber O-rings data from the accelerated degradation test and the nature storage test. Finally, the fitness of three equivalent temperature models are compared by the criterion of mean squared error (MSE).

Compared with accelerated degradation test, degradation test under normal temperature could obtain more precise results but become increasingly difficult owing to the characteristics of long-period and high-cost which conflicts with the marketplace demands for decreasing development time. In this case, equivalent temperature model establishes an effective connection between accelerated degradation test and degradation test at normal temperature. In terms of actual usage, the equivalent temperature of nature storage test can be obtained by directly substituting the statistical data of temperatures in normal condition and the relevant parameters derived by accelerated degradation test into the model. By means of this model, the life or degradation performance information of the product under normal non-constant temperature can be predicted directly without the time-consuming and costly degradation test. The model can be applied to all of the products whose degradation follows the power law degradation path.

The remaining segments of this paper are organized as follows. Section 2 introduces the derivation of the empirical equivalent temperature model and the generalized equivalent temperature model. In Section 3, parameters in the model are estimated based on accelerated degradation data. Section 4 introduces the sensitivity analysis of parameters to equivalent temperature. A case study of rubber O-rings is presented to verify the high performance of generalized equivalent temperature model compared with other two equivalent temperature models in Section 5. Section 6 presents the conclusion of the paper and application of the model.

2. Equivalent temperature model

2.1. Empirical equivalent temperature model

The relationship between the degradation rate and temperature T could be described as Arrhenius function which is written as [14, 20]:

$$\frac{dy}{dt} = Ae^{\left[\frac{-E}{RT}\right]} \quad (1)$$

where y is the degradation, dy/dt is the degradation rate, A is the pre-exponential factor of the Arrhenius rate constant, E represents material apparent activation energy, R is the universal gas constant ($8.3145 \text{ kJ kmol}^{-1} \text{ K}^{-1}$) and T is the absolute temperature in Kelvin. Specify $\omega = \ln A$, $\beta = -E/R$ to get the simple model of the function and it is obviously that $\beta < 0$ from its physics meaning. Then the degradation can be derived as:

$$y = e^{[\omega + \beta/T]} t^\alpha + y_0 \quad (2)$$

where y_0 is a constant.

Accelerated degradation test accelerates the degradation of products by means of increasing the temperature stress to obtain the degradation at accelerated temperature and the temperature stress is assumed constant during the test [3]. However, the temperature stress is often changing during nature storage test which is recognized that $T=f(t)$. It is assumed that there is an equivalent temperature T_0 during which the degradation in a certain time ζ is equal to the degradation at a time-varying temperature $T(t)$ in the same time. As such, the following equation must be satisfied:

$$e^{[\beta/T_0]} \zeta = \int_0^\zeta e^{[\beta/T(t)]} dt \quad (3)$$

For a giving $T(t)$, the equivalent temperature T_0 can be simplified as:

$$T_0 = \frac{\beta}{\ln \left\{ \frac{1}{\zeta} \int_0^\zeta e^{[-\beta/(T(t))]} dt \right\}} \quad (4)$$

However, in real environment, the equation of T_0 is difficult to have its analytical expression because $T(t)$ is unavailable to model continuously. So the integral expression in denominator is discretized, then:

$$\mathbf{M}_1: T_0 = \frac{\beta}{\ln \left\{ \frac{1}{\zeta} \sum_{i=1}^n t_i \cdot \exp \left(\frac{\beta}{T_i} \right) \right\}} \quad (5)$$

where $t_i (i=1, 2, \dots, n)$ is the time at temperature T_i and the total storage time is $\zeta = \sum_{i=1}^n t_i$.

2.2. Generalized equivalent temperature model

However, sometimes the degradation rate k is correlative with test time t , then Eq. (1) and Eq. (2) can be rewritten as:

$$\begin{cases} \frac{dy}{dt} = \frac{1}{\alpha} e^{[\omega + \beta/T]} t^{\alpha-1} \\ y = e^{[\omega + \beta/T]} t^\alpha + y_0 \end{cases} \quad (6)$$

This form of degradation process is also called power law model where time index α is a constant related to the degradation mechanism with $\alpha > 0$. The degradation path is an up-convex curves while $0 < \alpha < 1$, and a straight line while $\alpha = 1$, and an up-concave curves while $\alpha > 1$.

It is also assumed that $y_0=0$ without loss of generality by certain corresponding transformation.

Similarly, in time-varying environment, the equivalent temperature T_0 can be expressed as:

$$T_0 = \frac{\beta}{\ln \left\{ \frac{1}{\alpha \zeta^\alpha} \int_0^\zeta e^{\left[\beta/T(t) \right]} t^{\alpha-1} dt \right\}} \quad (7)$$

For equation (4) and (7), the expression of T_0 in equation (7) can be reduced to the equation (4) exactly while $\alpha=1$. Undoubtedly, the analytical expression of T_0 in equation (7) is hard to handle even by discretizing. Based on the equal degradation measure principle, a generalized equivalent temperature model is derived.

For the requirement of derivation of the generalized equivalent temperature model, the following assumption is introduced.

The increments of the degradation under same magnitude, same time and different orders of temperature stresses are same in natural environment.

To illustrate the idea of the above assumption, Fig. 1 is given. As shown in Fig. 1, it is assumed that a product is aged at the temperature T_1 for time t_1 first and the increment of the degradation is y^{11} . Then, the product is aged at temperature T_2 for time t_2 , and the increment of the degradation is y^{12} . The degradation path is shown as the short dash line in Fig. 1. In addition, the degradation rate is k_1 at the temperature T_1 , and k_2 at the temperature T_2 . In general, t_1, t_2, k_1, k_2 are greater than 0.

Fig. 1. The degradation path under different stress orders

From Eq. (6), we can get:

$$k_1 = \exp(\omega + \beta / T_1) \quad (8)$$

$$k_2 = \exp(\omega + \beta / T_2) \quad (9)$$

$$y^{11} = k_1 t_1^\alpha \quad (10)$$

The degradation is continuous and only depends on the accumulated degradation and the current temperature stress. As a result, the degradation y^{11} should be converted to the equivalent time τ_{12} at temperature T_2 :

$$y^{11} = k_2 \tau_{12}^\alpha \quad (11)$$

Put Eq.(8), Eq.(9) and Eq.(10) into Eq.(11) and we can obtain:

$$\tau_{12} = \left(\frac{k_1}{k_2} \right)^{1/\alpha} = \left\{ \exp \left[\beta \left(\frac{1}{T_1} - \frac{1}{T_2} \right) \right] \right\}^{1/\alpha} \quad (12)$$

Through the derivation above, y^{12} can be formulated as:

$$y^{12} = k_2 \left(t_2 + t_1 \left(\frac{k_1}{k_2} \right)^{1/\alpha} \right)^\alpha$$

On the contrary, the product is aged at the temperature T_2 for time t_2 firstly and the increment of the degradation is y^{21} . Then, the product is aged at the temperature T_1 for time t_1 , and the increment of the degradation is y^{22} . The degradation path is shown as the heavy line in Fig. 1. Similarly, we have:

$$y^{21} = k_2 t_2^\alpha$$

$$y^{22} = k_1 \left(t_1 + t_2 \left(\frac{k_2}{k_1} \right)^{1/\alpha} \right)^\alpha$$

It is proved that $y^{12}=y^{22}$ and the proof is given as follows:

$$\frac{y^{12}}{y^{22}} = \frac{k_2 \left(t_2 + t_1 \left(\frac{k_1}{k_2} \right)^{1/\alpha} \right)^\alpha}{k_1 \left(t_1 + t_2 \left(\frac{k_2}{k_1} \right)^{1/\alpha} \right)^\alpha} = \frac{\left((k_2)^{1/\alpha} t_2 + t_1 (k_1)^{1/\alpha} \right)^\alpha}{\left((k_1)^{1/\alpha} t_1 + t_2 (k_2)^{1/\alpha} \right)^\alpha} = 1$$

In the nature storage test, the storage time is $t_i (i=1,2,\dots,n)$ at temperature T_i and the total storage time is $\zeta = \sum_{i=1}^n t_i$. The degradation after

the whole nature storage test is equivalent to the degradation at temperature T_0 for time ζ . The generalized equivalent temperature model can be written as:

$$\mathbf{M}_0: \begin{cases} T_0 = \frac{1}{\frac{1}{T_n} + \frac{\alpha}{\beta} \ln(\tau_n) - \frac{\alpha}{\beta} \ln(\zeta)} \\ \tau_n = t_n + (\tau_{n-1}) \left(\frac{k_{n-1}}{k_n} \right)^{1/\alpha} \\ \tau_{n-1} = t_{n-1} + \tau_{n-2} \left(\frac{k_{n-2}}{k_{n-1}} \right)^{1/\alpha} \\ \dots \\ \tau_2 = t_2 + \tau_1 \left(\frac{k_1}{k_2} \right)^{1/\alpha} \\ \tau_1 = t_1 \end{cases} \quad (13)$$

where $\tau_1, \tau_2, \dots, \tau_n$ are equivalent time, and $\frac{k_{n-1}}{k_n} = \exp \left(\beta \left(\frac{1}{T_{n-1}} - \frac{1}{T_n} \right) \right)$.

3. Parameter estimation based on accelerated degradation data

In constant accelerated degradation test, there are N accelerated temperature stresses, and the number of the samples under each stress are n_i . The degradation path is described by Eq (14).

$$y_{ijl}(t_{ijl} | T_i) = y_{ij0} + k_{ij} t_{ijl}^\alpha, \quad (14)$$

$$i = 1, 2, \dots, N, j = 1, 2, \dots, n_i, l = 1, 2, \dots, L_{ij}$$

where $y_{ijl}(t_{ijl} | T_i)$ is the measured degradation of the j^{th} sample at time t_{ijl} under temperature T_i , y_{ij0} is the initial value of the degradation and it is assumed that $y_{ij0}=0$ without loss of generality, k_{ij} is the degradation rate of the j^{th} sample at temperature T_i and $k_{ij}>0$, L_{ij} is the number of measurement time for each sample.

First, the value of α is calculated by a least square method which minimize the value of I and the expression of I is shown as the formula below

$$\begin{cases} I(\alpha) = \sum_{i=1}^N \sum_{j=1}^{n_i} \sum_{l=1}^{L_{ij}} (y_{ijl} - \hat{y}_{ijl})^2 \\ \hat{y}_{ijl} = \hat{k}_{ij} t_{ijl}^\alpha \end{cases} \quad (15)$$

where the value of $\hat{k}_{ij}$ can be estimated by linear fit with an exact value of α and $\hat{y}_{ijl}$ is the estimated degradation.

Then, combining Eq.(6) and Eq.(15), the following relationship can be obtained

$$k_{ij}(T_i) = e^{[\omega + \beta/T_i]} \quad (16)$$

Taking the logarithm of the left and right sides of the above formula, the parameter ω, β can be estimated by a least square method.

4. Sensitivity analysis of parameters

The equivalent temperature in M_0 is closely related to the parameters β and α . Next, we would discuss how equivalent temperature varies with them.

4.1. Sensitivity analysis of parameter β

Taking the first partial derivative of Eq. (13) with respect to β , the following equations can be given.

$$\begin{cases} \frac{\partial T_0}{\partial \beta} = - \left(\frac{1}{\frac{1}{T_n} + \frac{\alpha}{\beta} \ln(\tau_n) - \frac{\alpha}{\beta} \ln(\zeta)} \right)^2 \cdot \left[\frac{\alpha}{(\beta)^2} (\ln(\zeta) - \ln(\tau_n)) + \frac{\alpha}{\beta \tau_n} \frac{d\tau_n}{d\beta} \right] \\ \frac{d\tau_n}{d\beta} = \frac{d\tau_{n-1}}{d\beta} \left(\frac{k_{n-1}}{k_n} \right)^{1/\alpha} + \tau_{n-1} \cdot \frac{1}{\alpha} \left(\frac{k_{n-1}}{k_n} \right)^{\frac{1}{\alpha}} \cdot \left(\frac{1}{T_{n-1}} - \frac{1}{T_n} \right) \\ \dots \\ \frac{d\tau_2}{d\beta} = \frac{d\tau_1}{d\beta} \left(\frac{k_1}{k_2} \right)^{1/\alpha} + \tau_1 \cdot \frac{1}{\alpha} \left(\frac{k_1}{k_2} \right)^{\frac{1}{\alpha}} \cdot \left(\frac{1}{T_1} - \frac{1}{T_2} \right) \\ \frac{d\tau_1}{d\beta} = 0 \end{cases} \quad (17)$$

It could be obtained that $\lim_{\beta \rightarrow -\infty} \frac{\partial T_0}{\partial \beta} = 0$, $\lim_{\alpha \rightarrow \infty} \frac{\partial T_0}{\partial \beta} = 0$ by

Eq. (17). Because of the complexity of $\frac{\partial T_0}{\partial \beta}$, it is achieved by numerical calculation rather than analytical approaches.

Take a storehouse in Nanjing for example. Its daily average temperature in 2014 is shown as Fig.2. The partial derivative of equivalent temperature to β is shown in Fig.3 in which the range of β is -10000~0 and the range of α is 0~5. The partial derivative of equivalent temperature to β is less than 0 in the numerical range of Fig.4 and it increases with the rise of α but decreases with the rise of β . Combined with the limit value, we can get the conclusion that equivalent temperature would decrease with the increase of β but its descent rate would gradually increase while the value of α is generally taken as a constant.

Fig. 2. Daily average temperature in 2014 of a storehouse in Nanjing

Fig. 3. The partial derivative of equivalent temperature to β

4.2. Sensitivity analysis of parameter α

Taking the first partial derivative of Eq. (13) with respect to α , it can be obtained

$$\begin{cases}
 \frac{\partial T_0}{\partial \alpha} = - \left(\frac{1}{T_n + \frac{\alpha}{\beta} \ln(\tau_n) - \frac{\alpha}{\beta} \ln(\zeta)} \right)^2 \left[\frac{1}{\beta} (\ln(\tau_n) - \ln(\zeta)) + \frac{\alpha}{\beta \tau_n} \frac{d\tau_n}{d\alpha} \right] \\
 \frac{d\tau_n}{d\alpha} = \frac{d\tau_{n-1}}{d\alpha} \left(\frac{k_{n-1}}{k_n} \right)^{1/\alpha} + (\tau_{n-1}) \left(\frac{k_{n-1}}{k_n} \right)^{1/\alpha} \ln \left(\frac{k_{n-1}}{k_n} \right) \left(-\frac{1}{\alpha^2} \right) \\
 \frac{d\tau_{n-1}}{d\alpha} = \frac{d\tau_{n-2}}{d\alpha} \left(\frac{k_{n-2}}{k_{n-1}} \right)^{1/\alpha} + (\tau_{n-2}) \left(\frac{k_{n-2}}{k_{n-1}} \right)^{1/\alpha} \ln \left(\frac{k_{n-2}}{k_{n-1}} \right) \left(-\frac{1}{\alpha^2} \right) \\
 \dots \\
 \frac{d\tau_2}{d\alpha} = \frac{d\tau_1}{d\alpha} \left(\frac{k_1}{k_2} \right)^{1/\alpha} + \tau_1 \left(\frac{k_1}{k_2} \right)^{1/\alpha} \ln \left(\frac{k_1}{k_2} \right) \left(-\frac{1}{\alpha^2} \right) \\
 \frac{d\tau_1}{d\alpha} = 0
 \end{cases} \quad (18)$$

$$\text{where } \ln \left(\frac{k_{n-1}}{k_n} \right) = \beta \left(\frac{1}{T_{n-1}} - \frac{1}{T_n} \right).$$

Similarly, the partial derivative of equivalent temperature to α is shown in Fig.5 in which the range of β is -10000~0 and the range of α is 0~5. The partial derivative of equivalent temperature to α is less than 0 in the numerical range of Fig.4 and it increases with the rise of both α and β . So we can get the conclusion that equivalent temperature would decrease with the increase of α but its descent rate would gradually reduce while the value of β is generally taken as a constant.

Fig. 4. The partial derivative of equivalent temperature to α

5. Practical Calculation

5.1. Experiment

5.1.1. Accelerated degradation test

Accelerated degradation test of nitrile rubber O-rings was carried out at 50°C, 60°C, 70°C and 80°C, with 8 samples at each temperature [22]. In order to simulate the stress state of nitrile rubber O-rings in practical application environment, the sample was placed in a specially-made jig which was composed of two compression plates and a

spacer ring to control the compression ratio, as shown in Fig. 5. Tight the four bolts to compress the rubber O-rings to achieve the specified thickness while putting the rubber O-rings in the jigs.

When rubber O-rings are held under compression, physical or chemical changes could occur that prevent the rubber returning to its original dimensions after the release of the deforming force. The result is a compression set and the magnitude of which depends on the time and temperature of compression as well as on the time and temperature of recovery. At elevated temperature, chemical changes become increasing more important and lead to a permanent [6]. As such, the compression set ε was chosen as the performance parameter, which is formulated as:

$$\varepsilon = \frac{D_0 - D_t}{D_0 - D_x} \times 100\%$$

Fig. 5. The specially-made jig

where D_0 is the initial thickness of the test sample, D_x is the height of the spacer ring, D_t is the thickness after time t in the aging chamber which is measured by releasing the deforming force on the samples and allowing a 1 hour recovery period at 25°C. $\varepsilon=31\%$ is set as the failure threshold for rubber O-rings since the internal air tightness of the structure will be destroyed along with the reduction of safety and reliability while ε is greater than 31%.

Place the loaded rubber O-rings with jigs to aging chamber, after the required test duration, remove the compression assembly from the aging chamber, allow it to cool to a standard laboratory temperature 25°C for 1 h, then release the O-rings, and after a further 1 h at standard laboratory temperature, measure their thickness. Each O-ring was measured at four points around the circumference and the results were averaged. Put the measured thickness at each time point to Eq. (12) and get the compression set curve as shown in Fig. 6.

Fig. 6. The compression set curve of accelerated degradation test

5.1.2. Nature storage test

The nature storage test of 8 nitrile rubber O-rings was carried out at laboratory in June 2013. The rubber O-rings were preserved at room temperature and prevented from light, refrigerating equipment in summer and heating equipment in winter. The jigs, samples, installation method and measurement method were same with the accelerated degradation test. The daily average temperatures during the test are shown in Fig. 7. By the end of the last measurement, the test was carried out for 856 days and there were 12 data were measured for each sample. Fig.8 shows the connection of compression set between adjacent measuring points. Non-monotonicity is observed from Fig. 8 compared with the data from accelerated degradation test.

Fig. 7. Comparison of equivalent temperature under M_0 , M_1 and M_2 during the nature storage test

Fig. 8. The compression set curve of nature storage test

5.2. Parameter estimation of accelerated degradation test

The degradation of nitrile rubber O-rings is a function of compression set which can be formulated as:

$$y = -\ln(1 - \varepsilon). \quad (19)$$

The data obtained from accelerated degradation test can be fitted by Eq.(14). The estimate of the parameter α is 0.37 and the results of the parameter k_{ij} are shown as Table 1.

Table 1. Degradation rate $\hat{k}_{ij}$ under accelerated degradation test

Stress number i	Temperature stress T_i	Sample serial number (j)	Degradation rate $\hat{k}_{ij}$
1	323.15	1	0.0315
		2	0.0310
		3	0.0255
		4	0.0305
		5	0.0296
		6	0.0244
		7	0.0207
		8	0.0213
2	333.15	1	0.0537
		2	0.0594
		3	0.0598
		4	0.0484
		5	0.0604
		6	0.0551
		7	0.0465
		8	0.0424
3	343.15	1	0.0858
		2	0.0922
		3	0.0795
		4	0.0889
		5	0.1053
		6	0.0757
		7	0.0700
		8	0.0610
4	353.15	1	0.1280
		2	0.1455
		3	0.1415
		4	0.1249
		5	0.1567
		6	0.1242
		7	0.1148
		8	0.1007

Then the value of ω and β are estimated by a least square method as Table 2 and the correlation coefficient r is greater than critical value at 1% level which validates the good fit to Arrhenius function.

$$\ln k_i = 14.7218 - 5913.0224 / T_i$$

Table 2. The parameters and correlation coefficients of accelerated model

Distribution parameters	ω	β	r
	14.7218	-5913.0224	0.9665

5.3. Model comparison

For comparative studies, we refer to the model M_1 for comparison which is expressed as Eq. (15). In addition we choose the average temperature model as M_2 which is shown as Eq. (20). And the proposed model is written as M_0 .

$$M_2: T_0 = \frac{1}{5} \sum_{i=1}^n t_i T_i \quad (20)$$

Based on M_0 , M_1 , M_2 , we respectively obtain the equivalent temperatures of these models as shown in Table 4. The 2.36°C temperature difference between M_0 and M_1 and the 3.84°C temperature difference between M_0 and M_2 are observed from Table 3.

Table 3. Comparison of T_0 under M_0 , M_1 , M_2

	M_0	M_1	M_2
$T_0/^\circ\text{C}$	23.32	20.96	19.48

5.3.1. The effect of β on equivalent temperature for the three models

The effect of parameter β on equivalent temperature is shown in Fig.9 that the range of β is -100000~0. From Fig. 9, we can see that both T_0 from M_0 and M_1 both increase with the decrease of β . At the same time, their increase rates decrease gradually which is consistent with our discussion in 4.1.

Fig. 9. The effect of parameter β on T_0

5.3.2. The effect of α to equivalent temperature for the three models

The effect of parameter α on equivalent temperature is shown in Fig.10 that the range of α is 0~10. From Fig. 10, we can see that T_0 decreases with the increase of α in M_0 . At the same time, its decrease rate decrease gradually which is consistent with our discussion in 4.2. Moreover, the equivalent temperatures in M_0 and M_1 are equal while $\alpha=1$ which indicates that the proposed model is more general and M_1 is a special case of the proposed model in the linear case.

5.3.3. Model verification

Substitute the daily average temperature in Fig. 8 to M_0 , M_1 and M_2 , respectively. The equivalent temperatures of these three models

Fig. 10. The effect of parameter α on T_0

are shown in Fig.7 during the nature storage test. From Fig.7, we can see that the change range of equivalent temperature along with the change of daily average temperature is relatively large in the first year of nature storage test. However, the equivalent temperature will not produce significant motion following daily average temperature during the second year of the test. Accordingly, there is a more slight fluctuation of equivalent temperature in the third year compared with the second year. And it would be predicted that equivalent temperature will eventually approach to a constant temperature while testing time is long enough.

Degradation rates during the nature storage test can be calculated by substituting equivalent temperature in Fig.7 to accelerated model shown as Eq.(6). To quantitatively compare the above models, the MSE is used to assess the estimation accuracy of the model. Let $\hat{y}_j(t_i)$ denote the estimated value of degradation of rubber O-rings at time t_i for the j^{th} sample, and $y_j(t_i)$ can be calculated by the measured compression set in Fig.8.

$$MSE = \frac{1}{N} \sum_{j=1}^N \frac{1}{L_j} \sum_{l=1}^{L_j} (\hat{y}_j(t_l) - y_j(t_l))^2$$

Using the MSE, the smallest MSE value corresponds to the best estimation accuracy [15], and thus it could serve as the criteria for model selection. For comparison, the MSEs under M_0 , M_1 and M_2 is listed in Table 4. From Table 4, it can be observed that MSE of M_0 is the smallest and the result of M_0 has higher precision in estimating the degradation of rubber O-rings in nature storage condition.

Table 4. MSEs under M_0 , M_1 , M_2

	M_0	M_1	M_2
MSE	0.0005676	0.0008791	0.0015947

6. Conclusion

Motivated by the need of practical engineering, a generalized equivalent temperature model is proposed in this paper. Therefore, the degradation at normal temperature which is ever-changing rather than constant can be derived by accelerated degradation test. The degradation at changing temperature for quite a time is equal

to the degradation at equivalent temperature for the same time. And the influences of the two key parameters in the model are analyzed by taking the first partial derivative of the equivalent temperature model.

It is demonstrated that the empirical equivalent temperature model is a special case of the proposed model by numerical computation. Meanwhile, the excellent performance of the proposed model is verified by nitrile rubber O-rings nature storage test. The result shows that the proposed model has the minimum MSE compared with other two equivalent temperature models.

The proposed model can be widely used in other products whose degradation at time t is proportional to a power law degradation model. For the long-lifetime products, the parameter α and parameter β in the model can be calculated by accelerated temperature stress test firstly. Then extrapolate the information in accelerated condition to equivalent temperature condition to obtain the information of life or degradation parameters in nature storage condition or real-use condition directly. It is quite time-saving and could get results with good accuracy.

References

1. Chan C K, Boulanger M, Tortorella M. Analysis of parameter-degradation data using life-data analysis programs. Proceedings Annual Reliability and Maintainability Symposium 1994; 288-291, <http://dx.doi.org/10.1109/RAMS.1994.291122>.
2. Cui Z, Liou J J, Yue Y. A new extrapolation method for long-term degradation prediction of deep-submicron MOSFETs. IEEE Transactions on Electron Devices 2003; 50(5): 1398-1401, <http://dx.doi.org/10.1109/TED.2003.813473>.
3. Escobar L A, Meeker W Q. A review of accelerated test models. Statistical science 2006; 552-577, <http://dx.doi.org/10.1214/088342306000000321>.
4. Huang J, Gao C, Cui W, et al. Lifetime prediction for tantalum capacitors with multiple degradation measures and particle swarm optimization based grey model. Journal of Central South University 2012; 19: 1302-1310, <http://dx.doi.org/10.1007/s11771-012-1142-y>.
5. Iben I E T. Head reliability of AMR sensors based on thermal stress tests. IBM Journal of Research and Development 2003; 47(4): 415-428, <http://dx.doi.org/10.1147/rd.474.0415>.
6. ISO/PRF Rubber, vulcanized or thermoplastic Determination of compression set Part At ambient or elevated temperatures. ISO/PRF 815-1 - Rubber, vulcanized or thermoplastic -- Determination of compression set -- Part 1: At ambient or elevated temperatures. ISO CT 61 - Plastics.
7. Lee H M, Wu J W, Lei C L. Assessing the lifetime performance index of exponential products with step-stress accelerated life-testing data. IEEE Transactions on Reliability 2013; 62(1): 296-304, <http://dx.doi.org/10.1109/TR.2013.2241197>.
8. Leong Gan C, Classe F, Hashim U. Superior performance and reliability of copper wire ball bonding in laminate substrate based ball grid array. Microelectronics International 2013; 30(3): 169-175, <http://dx.doi.org/10.1108/MI-11-2012-0074>.
9. Liu W, He G. Storage life of silicone rubber sealing ring used in solid rocket motor. Chinese Journal of Aeronautics 2014; 27(6): 1469-1476, <http://dx.doi.org/10.1016/j.cja.2014.10.013>.
10. Lu C J, Meeker W O. Using degradation measures to estimate a time-to-failure distribution. Technometrics 1993; 35(2): 161-174, <http://dx.doi.org/10.2307/1269661>.
11. Marioli M, Meneghini M, Rossi F, et al. Degradation mechanisms and lifetime of state-of-the-art green laser diodes. physica status solidi 2015; 212(5): 974-979, <http://dx.doi.org/10.1002/pssa.201431714>.
12. Meeker W Q, Escobar L A, and Lu C J. Accelerated degradation tests: modeling and analysis. Technometrics 1998; 40(2): p. 89-99, <http://dx.doi.org/10.2307/1270643>.
13. Naikan V N A, Rathore A. Accelerated temperature and voltage life tests on aluminium electrolytic capacitors: a DOE approach. International Journal of Quality & Reliability Management 2016; 33(1): 120-139, <http://dx.doi.org/10.1108/IJQRM-12-2014-0201>.
14. Purnell P. Interpretation of climatic temperature variations for accelerated ageing models. Journal of materials science 2004; 39(1): 113-118, <http://dx.doi.org/10.1023/B:JMSC.0000007734.71945.93>.
15. Si X S, Wang W, Hu C H, et al. Remaining useful life estimation based on a nonlinear diffusion degradation process. IEEE Transactions on Reliability 2012; 61(1): 50-67, <http://dx.doi.org/10.1109/TR.2011.2182221>.
16. Tajima K, Hotta H, Yamada Y, et al. Accelerated test on electrochromic switchable mirror based on magnesium alloy thin film in simulated environment of various relative humidities. Solar Energy Materials and Solar Cells 2012; 99: 76-83, <http://dx.doi.org/10.1016/j.solmat.2011.06.018>.
17. Takeda E, Suzuki N. An empirical model for device degradation due to hot-carrier injection. IEEE Electron Device Letters 1983; 4(4): 111-113, <http://dx.doi.org/10.1109/EDL.1983.25667>.
18. akulov N V, Myshlyavtsev A V, Malyutin V I. Estimation of in-use Guaranteed Rubber Lifetime test methods. Procedia Engineering 2015; 113: 479-483, <http://dx.doi.org/10.1016/j.proeng.2015.07.339>.
19. Van Noortwijk J M, Pandey M D. A stochastic deterioration process for time-dependent reliability analysis. Proceedings of the Eleventh IFIP WG 7.5 Working Conference on Reliability and Optimization of Structural Systems 2004; 259-265.
20. Wang C H, Hsu C H. New nonisothermal Arrhenius temperature integral approximate formula. Environmental Engineering Science 2012; 29(10): 964-971, <http://dx.doi.org/10.1089/ees.2011.0292>.
21. Wang Z, Shang S, Zhai G, et al. Research on storage degradation testing and life prediction based on ARMA and wavelet transform model for aerospace electromagnetic relay. IEEE 60th Holm Conference on Electrical Contacts (Holm) 2014; 1-8, <http://dx.doi.org/10.1109/HOLM.2014.7031021>.
22. Xiao K, Gu X, Peng C. Reliability evaluation of the O-type rubber sealing ring for fuse based on constant stress accelerated degradation testing. Journal of Mechanical Engineering 2014; 16: 62-69, <http://dx.doi.org/10.3901/JME.2014.16.062>.
23. Yang G. Accelerated life tests at higher usage rates. IEEE Transactions on Reliability 2005; 54(1): 53-57, <http://dx.doi.org/10.1109/TR.2004.841730>.
24. Yu H F. Designing an accelerated degradation experiment by optimizing the estimation of the percentile. Quality and Reliability Engineering International 2003; 19(3): 197-214, <http://dx.doi.org/10.1002/qre.518>.

25. Yu H F. Designing an accelerated degradation experiment with a reciprocal Weibull degradation rate. *Journal of statistical planning and inference* 2006; 136(1): 282-297, <http://dx.doi.org/10.1016/j.jspi.2004.06.030>.

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