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RESIDUAL LIFE PREDICTION FOR HIGHLY RELIABLE PRODUCTS WITH PRIOR ACCELERATED DEGRADATION DATA

PROGNOZOWANIE TRWAŁOŚCI RESZTKOWEJ WYSOCE NIEZAWODNYCH PRODUKTÓW NA PODSTAWIE DANYCH HISTORYCZNYCH Z PRZYSPIESZONYCH BADAŃ DEGRADACJI

To precisely predict the residual life for functioning products is a key of carrying out condition based maintenance. For highly reliable products, it is difficult to obtain abundant degradation data to precisely predict the residual life under normal stress levels. Thus, how to make use of historical degradation data to improve the accuracy of the residual life prediction is an interesting issue. Accelerated degradation testing, which has been widely used to evaluate the reliability of highly reliable products, can provide abundant accelerated degradation data. In this paper, a residual life prediction method based on Bayesian inference that takes accelerated degradation data as prior information was studied. A Wiener process with a time function was used to model degradation data. In order to apply the random effects of all the parameters of a Wiener process, the non-conjugate prior distributions were considered. Acceleration factors were introduced to convert the parameter estimates from accelerated stress levels to normal stress levels, so that the proper prior distribution types of the random parameters can be selected by the Anderson-Darling statistic. A Markov Chain Monte Carlo method with Gibbs sampling was used to evaluate the posterior means of the random parameters. An illustrative example of self-regulating heating cable was utilized to validate the proposed method.

Keywords: Residual life; Bayesian inference; random parameters; Wiener process; acceleration factor.

Precyzyjne przewidywanie trwałości resztkowej użytkowanego produktu stanowi klucz do prawidłowego utrzymania ruchu w oparciu o bieżący stan techniczny (condition-based maintenance). W przypadku produktów o wysokiej niezawodności, trudno jest uzyskać ilość danych degradacyjnych, która umożliwiałaby precyzyjne prognozowanie trwałości resztkowej przy normalnym poziomie obciążeń. Dlatego też bardzo ważnym zagadnieniem jest wykorzystanie historycznych danych degradacyjnych umożliwiających zwiększenie trafności prognozowania trwałości resztkowej. Przyspieszone badania degradacyjne, które powszechnie wykorzystuje się do oceny niezawodności wysoce niezawodnych produktów, mogą dostarczać bogatych danych o przyspieszonej degradacji. W przedstawionej pracy badano metodę prognozowania trwałości resztkowej opartą na wnioskowaniu bayesowskim, w którym jako uprzednie informacje wykorzystano dane z przyspieszonych badań degradacji. Dane degradacyjne modelowano za pomocą procesu Wienera z funkcją czasu. Aby móc zastosować efekty losowe wszystkich parametrów procesu Wienera, rozważano niesprężone rozkłady a priori. Wprowadzono współczynniki przyspieszenia, które pozwoliły na przekształcenie szacowanych wartości parametrów z poziomu obciążeniowego w próbie przyspieszonej do poziomu obciążeń normalnych, co umożliwiło wybór odpowiednich typów parametrów losowych rozkładu a priori z wykorzystaniem statystyki testowej Andersona-Darlinga. Metodę Monte Carlo opartą na łańcuchach Markowa z próbnikiem Gibbsa wykorzystano do oceny średnich a posteriori parametrów losowych. Proponowaną metodę zweryfikowano na podstawie przykładu samoregulującego przewodu grzejnego.

Słowa kluczowe: trwałość resztkowa; wnioskowanie bayesowskie; parametry losowe; proces Wienera; współczynnik przyspieszenia.

1. Introduction

Recently, more and more highly reliable products have emerged in military and aerospace fields. However, it is very time-consuming to obtain enough failure data to evaluate the reliability of highly reliable products under normal stress levels. To resolve this problem, accelerated life test (ALT) is widely adopted to obtain product failure data in an acceptable time period. But for highly reliable products, even accelerated life test is not effective because little failure data can be obtained within limited time and budget. For some highly reliable product, their certain performance indexes will degrade over time. If the performance degradation can be observed, product lifetime information is likely to be extrapolated. Therefore, accelerated degradation test (ADT), under which products are put into accelerated stress levels to accelerate degradation process, has become a popular approach to

the lifetime prediction for highly reliable products [13, 21, 26, 32, 33]. However, the aim of ADT is to extrapolate the lifetime information for the population rather than the residual life for an individual.

The residual life of a product is defined as the length from the current time to the failure time, and precisely predicting the residual life is important to carry out condition based maintenance (CBM), and prognostics and health management [1, 22]. For an individual with high reliability, the degradation data observed under normal stress levels cannot show a distinct degradation trend, therefore it is difficult to precisely predict the residual life. In order to precisely predict the residual life for an individual with limited real-time degradation data, the prediction methods based on Bayesian inference have been popularly studied [3, 7, 8]. Gebraeel *et al* [5] developed an exponential degradation model with random parameters to model bearing degradation signals. They assumed the random parameters to obey conjugate prior

distributions for mathematical tractability, and obtained the prior distributions of random parameters using the historical degradation data of the population of devices, then predicted the residual life for an individual with the real-time degradation data. Chakraborty *et al* [2] also studied the exponential degradation model with random parameters, but they assumed the random parameters to obey non-conjugate prior distributions, and adopted the Metropolis-Hasting algorithm to estimate the posterior means of random parameters. Gebräel *et al* [6] proposed a Bayesian method, which takes the failure time data as historical information while takes the real-time degradation data of an individual as filed information, to predict the residual life distribution for a rotating machinery. Karandikar *et al* [10] developed a Bayesian method, which applied Monte Carlo simulations to evaluate the posterior means of random parameters, to predict the residual life for an aircraft fuselage panel. Baraldi *et al* [1] also developed a Bayesian method, which combines the degradation data of historical equipment and the real-time observing degradation data, to predict the residual life for nuclear power plants. Jin *et al* [9] presented a Bayesian framework based on a Wiener process with random parameters. The framework utilizes off-line population degradation data and on-line individual degradation data to predict residual life. Wang *et al* [28] also studied the Bayesian method based on Wiener degradation processes.

From the above analysis, it can be concluded that the residual life prediction methods based on Bayesian inference usually take the historical degradation data or failure data under normal stress levels as prior information. However, there is a lack of research on how to use accelerated degradation data as prior information. For some high reliable products, the accelerated degradation data is the only prior information source. Thus, how to make full use of historical accelerated degradation data to predict residual life is an interesting and significant issue. In the paper, we proposed a Bayesian method, which takes accelerated degradation data as prior information while considers the real-time degradation data observed under normal stress level as field information, to improve the prediction accuracy of the residual life for a highly reliable product.

The rest of the paper is organized as follows: Section 2 describes the residual life prediction model based on a Wiener process with random parameters. Section 3 discusses how to evaluate the prior distributions of the random parameters from accelerated degradation data. We deduced the expression of the acceleration factor based on Wiener degradation process, and converted parameter estimates from accelerated stress levels to normal stress level through acceleration factors. Section 4 develops simulation tests to validate the proposed method of converting parameter estimates. Section 5 provides a case study of self-regulating heating cable to investigate the effectiveness of the proposed Bayesian method. Section 6 draws some conclusions.

2. Residual life prediction model

It is well known that the main characteristic of degradation process is uncertain over time, therefore stochastic process models, such as Wiener process, Gamma process, Poisson process, *et al*, are naturally appropriate to model the evolution of degradation process. Wiener process can characterize both monotonic and non-monotonic degradation data, and describe unit-to-unit variance of products if random parameters are considered. Hence, Wiener process has been widely applied to model stochastic degradation process, see Whitmore and Schenkelberg [31], Wang [29], Peng and Tseng [19], Wang *et al* [30], Huang *et al* [4].

In mathematics, a Wiener process $\{X(t) = \mu \cdot \Lambda(t) + \sigma \cdot B(\Lambda(t))\}$ has independent increments $\Delta X(t) = X(t + \Delta t) - X(t)$ that follow a normal distribution as:

$$\Delta X(t) \sim N(\mu \Delta \Lambda(t), \sigma^2 \Delta \Lambda(t)) \quad (1)$$

where μ is a drift parameter, $\sigma (\sigma > 0)$ is a diffusion parameter, $\Lambda(t)$ is a monotone increasing function of time t with $\Lambda(0) = 0$, $\Delta \Lambda(t) = \Lambda(t + \Delta t) - \Lambda(t)$ denotes a time increment and $B(\cdot)$ denotes a standard brown motion.

Let a constant value l represent the failure threshold, the failure time ξ can be considered to be the time when $X(t)$ firstly reaches l . So the failure time ξ can be described as:

$$\xi = \inf \{t : X(t) \geq l\} \quad (2)$$

For a Wiener process, it is well known that the failure time ξ follows an inverse Gaussian distribution. So the probability density function (PDF) of ξ can be expressed as:

$$f_{\xi}(t) = \frac{l}{\sqrt{2\pi\sigma^2\Lambda^3(t)}} \exp \left[-\frac{(l - \mu\Lambda(t))^2}{2\sigma^2\Lambda(t)} \right] \frac{d\Lambda(t)}{dt} \quad (3)$$

Given that the degradation $X(t_i)$ at time t_i does not reaches l , the residual life RL_{t_i} at t_i can be defined as $RL_{t_i} = \inf \{t : X(t_i + t) \geq l \mid X(t_i) < l\}$ [7,24]. Therefore, the PDF of RL can be written as:

$$f_{RL}(t) = \frac{l'}{\sqrt{2\pi\sigma^2(\Lambda(t))^3}} \exp \left[-\frac{(l' - \mu\Lambda(t))^2}{2\sigma^2\Lambda(t)} \right] \frac{d\Lambda(t)}{dt} \quad (4)$$

where $l' = l - X(t)$. Furthermore, the expectation of RL can be deduced from:

$$\overline{RL} = E(RL) = \int_0^{+\infty} t \cdot f_{RL}(t) dt \quad (5)$$

where $\Lambda^{-1}(\cdot)$ denotes the inverse function of $\Lambda(\cdot)$. In order to obtain better flexibility and tractability, the form of $\Lambda(\cdot)$ was specified as

$\Lambda(t) = t^r$. If $r=1$, it is appropriate to model linear degradation processes. If $r>1$, it is able to model convex degradation processes. If $r<1$, it is capable of modeling concave degradation processes.

Thus, the RL can be evaluated once the estimates of μ, σ, r are obtained. However, the real-time degradation data of highly reliable product is commonly so little not to be straightforward used to obtain credible estimates $\hat{\mu}, \hat{\sigma}, \hat{r}$. To resolve the problem, we adopted a Bayesian method to obtain more credible posterior estimates of $\hat{\mu}, \hat{\sigma}, \hat{r}$ by making full use of historical accelerated degradation data. Although the application of Wiener processes in Bayesian inference has been widely studied in literature, most works assume that the random parameters of a Wiener process obey the following conjugate prior distributions: $\sigma^2 \sim \text{IG}(a, b)$, $\mu \mid \sigma^2 \sim N(c, d \cdot \sigma^2)$, where $\text{IG}(\cdot)$ represents an Inverse Gamma distribution, and a, b, c, d are hyper parameters. The above assumption has two limitations. First, the random effect of r can not be taken into account; Second, μ, σ are limited to

obey the above specified distributions, which may not be applicable to some cases. In this paper, the non-conjugate prior distributions of μ, σ, r are alternatively used in Bayesian inference, and μ, σ, r are assumed to be mutually independent. Let $\pi(\mu, \sigma^2, r)$ represent the joint prior PDF, $\pi(\mu)$, $\pi(\sigma^2)$, $\pi(r)$ be the corresponding PDF of μ, σ, r , respectively, then $\pi(\mu, \sigma^2, r) = \pi(\mu) \cdot \pi(\sigma^2) \cdot \pi(r)$.

Suppose that $\mathbf{X} = [X_1(t_1), X_2(t_2), \dots, X_{n+1}(t_{n+1})]$ is the real-time degradation data of an individual under normal stress level and $L(\Delta\mathbf{X} | \mu, \sigma^2, r)$ is a likelihood function, then the joint posterior PDF $\pi(\mu, \sigma^2, r | \Delta\mathbf{X})$ can be deduced according to the Bayesian theory [25, 34].

$$\pi(\mu, \sigma^2, r | \Delta\mathbf{X}) = \frac{L(\Delta\mathbf{X} | \mu, \sigma^2, r) \cdot \pi(\mu, \sigma^2, r)}{\int_{-\infty}^{+\infty} \int_0^{+\infty} \int_{-\infty}^{+\infty} L(\Delta\mathbf{X} | \mu, \sigma^2, r) \cdot \pi(\mu, \sigma^2, r) d\mu d\sigma^2 dr} \quad (6)$$

The marginal density functions of $\mu | \Delta\mathbf{X}$, $\sigma^2 | \Delta\mathbf{X}$, $r | \Delta\mathbf{X}$ can be expressed as:

$$\pi(\mu | \Delta\mathbf{X}) = \int_{-\infty}^{+\infty} \int_0^{+\infty} \pi(\mu, \sigma^2, r | \Delta\mathbf{X}) d\sigma^2 dr \quad (7)$$

$$\pi(\sigma^2 | \Delta\mathbf{X}) = \int_{-\infty}^{+\infty} \int_0^{+\infty} \pi(\mu, \sigma^2, r | \Delta\mathbf{X}) d\mu dr \quad (8)$$

$$\pi(r | \Delta\mathbf{X}) = \int_0^{+\infty} \int_{-\infty}^{+\infty} \pi(\mu, \sigma^2, r | \Delta\mathbf{X}) d\mu d\sigma^2 \quad (9)$$

Furthermore, the expectations of $\mu | \Delta\mathbf{X}$, $\sigma^2 | \Delta\mathbf{X}$ and $r | \Delta\mathbf{X}$ can be calculated as:

$$E(\mu | \Delta\mathbf{X}) = \int_{-\infty}^{+\infty} \mu \cdot \pi(\mu | \Delta\mathbf{X}) d\mu \quad (10)$$

$$E(\sigma^2 | \Delta\mathbf{X}) = \int_0^{+\infty} \sigma^2 \pi(\sigma^2 | \Delta\mathbf{X}) d\sigma^2 \quad (11)$$

$$E(r | \Delta\mathbf{X}) = \int_0^{+\infty} r \cdot \pi(r | \Delta\mathbf{X}) dr \quad (12)$$

Generally, it is difficult to evaluate $E(\mu | \Delta\mathbf{X})$, $E(\sigma^2 | \Delta\mathbf{X})$ and $E(r | \Delta\mathbf{X})$ through direct mathematical calculation. One alternative method is using Markov Chain Monte Carlo (MCMC) simulation with Gibbs sampling [16, 20], we implemented the method in WinBUGS.

Replace μ, σ, Λ with $E(\mu | \Delta\mathbf{X})$, $E(\sigma^2 | \Delta\mathbf{X})$ and $E(r | \Delta\mathbf{X})$ in Eq. (4) and Eq. (5), the posterior PDF of RL and the posterior expectation of RL can be obtained. There will be more field degradation data available as the individual product works over time. Once new degradation data is available, $E(\mu | \Delta\mathbf{X})$, $E(\sigma^2 | \Delta\mathbf{X})$ and $E(r | \Delta\mathbf{X})$ are immediately updated. So the RL of an individual can be real-time predicted.

3. Evaluate the prior distributions of random parameters

Before predicting RL using Bayesian method, the joint prior PDF $\pi(\mu, \sigma^2, r)$ need be evaluated from historical accelerated degradation data. In this section, we discussed the method of evaluating

$\pi(\mu, \sigma^2, r)$ from historical accelerated degradation data. Firstly, a Wiener process with 3 random parameters were used to model accelerated degradation data. Then, the acceleration factor constant principle was introduced to deduce the corresponding expression of acceleration factor for a Wiener process, and the parameter estimates were converted from accelerated stress levels to normal stress level through acceleration factors. Lastly, the Anderson-Darling goodness-of-fit method was utilized to select the proper distribution types for 3 random parameters.

3.1. Estimate parameters from accelerated degradation data

A constant stress accelerated degradation test was conducted to obtain the lifetime information of the population for a kind of highly reliable product. Suppose that S_0 denotes the normal stress level, S_k denotes the k th accelerated stress level, X_{ijk} denotes the i th degradation data of the j th product under S_k , t_{ijk} represents the corresponding observing time, $\Delta X_{ijk} = X_{ijk} - X_{(i-1)jk}$ represents the degradation data increment, and $\Lambda(t_{ijk}) = t_{ijk}^{\Lambda}$ represents the time increment, where $i = 1, 2, \dots, n_1$; $j = 1, 2, \dots, n_2$; $k = 1, 2, \dots, n_3$. According to the Wiener process features, ΔX_{ijk} follows a Normal distribution as:

$$\Delta X_{ijk} \sim N(\mu_{jk} \Delta \Lambda(t_{ijk}), \sigma_{jk}^2 \Delta \Lambda(t_{ijk})) \quad (13)$$

To obtain maximum likelihood estimates (MLEs) $\hat{\mu}_{jk}$, $\hat{\sigma}_{jk}^2$, $\hat{r}_{jk}$ for each product, the likelihood function can be established as:

$$L(\mu_{jk}, \sigma_{jk}^2, r_{jk}) = \prod_{i=1}^{n_1} \frac{1}{\sqrt{2\pi\sigma_{jk}^2 \left(t_{ijk}^{r_{jk}} - t_{(i-1)jk}^{r_{jk}} \right)}} \exp \left[-\frac{\left(\Delta X_{ijk} - \mu_{jk} \left(t_{ijk}^{r_{jk}} - t_{(i-1)jk}^{r_{jk}} \right) \right)^2}{2\sigma_{jk}^2 \left(t_{ijk}^{r_{jk}} - t_{(i-1)jk}^{r_{jk}} \right)} \right] \quad (14)$$

3.2. Convert parameter estimates from accelerated stress levels to normal stress level

Suppose $F_k(t_k)$, $F_0(t_0)$ represents the accumulative distribution function (CDF) under S_k , S_0 , respectively, if $F_k(t_k) = F_0(t_0)$, then the acceleration factor of S_k relative to S_0 can be defined as:

$$AF_{k,0} = t_0 / t_k \quad (15)$$

In practice, Eq. (15) is rarely applied to evaluate $AF_{k,0}$ since t_0 and t_k are difficult to accurately obtained. For a Wiener process, a widely adopted assumption is that μ should change with accelerated stress level varying while σ should be invariable, see Padgett and Tomlinson [17], Park and Padgett [18], Liao and Tseng [11], Lim and

Yum [14], et al. Based on the above assumption, the expression of $AF_{k,0}$ can be expressed as $AF_{k,0} = \mu_k / \mu_0$. However, there are different assumption that both μ and σ will change with accelerated stress level varying, see Whitmore and Schenkelberg[31], Liao and Elsayed[12]. Thus, the expression of acceleration factor for a Wiener process needs further research. In this paper, we applied the acceleration factor constant principle to deduce the expression of acceleration factor for a Wiener process.

Zhou et al [35] first pointed out the acceleration factor constant principle that $AF_{k,0}$ should be a constant which does not change with time t_0, t_k and depends only on S_k, S_0 under an effective accelerated test. Moreover, to guarantee that $AF_{k,0}$ is a constant, the failure mechanism of products must remain consistent under S_k, S_0 . According to the accelerated factor constant principle, for any $t_0, t_k > 0$, the following expression is always identical:

$$F_k(t_k) = F_0(AF_{k,0}t_k) \quad (16)$$

From Eq. (16), the following identical equation can be deduced:

$$f_k(t_k) = AF_{k,0} \cdot f_0(t_0). \quad (17)$$

The deducing process was illustrated as:

$$\begin{aligned} f_k(t_k) &= \frac{dF_k(t_k)}{dt_k} = AF_{k,0} \frac{dF_0(AF_{k,0} \cdot t_k)}{d(AF_{k,0} \cdot t_k)} \\ &= AF_{k,0} \frac{dF_0(t_0)}{dt_0} = AF_{k,0} \cdot f_0(t_0) \end{aligned}$$

Substitute Eq. (3) into Eq. (17), and specify $\Lambda(t) = t$, then:

$$\begin{aligned} AF_{k,0} = \frac{f_k(t_k)}{f_0(t_0)} &= \frac{\sigma_0 AF_{k,0}^{3/2}}{\sigma_k} \exp \left[\left(\frac{l \cdot \mu_0}{\sigma_0^2} - \frac{l \cdot \mu_k}{\sigma_k^2} \right) \right. \\ &\quad \left. + \frac{1}{t_k} \left(\frac{l^2}{2\sigma_0^2 AF_{k,0}} - \frac{l^2}{2\sigma_k^2} \right) + t_k \left(\frac{\mu_0^2 AF_{k,0}}{2\sigma_0^2} - \frac{\mu_k^2}{2\sigma_k^2} \right) \right] \end{aligned} \quad (18)$$

To ensure $AF_{k,0}$ is a constant which does not change with t_k , the following relationships must be satisfied:

$$\begin{cases} \frac{l^2}{2\sigma_0^2 AF_{k,0}} - \frac{l^2}{2\sigma_k^2} = 0 \\ \frac{\mu_0^2 AF_{k,0}}{2\sigma_0^2} - \frac{\mu_k^2}{2\sigma_k^2} = 0 \end{cases} \quad (19)$$

So the following equation can be deduced:

$$AF_{k,0} = \mu_k / \mu_0 = \sigma_k^2 / \sigma_0^2 \quad (20)$$

Eq. (20) indicates that both μ and σ should change with accelerated stress level varying. When we specify $\Lambda(t) = t^r$, the following equation can be deduced:

$$AF_{k,0} = (\mu_k / \mu_0)^{1/r} = (\sigma_k^2 / \sigma_0^2)^{1/r}, \quad r_k = r_0 = r \quad (21)$$

Eq. (21) indicates that both μ and σ should change with accelerated stress level varying while r should remain invariable. The mathematical relationships between μ, σ and accelerated stresses can be modeled by reaction rate models [15]. Assume that temperature T is a stress variable and the Arrhenius relationship is correspondingly selected as the reaction rate model. According to Eq. (21), μ, σ^2 and r can be expressed as:

$$\mu_k = \exp(\lambda_1 - \lambda_2 / T_k), \quad \sigma_k^2 = \exp(\lambda_3 - \lambda_2 / T_k), \quad r_k = \lambda_4 \quad (22)$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are coefficients.

To obtain the MLEs of $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, a likelihood function that incorporates all the accelerated degradation data was constructed:

$$L(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = \prod_{i=1}^{n_1} \prod_{j=1}^{n_2} \prod_{k=1}^{n_3} \frac{1}{\sqrt{2\pi \exp(\lambda_3 - \lambda_2 / T_k) (t_{ijk}^{\lambda_4} - t_{(i-1)jk}^{\lambda_4})}} \exp \left[-\frac{(\Delta x_{ijk} - \exp(\lambda_1 - \lambda_2 / T_k) (t_{ijk}^{\lambda_4} - t_{(i-1)jk}^{\lambda_4}))^2}{2 \exp(\lambda_3 - \lambda_2 / T_k) (t_{ijk}^{\lambda_4} - t_{(i-1)jk}^{\lambda_4})} \right] \quad (23)$$

With $\hat{\lambda}_2$ and $\hat{\lambda}_2$, $AF_{k,0}$ was evaluated by:

$$AF_{k,0} = \exp(\hat{\lambda}_4 \cdot \hat{\lambda}_2 (1/T_0 - 1/T_k)) \quad (24)$$

To simply the computation, $(AF_{k,0})^{\hat{\lambda}_4}$ was represented by $Z_{k,0}$.

Thus, the $\hat{\mu}_{jk}, \hat{\sigma}_{jk}^2, \hat{r}_{jk}^2$ under T_k can be transformed into the corresponding values under T_0 by:

$$\hat{\mu}_{j(k0)} = \hat{\mu}_{jk} / Z_{k,0}, \quad \hat{\sigma}_{j(k0)}^2 = \sigma_{jk}^2 / Z_{k,0}, \quad r_{j(k0)} = r_{jk} \quad (25)$$

For the denotation convenience, $\hat{\mu}_{j(k0)}, \hat{\sigma}_{j(k0)}^2$ and $\hat{r}_{j(k0)}$ were replaced with $\hat{\mu}_m, \hat{\sigma}_m^2$ and $\hat{r}_m$, respectively, where $m=1,2,\dots,M$ and M denotes the total number of products under the accelerated test.

3.3. Select the best fitting distribution types and estimate hyper parameters

We utilized the Anderson-Darling goodness-of-fit test to select the best fitting distribution types for $\hat{\mu}_m$, $\hat{\sigma}_m^2$ and $\hat{r}_m$. Since the Exponential, Normal, Lognormal, Weibull and Gamma distribution have good statistical characteristics and can cover the majority of parameter distribution, they were selected to be candidates for the best fitting distribution type. The one with the smallest AD is considered to be the best fitting distribution type [24]. In addition, the Anderson-Darling statistic can also be used for hypothesis test. The null hypothesis is defined as H_0 : data follow the specified distribution type. The significance level is specified as 0.05, if the p value of Anderson-Darling is greater than 0.05, the null hypothesis is accepted.

Suppose that $\hat{\mu}_m \sim \text{Wbl}(\eta, \delta)$, $\hat{\sigma}_m^2 \sim \text{Ga}(\alpha, \beta)$ and $\hat{r}_m \sim \text{N}(a, b)$, where $\text{Wbl}(\cdot)$ denotes a Weibull distribution and η, δ is the scale parameter and shape parameter, $\text{Ga}(\cdot)$ denotes a Gamma distribution and α, β is the shape parameter and scale parameter, $\text{N}(\cdot)$ denotes a Normal distribution and a, b is the mean and variance. In order to estimate the hyper parameters, the following likelihood functions can be established:

$$L(\hat{\mu}_m; \eta, \delta) = \prod_{m=1}^M \delta \cdot \eta^{-\delta} (\hat{\mu}_m)^{\delta-1} \exp \left[- \left(\frac{\hat{\mu}_m}{\eta} \right)^{\delta} \right] \quad (26)$$

$$L(\hat{\sigma}_m^2; \alpha, \beta) = \prod_{m=1}^M \frac{\beta^{\alpha}}{\Gamma(\alpha)} (\hat{\sigma}_m^2)^{\alpha-1} \exp(-\beta \hat{\sigma}_m^2) \quad (27)$$

$$L(\hat{r}_m; a, b) = \prod_{m=1}^M \frac{1}{\sqrt{2\pi} \cdot b} \exp \left(- \frac{(\hat{r}_m - a)^2}{2 \cdot b} \right) \quad (28)$$

The prior distributions of random parameters μ, σ^2, r can be obtained as $\mu \sim \text{Wbl}(\hat{\eta}, \hat{\delta})$, $\sigma^2 \sim \text{Ga}(\hat{\alpha}, \hat{\beta})$, $r \sim \text{N}(\hat{a}, \hat{b})$.

4. Validation test

A simulation test was developed to validate the conversion method illustrated in section 3.2. The simulated degradation increment $\Delta x_{i0}, \Delta \Lambda(t_{i0})$ under S_0 was generated from a conditional Wiener process [27,28], that is:

$$\omega_0 \sim \text{Ga}(a, b), \quad \mu_0 | \omega_0 \sim \text{N}(c, d / \omega_0) \\ \Delta x_{i0} | (\mu_0, \omega_0) \sim \text{N}(\mu_0 \Delta \Lambda(t_{i0}), \omega_0^{-1} \Delta \Lambda(t_{i0})) \quad (29)$$

where $\omega_0 = \sigma_0^{-2}$ and $\Lambda(t) = t^r$. The parameters of the above model were specified as $(a, b, c, d) = (4, 2, 3, 1)$; $i = 1, 2, \dots, 10$; $h = 1, 2, \dots, 5$; $t_{i0} = 10, 20, \dots, 100$; $\Lambda(t_{i0}) = t_{i0}^r$; $r = 0.5, 1, 2$. The validation steps

were described as the following. First, the estimates $\hat{\mu}_0, \hat{\sigma}_0^2, \hat{r}_0$ were obtained from $\Delta x_{i0}, \Delta \Lambda(t_{i0})$. Then, specify the acceleration factor $AF_{k,0}$ to be 0.4, 4, respectively, and calculate the converted degradation data $\Delta x_{ik}, \Delta \Lambda(t_{ik})$ under S_k , where $x_{ik} = x_{i0}$, $t_{ik} = t_{i0} / AF_{k,0}$. Next, the estimates $\hat{\mu}_k, \hat{\sigma}_k^2, \hat{r}_k$ were obtained from $\Delta x_{ik}, \Delta \Lambda(t_{ik})$. Last, $\hat{\mu}_k / \hat{\mu}_0$, $\hat{\sigma}_k^2 / \hat{\sigma}_0^2$ and $\hat{r}_k / \hat{r}_0$ were computed. Table 1 shows that $\hat{r}_k / \hat{r}_0$ is nearly equal to 1 and both $\hat{\mu}_k / \hat{\mu}_0$ and $\hat{\sigma}_k^2 / \hat{\sigma}_0^2$ are nearly equal to $(AF_{k,h})^r$, so the result is consistent with Eq. (21).

Table 1. The results of simulation test

r	$AF_{k,h} = 0.4$			$AF_{k,h} = 4$		
	$\frac{\hat{\mu}_k}{\hat{\mu}_0}$	$\frac{\hat{\sigma}_k^2}{\hat{\sigma}_0^2}$	$\frac{\hat{r}_k}{\hat{r}_0}$	$\frac{\hat{\mu}_k}{\hat{\mu}_0}$	$\frac{\hat{\sigma}_k^2}{\hat{\sigma}_0^2}$	$\frac{\hat{r}_k}{\hat{r}_0}$
0.5	0.6325	0.6324	1.0002	2.0000	2.0002	1.0001
1	0.4000	0.4000	1.0000	4.0001	4.0001	1.0000
2	0.1600	0.1600	0.9999	15.9999	16.0002	1.0000

5. Numerical case

5.1. Modeling degradation data

The historical accelerated degradation data of self-regulating heating cable [31] was listed in Table 2. Because the type of heating cable experienced a cure process at the beginning of the degradation test, only the degradation data after the cure process were considered. There were 15 products that were averagely allocated to 3 accelerated temperature levels under a constant-stress ADT. It was assumed that the normal temperature level was $T_0 = 448.15\text{K}$. Degradation was measured as the natural logarithm of resistance, and the failure threshold l was specified as $l = \ln(2)$.

However, there was no field degradation data available under T_0 . We simulated the field degradation data which are listed in Table 3.

5.2. Determining prior distribution

Eq. (14) was used to obtain the MLEs of parameters for each product, and the estimates were listed in Table 4. For a Wiener degradation process, the independent degradation increments Δx_{ijk} of each product follows a Normal distribution, written as $\Delta x_{ijk} \sim \text{N}(\hat{\mu}_{jk} \Delta \hat{\Lambda}_{jk}(t_{ijk}), \hat{\sigma}_{jk}^2 \Delta \hat{\Lambda}_{jk}(t_{ijk}))$, where $\hat{\mu}_{jk}$, $\hat{\sigma}_{jk}^2$, $\hat{r}_{jk}$ are the estimates of the j th product under the k th accelerated temperature level T_k . The above Normal distribution can be converted to the following standard Normal distribution:

$(\Delta x_{ijk} - \hat{\mu}_{jk} \Delta \hat{\Lambda}_{jk}(t_{ijk})) / \sqrt{\hat{\sigma}_{jk}^2 \Delta \hat{\Lambda}_{jk}(t_{ijk})} \sim \text{N}(0, 1)$. If the above relationship holds, the null hypothesis that the degradation process of the j th product under T_k obeys a Wiener process is accepted. An Anderson-Darling test with significance level $\alpha = 0.05$ validates that the degradation processes of all products obey Wiener processes.

Table 2. Historical accelerated degradation data of heating cables

Measured time ($\times 10^3$ h)	Item				
	1	2	3	4	5
(a) $T_1 = 473.15\text{K}$					
0.496	-0.120682	-0.118779	-0.123600	-0.126501	-0.124359
0.688	-0.112403	-0.109853	-0.115186	-0.118941	-0.111966
0.856	-0.103608	-0.101593	-0.105657	-0.110288	-0.107869
1.024	-0.096047	-0.094567	-0.098569	-0.103419	-0.100304
1.192	-0.085673	-0.084698	-0.088613	-0.095465	-0.085916
1.360	-0.077677	-0.076070	-0.079332	-0.084769	-0.077947
2.008	-0.045218	-0.040623	-0.045835	-0.052268	-0.045597
2.992	0.000526	0.004237	0.000533	-0.008265	0.000524
4.456	0.059261	0.063742	0.061032	0.051139	0.059544
5.608	0.093394	0.095117	0.093612	0.082414	0.084912
(b) $T_2 = 513.15\text{K}$					
0.160	-0.005152	-0.019888	-0.045961	-0.023188	-0.044267
0.328	0.056930	0.046278	0.015198	0.040737	0.018173
0.496	0.112631	0.101628	0.067119	0.095504	0.072214
0.688	0.173202	0.162705	0.128670	0.156129	0.131555
0.856	0.214266	0.202604	0.168271	0.196349	0.171394
1.024	0.272668	0.257563	0.221611	0.250900	0.225281
1.192	0.311422	0.297875	0.260910	0.291937	0.266314
1.360	0.351988	0.338902	0.302126	0.332887	0.306105
2.008	0.489847	0.461855	0.440738	0.473130	0.443941
2.992	0.656780	0.629991	0.606275	0.638651	0.611724
4.456	0.851985	0.798431	0.834114	0.798457	-
(c) $T_3 = 533.15\text{K}$					
0.160	0.123360	0.127605	0.120759	0.105206	0.120115
0.328	0.251084	0.254944	0.247156	0.232389	0.247949
0.496	0.393107	0.394496	0.391516	0.375789	0.388406
0.688	0.517137	0.518485	0.513872	0.500556	0.511850
0.856	0.598797	0.599265	0.595704	0.583362	0.595220
1.024	0.693925	0.694445	0.688930	0.679117	0.690324
1.192	0.774347	0.774428	0.770313	0.758314	0.770782

* A period denotes a missing value.

Table 3. The simulated field degradation data for an individual heating cable at T_0

Time /h	1000	2000	3000	4000	5000
Degradation	-0.2209	-0.1984	-0.1815	-0.1651	-0.1555
Time /h	6000	7000	8000	9000	
Degradation	-0.1443	-0.1323	-0.1231	-0.1128	

With Eq. (23), we obtained the MLEs of coefficients as $\hat{\lambda}_1 = 16.950$, $\hat{\lambda}_2 = 9158.226$, $\hat{\lambda}_3 = 10.317$, $\hat{\lambda}_4 = 0.612$. According to the Eq. (23), conversion coefficients were calculated as $Z_{1,0} = 2.944$, $Z_{2,0} = 13.311$, $Z_{3,0} = 25.999$. Furthermore, we converted the MLEs

Table 4. MLEs of parameters for each product

	Item				
	1	2	3	4	5
(a) T_1					
$\hat{\mu}$	0.067029	0.070533	0.070463	0.064605	0.083435
$\hat{\sigma}^2$	0.000026	0.000039	0.000035	0.000039	0.000120
$\hat{r}$	0.770668	0.74692	0.754380	0.776351	0.662973
(b) T_2					
$\hat{\mu}$	0.402341	0.425767	0.367615	0.427607	0.369066
$\hat{\sigma}^2$	0.000442	0.000523	0.000230	0.000753	0.000214
$\hat{r}$	0.602857	0.553023	0.662489	0.552906	0.665158
(c) T_3					
$\hat{\mu}$	0.788653	0.781201	0.788428	0.791756	0.784433
$\hat{\sigma}^2$	0.00117	0.001096	0.001283	0.001255	0.001017
$\hat{r}$	0.661216	0.664650	0.659024	0.660437	0.666665

of parameters under T_k to the corresponding values $\hat{\mu}_m$, $\hat{\sigma}_m^2$, $\hat{r}_m$ under T_0 , respectively, as listed in Table 5.

Anderson-Darling statistic was applied to determining the best fitting distribution. Table 6 shows the values of AD . Since $\hat{\sigma}_m^2$ should be always large than zero, the Normal distribution is excluded. From Table 6, it was concluded that the Weibull distribution was the best fitting distribution type for both $\hat{\mu}_m$ and $\hat{\sigma}_m^2$, and the Normal distribu-

Table 5. The converted values $\hat{\mu}_m, \hat{\sigma}_m^2, \hat{r}_m$

	Item				
	1	2	3	4	5
(a) $T_1 \rightarrow T_0$					
$\hat{\mu}_m$	2.277E-2	2.396E-2	2.393E-2	2.194E-2	2.834E-2
$\hat{\sigma}_m^2$	8.832E-6	1.325E-5	1.189E-5	1.325E-5	4.076E-5
$\hat{r}_m$	0.771	0.747	0.754	0.776	0.663
(b) $T_2 \rightarrow T_0$					
$\hat{\mu}_m$	3.023E-2	3.199E-2	2.762E-2	3.213E-2	2.773E-2
$\hat{\sigma}_m^2$	3.321E-5	3.929E-5	1.728E-5	5.657E-5	1.608E-5
$\hat{r}_m$	0.603	0.553	0.662	0.553	0.665
(c) $T_3 \rightarrow T_0$					
$\hat{\mu}_m$	3.033E-2	3.005E-2	3.033E-2	3.045E-2	3.017E-2
$\hat{\sigma}_m^2$	4.500E-5	4.216E-5	4.935E-5	4.827E-5	3.912E-5
$\hat{r}_m$	0.661	0.665	0.659	0.660	0.667

Table 6. The values of AD

	Exponential	Normal	LogNorm	Gamma	Weibull
$\hat{\mu}_m$	5.449	0.905	1.016	1.032	0.806
$\hat{\sigma}_m^2$	1.733	--	0.985	0.980	0.952
$\hat{r}_m$	5.682	0.878	0.894	0.914	0.982

Table 7. The posterior means of random parameters after the 5th field degradation data was got

Parameter	Posterior mean	Standard deviation	MC error	Samples
μ	0.02930	0.00299	2.86E-5	9000
σ^2	4.149E-5	1.568E-5	1.59E-7	9000
r	0.6124	0.0601	6.391E-4	9000

Fig. 1. The fitted posterior distributions of μ, σ^2, r

Fig. 2. The values of autocorrelation function in the iterative processes

Fig. 3. The changing curves of the posterior means of random parameters

tion was the best fitting distribution type for $\hat{r}_m$. The MLEs of hyper parameters were obtained according to Eq. (26) and Eq. (27). So the prior distributions of 3 random parameters were evaluated from historical accelerated degradation data, they are $\mu \sim \text{Wbl}(2.952\text{E-}2, 11.464)$, $\sigma^2 \sim \text{Wbl}(3.580\text{E-}5, 2.150)$, $r \sim \text{N}(0.671, 4.774\text{E-}3)$.

After the field degradation data was obtained, the MCMC method with Gibbs sampling was used to fit the posterior distributions of random parameters in WinBUGS software. After the 5th field degradation data was got, the fitted posterior distributions were shown in figure 1. From figure 2, it can be seen that all the iterative processes are convergent since the autocorrelation functions quickly tend to 0. The

Fig. 4. The changing curves of $\overline{RL}_1$, $\overline{RL}_2$, $\overline{RL}_3$ and the PDF of RL_3

posterior means $E(\mu | \Delta \mathbf{X})$, $E(\sigma^2 | \Delta \mathbf{X})$ and $E(r | \Delta \mathbf{X})$ were evaluated in WinBUGS, as listed in table 7. Furthermore, with new field degradation data was available, $E(\mu | \Delta \mathbf{X})$, $E(\sigma^2 | \Delta \mathbf{X})$ and $E(r | \Delta \mathbf{X})$ were corresponding updated, and their changing curves were shown in Figure 3.

Let $\overline{RL}_1$ denote the predictions that were evaluated from only the field degradation data of an individual, $\overline{RL}_2$ denote the predic-

Table 8. The predictions of residual life ($RL \times 10^5$ /h)

	Measuring time /h				
	5000	6000	7000	8000	9000
$\overline{RL}_1$	3.089 [1.885, 5.301]	2.513 [1.211, 4.018]	1.332 [0.978, 3.890]	1.341 [1.012, 3.805]	0.939 [0.762, 2.981]
$\overline{RL}_2$	2.145 [1.421, 2.876]	2.081 [1.450, 2.772]	2.004 [1.498, 2.624]	1.949 [1.519, 2.468]	1.874 [1.466, 2.303]
$\overline{RL}_3$	2.536 [1.758, 3.212]	2.116 [1.449, 2.801]	1.816 [1.209, 2.488]	1.802 [1.233, 2.317]	1.518 [1.087, 1.955]

Fig. 5. The goodness-of-fit tests for $\hat{\omega}_m$ and $\hat{\mu}_m | \hat{\omega}_m$

tions that were evaluated by the Bayesian method with conjugate prior distributions of random parameters, $\overline{RL}_3$ denote the predictions that were evaluated by the method proposed in this paper, the residual life predictions and the confidence intervals of the predictions at several measuring time points were summarized in table 8. The confidence intervals were obtained with 95% confidence level by Bootstrap sampling method. The prediction method for $\overline{RL}_2$ was detailed demonstrated in appendix. Figure 4 shows the changing curves of $\overline{RL}_1$, $\overline{RL}_2$, $\overline{RL}_3$ and the PDF of RL_3 .

5.4. Analysis and Conclusion

In table 7, the confidence intervals of $\overline{RL}_2$ and $\overline{RL}_3$ at each measuring point are obviously smaller than those of $\overline{RL}_1$. It is concluded that the prediction method taking accelerated degradation data as prior information improves the prediction precision of the residual life.

As more field degradation data was available, all the confidence intervals of $\overline{RL}_1, \overline{RL}_2, \overline{RL}_3$ became smaller. It suggests that the prediction precision was gradually improved, and the width of the PDF curve in figure 4 became narrower over time also supports the conclusion.

Compared with $\overline{RL}_2$ and $\overline{RL}_3$, the curve of $\overline{RL}_1$ was not smooth and presented comparatively big changes. The reason is that the field degradation data was short and had a nonlinear change. Thus, newly obtained degradation data may significantly changes the former estimates of the parameters, which led to the poor regularity of residual life predictions. In comparison, the prediction methods for $\overline{RL}_2$ and $\overline{RL}_3$ reduced the impact of the abrupt change of the field degradation and improved the reliability of the prediction results. Com-

pared with $\overline{RL}_3$, the curve of $\overline{RL}_2$ was more smooth and had smaller changes, because the prediction method for $\overline{RL}_2$ didn't consider the random effects of r . The estimates of r obtained from the prior information was used to the prediction model of $\overline{RL}_2$. Thus, $\overline{RL}_2$ was more conservative and greatly influenced by the prior information. In contrast, the prediction method for $\overline{RL}_3$ took the random effects of 3 parameters into account, which let $\overline{RL}_3$ more sensitively reflect the variation of the field degradation data.

In addition, the prediction method for $\overline{RL}_2$ requires $\hat{\mu}_z$ and $\hat{\sigma}_z^2$ to follow the conjugate prior distributions as

$$\hat{\omega}_m = \hat{\sigma}_m^{-2} \sim \text{Ga}(\hat{a}, \hat{b}),$$

$$\hat{\mu}_m | \hat{\omega}_m \sim N(\hat{c}, \hat{d} / \hat{\omega}_m),$$

where $\hat{a}, \hat{b}, \hat{c}, \hat{d}$ are estimates of hyper parameters and $z = 1, 2, \dots, 15$. The Anderson-Darling statistic was used to validate the null hypothesis

that both $\hat{\mu}_m$ and $\hat{\sigma}_m^2$ to follow the conjugate prior distributions.

$$\hat{\mu}_m | \hat{\omega}_m \sim N(\hat{c}, \hat{d} / \hat{\omega}_m) \text{ was converted to } (\hat{\mu}_m - \hat{c}) / \sqrt{\hat{d} / \hat{\omega}_m} \sim N(0, 1)$$

according to the characteristics of Normal distribution. The significance level was specified as 0.5, and the information of the goodness-of-fit test was shown in figure 5. The null hypothesis was rejected since the values of p were less than 0.05. It would cause the misspecification of distribution types of random parameters if the prediction method for $\overline{RL}_2$ was adopted in the case study. Therefore, it was appropriate and reliable to adopt the prediction method for $\overline{RL}_3$ in the case study.

6. Conclusions

In this paper, a residual life prediction method for degradation product based on Bayesian inference was proposed. A Wiener process with a time function was used to model degradation data.

Our proposed method made two potential novel contributions. a). Acceleration factor was introduced to transform the test data from accelerated stresses to normal use stress, so that accelerated degradation data can be considered as prior information. b). The non-conjugate prior distributions of random parameters were applied to Bayesian inference, which not only can consider the random effects of all the parameters but also can avoid the potential misspecifications of the parameter distributions when conjugate prior distributions were adopted.

Besides, there are several meaningful workings and conclusions about the study of the paper.

- 1) The residual life prediction method based on Bayesian inference that integrated the field degradation data and accelerated

degradation data can improve the prediction accuracy and real-time updating for the residual life.

- 2) The acceleration factor constant principle was used to deduce the relationships that the parameters of the Wiener process with a time function should satisfy. It offered a feasible approach to constructing the acceleration models for the parameters of the Wiener process.

Appendix

With $\hat{\mu}_m, \hat{\omega}_m$ the prior estimates of hyper parameters a, b, c, d can be obtained from:

$$L(\hat{\omega}_m; a, b) = \prod_{m=1}^M \frac{b^a}{\Gamma(a)} \hat{\omega}_m^{a-1} \exp(-b\hat{\omega}_m) \quad (\text{A.1})$$

$$L(\hat{\mu}_m | \hat{\omega}_m; c, d) = \prod_{m=1}^M \frac{\hat{\omega}_m^{0.5}}{\sqrt{2\pi d}} \exp\left(-\frac{\hat{\omega}_m(\hat{\mu}_m - c)^2}{2d}\right) \quad (\text{A.2})$$

From the field degradation data $\mathbf{X} = [X_1(t_1), X_2(t_2), \dots, X_{n+1}(t_{n+1})]$, the posterior estimates of hyper parameters can be deduced according to the Bayesian formula:

$$\pi(\mu, \omega | \Delta \mathbf{X}) = \frac{L(\Delta \mathbf{X} | \mu, \omega) \cdot \pi(\mu, \omega)}{\int_0^{+\infty} \int_{-\infty}^{+\infty} L(\Delta \mathbf{X} | \mu, \omega) \cdot \pi(\mu, \omega) d\mu d\omega} \quad (\text{A.3})$$

where $L(\Delta \mathbf{X} | \mu, \omega)$ is a likelihood function $\pi(\mu, \omega)$ is the joint prior PDF of μ and ω , denoted as $\pi(\mu, \omega) = \pi(\mu | \omega) \cdot \pi(\omega)$ and $\pi(\mu, \omega | \Delta \mathbf{X})$ is the joint posterior PDF.

Substitute:

$$\begin{aligned} \pi(\mu, \omega) &= \frac{\omega^{1/2}}{\sqrt{2\pi\hat{d}}} \exp\left(-\frac{\omega(\mu - \hat{d})^2}{2\hat{d}}\right) \cdot \frac{\hat{b}^{\hat{a}}}{\Gamma(\hat{a})} \omega^{\hat{a}-1} \exp(-\hat{b}\omega) \end{aligned}$$

and:

$$\begin{aligned} L(\Delta \mathbf{X} | \mu, \omega) &= \prod_{j=1}^n \frac{\omega^{1/2}}{\sqrt{2\pi\Delta\Lambda(t_j)}} \exp\left(-\frac{\omega(\Delta X_j - \mu \cdot \Delta\Lambda(t_j))^2}{2\Delta\Lambda(t_j)}\right) \end{aligned}$$

into (A.3), then:

$$\begin{aligned} \pi(\mu, \omega | \Delta \mathbf{X}) &\propto L(\Delta \mathbf{X} | \mu, \omega) \cdot \pi(\mu, \omega) \\ &\propto \omega^{(n+1)/2 + \hat{a} - 1} \exp\left\{-\frac{\omega}{2} \left(\mu^2 \sum_{j=1}^n \Delta\Lambda(t_j) - 2\mu \sum_{j=1}^n \Delta X_j + \sum_{j=1}^n \frac{\Delta X_j^2}{\Delta\Lambda(t_j)} \right) - \frac{\omega}{2} \left(\frac{(\mu - \hat{d})^2}{\hat{d}} \right) - \hat{b}\omega \right\} \\ &\propto \omega^{n/2 + \hat{a} - 1} \exp\left\{-\omega \left(\hat{b} + \frac{\hat{c}^2}{2\hat{d}} - \frac{\left(\hat{d} \sum_{j=1}^n \Delta X_j + \hat{c} \right)^2}{2 \left(\hat{d}^2 \sum_{j=1}^n \Delta\Lambda(t_j) + \hat{d} \right)} + \sum_{j=1}^n \frac{\Delta X_j^2}{2\Delta\Lambda(t_j)} \right)\right\} \cdot \exp\left\{-\frac{\omega}{2} \left(\frac{\mu - \frac{\hat{d} \sum_{j=1}^n \Delta X_j + \hat{c}}{\hat{d} \sum_{j=1}^n \Delta\Lambda(t_j) + 1}}{\frac{\hat{d}}{\hat{d} \sum_{j=1}^n \Delta\Lambda(t_j) + 1}} \right)^2\right\} \end{aligned}$$

- 3) The prediction method that adopts non-conjugate prior distributions can be applied to more cases, since it does not require parameters to follow the specified distribution types. Moreover, the assumption that parameters are mutually independent facilitates the engineering application of the method.

The posterior estimates of hyper parameters can be obtained as:

$$\begin{aligned} \hat{a} | \Delta \mathbf{X} &= \frac{n}{2} + \hat{a}, \\ \hat{b} | \Delta \mathbf{X} &= \hat{b} + \frac{\hat{c}^2}{2\hat{d}} - \frac{\left(\hat{d} \sum_{j=1}^n \Delta X_j + \hat{c} \right)^2}{2 \left(\hat{d}^2 \sum_{j=1}^n \Delta\Lambda(t_j) + \hat{d} \right)} + \sum_{j=1}^n \frac{\Delta X_j^2}{2\Delta\Lambda(t_j)} \\ \hat{c} | \Delta \mathbf{X} &= \frac{\hat{d} \sum_{j=1}^n \Delta X_j + \hat{c}}{\hat{d} \sum_{j=1}^n \Delta\Lambda(t_j) + 1}, \quad \hat{d} | \Delta \mathbf{X} = \frac{\hat{d}}{\hat{d} \sum_{j=1}^n \Delta\Lambda(t_j) + 1} \end{aligned}$$

The form of $\Lambda(t)$ was specified as $\Lambda(t) = t^{\hat{r}}$, where $\hat{r}$ was estimated from the accelerated degradation data. Furthermore, the posterior means $E(\mu | \Delta \mathbf{X})$, $E(\omega | \Delta \mathbf{X})$ of μ, ω were evaluated. Substitute $E(\mu | \Delta \mathbf{X})$, $E(\omega | \Delta \mathbf{X})$ and $\Lambda(t) = t^{\hat{r}}$ into Eq.(4) and (5), $\overline{RL}_2$ can be evaluated.

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