A numerical analysis of the wave phenomena in the spatial structure of a steel grid with a rubber filling

Izabela Major

ABSTRACT:
The paper presents a numerical analysis of mechanical wave propagation in the spatial structure of a steel grid with a rubber filling. Steel was adopted as an isotropic linear material while rubber was modeled as a Zahorski incompressible material. A properly prepared structure can reduce the dynamic effects resulting from mechanical wave propagation. Results obtained from the numerical analysis allow us to assess the impact of the material used with the dynamic interactions in the analyzed grid with a rubber filling. The results of the numerical analysis were presented graphically. Wave phenomena have been modeled using ADINA software.

KEYWORDS:
steel grid; wave phenomena; rubber

1. Introduction

Progress in measuring techniques has contributed to the development in research on wave phenomena in the material continuum, including the modeling of continuous compressible or incompressible hyperelastic material [1]. Compressible materials include steel, while incompressible materials consist of Mooney-Rivlin [2] and Zahorski [3] models (the authors have presented a mathematical model of rubber in their work). Many scientists use rubber or rubber-like materials in their research [4-8]. Nowadays, due to the pressure to reduce costs, experimental tests are often preceded by numerical analyzes that allow optimal model preparation. The analysis of behavior in nonlinear hyperelastic materials is possible thanks to the use of numerical programs that use the finite element method [9-12]. Numerical programs based on FEM mostly contain in their library, models of materials, including models of hyperelastic materials. By choosing one of these models, it is possible to perform a detailed numerical analysis of the nonlinear behavior of the designed element or structure, cf. [13].

In this study, a spatial steel grid model with rubber material filling empty areas of the grid has been adopted. The ADINA program was selected for the analysis of the phenomena of mechanical wave propagation in a structure made of steel and rubber. The grid was modeled as S235 structural steel. Zahorski’s hyperelastic material was adopted for the rubber. In the program, various rubber and rubber-like models are implemented, including a Mooney-Rivlin model. In order to make the analysis possible using Zahorski’s material, the Mooney-Rivlin material library in ADINA was modified (the whole procedure has been described in [14]), and thus the appropriate elastic potential for rubber was obtained.

1 Czestochowa University of Technology, Faculty of Civil Engineering, ul. Akademicka 3, 42-218 Częstochowa, Poland, e-mail: imajor@bud.pcz.pl, orcid id: 0000-0003-1234-9317
2. Materials adopted for analysis

For the analyzed structure, a grid was made of S235 steel, which was accepted as an isotropic linear material for the numerical analysis. The grid was filled with rubber, which is incompressible. Elastic energy for an incompressible isotropic hyperelastic material is linearly dependent on the invariants of the strain tensor. The constitutive relationship describing Zahorski’s material [3] can be written in the following form

\[ W(I_1, I_2) = C_1(I_1 - 3) + C_2(I_2 - 3) + C_3(I_3^2 - 9) \]  

(1)

where \( C_1, C_2, C_3 \) are the elastic constants, whereas \( I_1, I_2 \) are the invariants of the deformation tensor.

In order to obtain the relationship describing the Mooney-Rivlin material [2], the constant \( C_3 \) in equation (1) should be compared to zero. The nonlinear term \( C_3(I_3^2 - 9) \) in equation (1) allows a more accurate analysis and better quality results, useful for describing wave processes. Zahorski’s constitutive relationship very well reflects the rubber’s behavior in the case of major deformation even for \( \lambda = 3 \), while satisfactory results for the Mooney-Rivlin and neo-Hookean materials are obtained only for \( \lambda \leq 1.4 \) [15]. Elastic constants for both the Zahorski and Mooney-Rivlin’s materials are shown in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Constant</th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value [Pa]</td>
<td>2.099·10⁵</td>
<td>1.275·10⁴</td>
<td>3.924·10³</td>
</tr>
</tbody>
</table>

3. Analyzed model

For a numerical analysis, a cross-sectional model of a steel grid with rubber material filling the gaps was adopted. The dimensions of the adopted model are shown in Figure 1. The results for this cross-section were compared with a full steel cross-section (cross-section B-B in Fig. 1).

It was assumed that the dynamic load acts perpendicular to the upper surface of the presented steel grid model with the filling, as an evenly distributed load with the value of \( q(t) = 1000 \) N/m, according to Figure 1. The assumed load reaches the declared maximum value for time \( t = 1·10^{-7} \) s, followed by its removal. The release of external impulse causes propagation of the disturbance, which is hereinafter referred to as mechanical wave propagation.
In the adopted model, boundary conditions were imposed by blocking both the displacement along the Y and Z axes, as well as the rotation relative to the X axis, according to Figure 1. It was also assumed that the combination of steel and rubber occurred as a result of vulcanization. A similar analysis was carried out in [17, 18].

The discretization of the considered section in the grid model was carried out using finite elements “2D-Solid” (rectangular elements). The size of each element was ~0.002 m. In the ADINA program, the “Nonlinear-Elastic” material model based on the stress-strain curve was used to describe the steel elements. In order to implement Zahorski’s material, which was used to describe the rubber fillings, the libraries for the Mooney-Rivlin material were modified [14].

4. Numerical test results

Due to the fact that mechanical waves transfer energy, wave propagation can be observed in subsequent time steps using an effective stress diagram. Effective stresses obtained from the numerical analysis of the steel grid (cross-section A-A in Fig. 1) are presented for six time steps: $t_1 = 2.6\cdot10^{-6}\text{s}$, $t_2 = 5.267\cdot10^{-6}\text{s}$, $t_3 = 6.433\cdot10^{-6}\text{s}$, $t_4 = 6.933\cdot10^{-6}\text{s}$, $t_5 = 8.6\cdot10^{-6}\text{s}$ and $t_6 = 1.1\cdot10^{-5}\text{s}$. Propagation of the disturbance in the structure of the steel grid with rubber filling is shown in Figure 2.

Fig. 2. Propagation of a mechanical wave in a steel grid with rubber filling.
Table 2 shows a comparison of the effective stress values obtained for the cross section of a steel grid with a rubber filling (A-A cross section, Fig. 1) and for a full steel cross section (B-B cross section, Fig. 1). The values were read at a height of 8 cm from the bottom surface of the grid directly at its edge (in Fig. 2 respectively for 1) to 6) on the left.

Figure 2 shows that the disturbance in subsequent time steps, i.e. from 1) to 4), propagates almost completely bypassing the spaces filled with rubber. Then the wave reaches the bottom surface of the grid structure. Figure 2 shows the effective stress only in the places between the rubber fillings at \( t_4 \), followed by the reflected wave. In the next time steps, i.e. 5) and 6), it can be seen how the reflected wave propagates in the elements of the steel grid directly under the rubber filling. The wave propagation time for the full steel cross-section corresponds to the time in the steel grid.

### Table 2

<table>
<thead>
<tr>
<th>Effective stress [Pa]</th>
<th>( t_1 ) = 2.6·10^{-6} [s]</th>
<th>( t_2 ) = 5.267·10^{-6} [s]</th>
<th>( t_3 ) = 6.433·10^{-6} [s]</th>
<th>( t_4 ) = 6.933·10^{-6} [s]</th>
<th>( t_5 ) = 8.6·10^{-6} [s]</th>
<th>( t_6 ) = 1.1·10^{-5} [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>grid with rubber filling</td>
<td>0.00E+00</td>
<td>2.5E+01</td>
<td>7.81E+01</td>
<td>1.8E+02</td>
<td>2.0E+01</td>
<td>6.5E+01</td>
</tr>
<tr>
<td>steel</td>
<td>0.00E+00</td>
<td>5.68E+01</td>
<td>1.25E+02</td>
<td>6.5E+01</td>
<td>7.0E+01</td>
<td>8.0E+01</td>
</tr>
</tbody>
</table>

Based on the results presented in Table 2, it can be stated that Zahorski’s material reduced the value of effective stresses of the propagating mechanical wave in a grid filled with rubber compared to the steel cross-section. The differences in the effective stress values are visible both in the time steps before the wave reaches the lower surface, i.e. before the time \( t_4 = 6.933·10^{-6} \) s, as well as after this time.

### 5. Conclusions

The paper presents a numerical analysis of the phenomenon of damping mechanical waves in an innovative spatial structure made of a steel grid filled with rubber. Based on the results obtained in the numerical analysis, it can be concluded that the Zahorski material model adopted for analysis has worked well to describe rubber behavior and has well defined its reduction properties as a result of the propagation of a disturbance caused by external impact.

The conducted numerical analysis showed that the presented solution can be used as additional protection against external sources of vibration transmitted to the structure. The proposal for such a solution can be accepted as innovative, due to the low cost of implementation in the technological process and materials that can be easily recycled. The solution discussed in this article is characterized by spatial stiffness and good damping properties, which allows the application of the adopted solution in areas of significant surface loads, where the vibro-isolating rubber mats will not be able to meet construction-technology requirements. Despite the many advantages of the adopted solution, prior to its application, experimental tests should be carried out to determine the real mechanical properties.

### References

A numerical analysis of the wave phenomena in the spatial structure of a steel grid

STRESZCZENIE:
Przedstawiono numeryczną analizę propagacji fal mechanicznych w przestrzennej strukturze rusztu stalowego z wypełnieniem gumowym. Stal przyjęto jako izotropowy materiał liniowy, natomiast gumę zamodellowano jako nieściśliwy materiał Zahorskiego. Przygotowana w odpowiedni sposób struktura może zmniejszać efekty dynamiczne wynikające z propagacji fal mechanicznej. Uzyskane z analizy wyniki pozwalają ocenić wpływ zastosowanego materiału na oddziaływania dynamiczne w analizowanym ruszcie z wypełnieniem gumowym. Wyniki analizy numerycznej przedstawiono w formie graficznej. Zjawiska falowe zamodelowane zostały z użyciem programu ADINA.

SŁOWA KLUCZOWE:
ruszt stalowy; zjawiska falowe; guma