An analysis of limit states ULS and SLS in a cross-section prefabricated ceiling, made of pre-tensioned pre-stressed TT slabs

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ABSTRACT:
Designing industrial buildings that use prefabricated ceilings made of TT slabs, is a complex issue. In professional practice, typical (catalog) TT floor solutions are used in production lines, warehouse buildings and cement plants. The article presents a comparative analysis of a cross-section of the prefabricated ceiling span, pre-tensioned pre-stressed, slab-rib type TT ceiling in an industrial building with a mullion and transom structure. The prefabricated TT slabs are produced from reinforced concrete beams. The slabs width is constant at 240 cm and the span is assumed to be 12.0 meters. The subject of structural analysis is the selection of an appropriate cross-section of the prefabricated element, the amount of pre-stressing reinforcement required, the concrete class, and on this basis, check the load limits ULS and serviceability limit SLS according to applicable Eurocodes. The analysis was conducted on the basis of the adopted computational model and carried out detailed strength calculations.

KEYWORDS:
TT type prefabricated ceilings; TT prefabricated slabs; pre-tensioned pre-stressed elements

1. Introduction
The use of prefabricated elements in the design of industrial buildings is very popular due to the speed of execution and similar durability compared to monolithic technology. It should be added that formwork and temporary supports are often omitted, which reduces investment outlays. Prefabricated elements are usually used in situations where the decisive factor is the speed of construction (spans of industrial or bridge ceilings [1] are made within a few days). The most commonly used solutions, in the main load-bearing elements, are pre-stressed concrete slabs prefabricated with ribs, marked in the literature and catalogs as 2T, TT or Π slabs. One prefabricated element consists of two main ribs longitudinally stressed with high tensile strength tendons (main tendons in the element's tension zone and usually one tendon in the compression zone due to the assembly phase) and ordinary transverse reinforcement in the form of stirrups. The ribs are connected by a thin reinforced concrete slab 3 to 8 cm thick with reinforcement in the form of a welded steel mesh. The cross-sectional height of the TT slabs (including concrete slab) is assumed in factories in the range of 44 to 106 cm. It gives the possibility to reach elements spanning up to 30.0 meters with high usage loads. Standard cross-sectional widths of the TT slabs range from 240 to 300 cm. The TT slabs are used in industrial buildings, multi-level car parks and in commercial and service facilities. There are also varieties of slabs with one pre-stressed rib (T type prefabricated slabs) in production.

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In Figure 1 there is an example of the use of prefabricated TT slabs, based on both sides by pre-stressed girders, in an industrial building.

![Fig. 1. View of the industrial ceiling with TT type pre-stressed concrete slab [2]](image)

### 2. Geometric data and compression parameters of a TT type floor slab

The analysis of the load capacity of an industrial floor span made of prefabricated TT concrete pre-tensioned slabs was carried out on a selected example of a ceiling in an industrial building (Fig. 1). The cross-section of the TT type floor slab as in (Fig. 2) was adopted for analysis. The values of the geometrical parameters of the cross section are given in Table 1.

![Fig. 2. Cross section of a TT type pre-tensioned concrete slab](image)

<table>
<thead>
<tr>
<th>Cross section of slab TT</th>
<th>b₁ [mm]</th>
<th>b₂ [mm]</th>
<th>h₁ [mm]</th>
<th>h₂ [mm]</th>
<th>h₃ [mm]</th>
<th>h [mm]</th>
<th>bₑ [mm]</th>
</tr>
</thead>
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<td>480</td>
<td>960</td>
<td>15</td>
<td>300</td>
<td>70</td>
<td>440</td>
</tr>
<tr>
<td>TT650</td>
<td>205</td>
<td>460</td>
<td>960</td>
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<td>500</td>
<td>80</td>
<td>650</td>
</tr>
<tr>
<td>TT860</td>
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<td>440</td>
<td>960</td>
<td>35</td>
<td>700</td>
<td>90</td>
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<td>420</td>
<td>960</td>
<td>45</td>
<td>900</td>
<td>90</td>
<td>1060</td>
</tr>
</tbody>
</table>

**LIMITATIONS:**
- min. 205
- max. 745
- 6,05×h₁
- max. 900
- min. 50
- min. 50
- max. 3000

Basic parameters of the prefabricated ceiling made from the TT slabs were adopted in static and strength calculations in accordance with [3]:
- Effective span in the axes of support of the TT slab: \( L_{eff} = 11.6 \) m
- Total length of the TT slab: \( L_b = 12.0 \) m
- Total width of the TT slab: \( b_{eff} = 2.39 \) m
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- Static scheme: single span structure
- Live load on the ceiling: 7.50 kN/m²

Fig. 3. Variants of cross sections of the TT floor slab: a) W-01 variant of the TT440 slab; b) W-02 variant of the TT650 slab; c) W-03 variant of the TT860 slab; d) W-04 variant of the TT1060 slab

Fig. 4. Calculation models for cross-sectional analysis of the TT floor slab: a) model for the TT440 slab; b) model for the TT650 slab; c) model for the TT860 slab; d) model for the TT1060 slab

Static calculations were made on the 3D Frame + Shell model in the Autodesk Robot Structural Analysis Professional 2018 software. The model uses beam elements - slabs with cross-sections as in Figure 3 and shell elements - a reinforced concrete slab made from reinforced concrete in a prefabricated ceiling of TT slabs. Diagrams of the calculation models are presented in Figure 4.

In the analysis of the cross-sectional structure ceiling span constructed by the TT slabs, results of static and strength calculations were made for variants W-01, W-02, W-03 and W-04. In the designed industrial building with TT type pre-tensioned concrete ceiling, fixed parameters were adopted: a slab span equal to 11,6 m in column axes, width of the TT slab equal to 2.4 m, width of the rib equal to 20.5 cm and rib spacing in light of the side walls - b on (Fig. 2) and (Fig. 3) equal to 96.0 cm. A non-structural constant load equal to 4.83 kN/m² and a live load equal to 7.50 kN/m² were adopted. Optionally, a characteristic load from a forklift truck was adopted in accordance with [4-6]. In the calculation models, two classes of concrete in the prefabricated slabs C45/55 and C50/60 were adopted in accordance with [6] and two classes of pre-stressing tendons Y1770S7 and Y1860S7. The exposure class was XC4 where $c_{\text{min,curv}} = 30$ mm. The poured concrete was made of C30/37 class concrete reinforced with welded steel mesh.

3. Analysis of the load capacity of a road viaduct

Several optimization criteria were assumed in order to carry out a detailed cross-sectional analysis of the TT prefabricated floor slabs, which were used for the 16 calculation models. Figure 3 presents four variants of the TT slab cross-sections W-01; W-02; W-03; W-04. In each
variant, four models were adopted, in which the variables were concrete and pre-stressed steel class according to the formula:

\[
W-01; W-02; W-03; W-04
\]

M-01 (C45/55 and Y1770S7); M-02 (C45/55 and Y1860S7);
M-03 (C50/60 and Y1770S7); M-04 (C50/60 and Y1860S7);

Criteria for cross-section optimization:

- **KR-1**: due to the safety condition for the ultimate fracture state of the bending TT slab according to [6] and [7]

\[
M_{R_{u11}} \leq M_{Rd}
\] (1)

- **KR-2**: checking the condition for exceeding the allowable edge stresses for concrete in a transient design situation according to [6] and [7]

\[
\delta_{ec} \leq \sigma_{dop} = 0.7 \times f_{cm}
\] (2)

- **KR-3**: due to the pre-stressing force condition for the scratch resistance category 1b (fully pre-stressed) for the tension zone (safety condition in the cracking limit state for the lower edge of the cross section) according to [6] and [7]

\[
P_{cr1} \leq P_{t1}
\] (3)

- **KR-4**: due to the pre-stressing force condition for the scratch resistance category 1b (fully pre-stressed) for the compressed zone (safety condition in the limit of cracking condition for the upper edge of the cross section) according to [6] and [7]

\[
P_{cr2} \leq P_{t2}
\] (4)

- **KR-5**: due to the load capacity of the concrete slab due to the reinforcement of the TT section in three places: at the base of the slab support; in the slab reinforcement cross-section and in the span section according to [6] and [7]

\[
M_{Sd} = \max (M_{Sd1}; M_{Sd2}; M_{Sd3}) \leq M_{Rd}
\] (5)

- **KR-6**: due to the pre-stressing force condition for the scratch resistance category 1b (fully pre-stressed) after immediate losses according to [6] and [7]

\[
P_{i1} \leq P_{i2}
\] (6)

- **KR-7**: due to the pre-stressing force condition for the 1b scratch resistance category (fully pre-stressed) after immediate and rheological losses according to [6] and [7]

\[
\text{KR-7.1: } P_{i1} \leq P_{i2} \text{ and KR-7.2: } P_{i1} \geq P_{cr1}
\] (7)

- **KR-8**: due to reverse deflection (dead load and pre-stressed) according to [6] and [7]

\[
a_0 \leq a_{lim1}
\] (8)

- **KR-9**: due to maximum deflection according to [6] and [7]

\[
a \leq a_{lim}
\] (9)

- **KR-10**: due to momentary deflection according to [6] and [7]

\[
a_{kt} \leq a_{limk}
\] (10)
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The calculation results for the 16 models are presented in Table 2. After a detailed analysis, the optimal cross-section of the TT type floor slab is marked in Table 2 under item 13.

**Table 2**
Summary of results of the TT slab cross-sectional optimization criteria

<table>
<thead>
<tr>
<th>No.</th>
<th>Calculation model code</th>
<th>KR-1 ≤ 1.0</th>
<th>KR-2 ≤ 1.0</th>
<th>KR-3 ≤ 1.0</th>
<th>KR-4 ≤ 1.0</th>
<th>KR-5 ≤ 1.0</th>
<th>KR-6 ≤ 1.0</th>
<th>KR-7.1 ≤ 1.0</th>
<th>KR-7.2 ≥ 1.0</th>
<th>KR-8 ≤ 1.0</th>
<th>KR-9 ≤ 1.0</th>
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<td>0.95</td>
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<td>0.41</td>
<td>0.11</td>
<td>0.32</td>
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<td>3.19</td>
<td>0.05</td>
<td>0.66</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Fig. 5. Final analysis of the TT1060 slab cross-section optimization

Analyzing all calculation results from the 16 criteria, it should be stated that only cross-sections of the slab with a total height of 1060 mm - items 13, 14, 15 and 16 in (Table 2) - meet all the criteria. The conditions that determined the selection of the optimal cross-section were criteria KR-7.2 and KR-9 related to the pre-stressing force condition for the 1b scratch resistance category (fully pre-stressed) after immediate and rheological losses and due to the maximum deflections of the TT type floor slab.

Figure 5 presents a graphical method of analyzing the final optimization of the cross-section of the TT concrete pre-stressed ceiling slab with a total height of 1060 mm.
4. Final conclusions

The Ultimate Limit State (ULS) and Service Limit States (SLS) were checked computationally for the cross-section of the TT pre-stressed concrete slab. 16 TT slabs models were tested based on 10 criteria. Based on the analysis, the optimal cross-section of the TT pre-stressed concrete slab was chosen, i.e. the TT1060 slab with 8 pre-stressed tendons of Y1770S7 steel (in each rib in the tension zone there are 2×4 pre-stressed tendons) and one pre-stressed tendon in the compression zone. The presented TT1060 slab met all optimization criteria and the detailed static and strength calculations according to [7] and [8] confirmed the selection of the above-mentioned floor slab in the industrial building.

References