THE USE OF LINEAR ALGEBRA AND ALGEBRA STRUCTURE TEACHING MATERIALS BASED CONCEPT MAPS TO OVERCOME STUDENTS' DIFFICULTIES IN MATHEMATICAL PROOF

Rahayu Kariadinata^{1*}, R.Poppy Yaniawati² and Ida Nuraida¹

¹*Department of Mathematic Education Faculty of Education and Teacher Training,
State Islamic University (UIN) Sunan Gunung Djati Bandung, Indonesia*

²*Department of Mathematic Education, Pasundan University, Bandung Indonesia*

Abstract: This study aims to design the Linear Algebra and Algebra Structure teaching materials based concept maps to overcome students' difficulties in mathematical proof. The method used was research and development, Four-D model (Define, Design, Develop, Disseminate). The subjects of the study were students of Mathematics Education, UIN Sunan Gunung Djati Bandung-Indonesia. Define stage was addressed to analyse the problems faced by students, especially in mathematical proof. In design stage, prototype of teaching materials based concept maps was designed. Develop stage was the stage of making teaching materials draft which was further validated by the expert. Based on the validation of the experts in term of content properness got 80,56%, presentation properness got 71,67%, language properness got 80,00%, and graphic properness got 73,90%. Therefore the average result is 76,53.% with very appropriate criterion, so that the draft is ready for a limited and classical test. The limited and classical test showed that the concept map can aid students in the process of proofing, the result of the test at the end of the meeting shows that 75% of students got a score higher than 70, this means that has achieved mastery learning in classical. Furthermore, the disseminate stage was the stage of spreading the revised teaching materials product. The result of the study concludes that the Linear Algebra and Algebra Structure teaching materials based concept maps have met the standard, hence it is effective to assist students in overcoming their difficulties in mathematical proof.

Keywords: Four-D Model, Linear Algebra and Algebra Structures teaching materials, Concept maps

Introduction

The proof ability is one of the five standard mathematical learning processes recommended by the National Council of Teachers of Mathematics (2000). Mathematics as a deductive science requires a method of truth-seeking process; this method is known as a mathematical proof method. The method of proof is necessary to convince the truth of statements or theorems that are generally in the form of implications or biimplications

In mathematics, the proof is a series of logical arguments that explain the truth of a statement (Hernadi, 2008: 1). Required abilities in mathematical proof according to Selden & Selden (Tall, 1999: 30) are the ability to construct evidence and the ability to validate evidence. Mathematical proof can serve as an actual process through the construction of evidence and as the final phase. As Hadamard (Tall, 1999: 32) points out that mathematical proof is the final phase in mathematical thinking. Based on the opinions of the experts above, mathematical proof is a logical way of thinking that starts from axioms or definitions and moves forward through logical steps to a conclusion.

Some subjects that require the ability in proofing process are Linear Algebra and Algebra Structure. Based on the experience when researchers teaching Linear Algebra and Algebra Structure, it shows that the weakness of the students' ability is in proofing process. These weaknesses include the difficulty of students in understanding

the definitions, the theorems that support the proof, linking between concepts that support the proof, and difficulty in giving arguments to obtain conclusions.

Study on student difficulties in mathematical proof done by some experts. Recio & Godino (2001) found that students are weak in producing evidence deductively, Weber (2001), found that lack of strategic knowledge, was the cause of early student difficulties in mathematical proof, and Selden & Selden's (Weber, 2003) found that students could not determine whether or not a proof is valid.

Grob, et al (2008) argues that many students face serious difficulties with consistent reasoning and argument, especially on mathematical proof. Furthermore, Moore (Knapp, 2006) finds seven student difficulties in proofing: 1) students do not understand the definition, it means that they cannot state a definition of a concept in their own way, 2) the students have little intuitive understanding of concepts, 3) students' image concepts are inadequate to help proofing, 4) students are unable to produce and use their own example, 5) students do not know how to use definitions to construct evidence, 6) students cannot understand and use mathematics and notation, 7) students do not know how to start evidence.

In accordance to that, it is necessary to make efforts from teacher to overcome student difficulties in mathematical proofing especially on the Linear Algebra and Algebra Structure subjects. Students need to be equipped with various sources of learning in achieving good results. One of the learning resources that need to be developed is teaching materials that can develop students' logical thinking, so that mathematical concepts are easy to understand by students

Teaching materials are any forms of materials that can be used to assist teachers in carrying out teaching and learning activities in the classroom. The intended teaching materials can be either written or unwritten (National Centre for Competency Based Training, 2007). Teaching materials provide information or a relatively operational picture for the management of the learning process. The other experts say that teaching materials is a set of materials that are arranged systematically, both written and unwritten, so as to create an environment or atmosphere that allows students to learn. Based on these definitions, it can be concluded that teaching materials is a learning component used by teachers as learning materials for students and assist teachers in carrying out teaching and learning activities in the classroom. Teaching materials can support student learning and improve student success. Of course, teaching materials should be in accordance with the content that students and teachers will use.

Interrelation of concepts should be presented clearly within teaching materials in order to facilitate students in constructing the evidence for proofing purposes which is in accordance with mathematic characteristics that has an interrelated concept. The interrelation between concepts can be delivered through concept maps integrated in teaching materials. This form of concept might help students in identifying key ideas related between topics and rearranging them into a logical pattern. Concept maps can accurately depict all the key concepts and principles that exist within the curriculum and the relation between the concepts and principles. In teacher's perspective, concept mapping serves benefit as aids in preparing the lesson. In student's perspective, concept map helps them in gaining deep understanding

The use of concept map as a teaching strategy was first developed by J. D. Novak of Cornell University in the early 1980's. It was derived from Ausubel's learning theory which places central emphasis on the influence of students' prior knowledge on subsequent meaningful learning. A concept maps is a diagram showing the relationships among concepts. They are graphical tools for organizing and representing knowledge. (Iona, et al, 2010:568)

Novak & Canas (Radix & Abdool, 2013) state that concept maps are hierarchical in graphical structures including nodes expressing concepts, and connecting arrows labeled with relationships between concepts.

Graphic organisers are tools and techniques that allow the user to visually depict the organisation of information.

The concept maps will serve as an aid for students in connecting any related concepts, as showed in Cardemone, Minemier and Fuata'i (Novak and Cañas, 2009:7) research, key ideas in learning mathematics can be represented by using concept map. In its application, concept maps help teachers in designing topics sequentially and help students to see the relationships between them. Students can create a concept maps from the topics that they have learned. Moreover, students not only show a better performance in problem-solving tests but also gain confidence in their ability in solving mathematics tasks. The use of concept maps through Vee graph with Form Five students in Western Samoa found that students became more autonomous in learning and better at solving problems than students who did not use concept maps.

This study aims to produce the Algebra Linear and Algebra Structure teaching materials based concept maps as an effort to overcome student difficulties in mathematical proof and examine the effectiveness of the teaching materials.

Research Method

This study is a development research with Four-D model (Define, Design, Develop and, Disseminate) suggested by oleh Sivasailam Thiagarajan, Dorothy S. Semmel, dan Melvyn I. Semmel (1974). The sample study was taken based on purposive sampling. The sample was students of Mathematics Education, Faculty of Education and Teacher Training, UIN Sunan Gunung Djati Bandung Indonesia, 3rd and 5th semesters of 2017/2018 academic year taking Linear Algebra and Algebra Structure subjects.

Teaching Materials Development Procedures

Teaching materials development procedures were based on development model which is 4-D model (Define, Design, Development, Disseminate).

Define Stage

At this stage, an assessment of standard competence and basic competence that will be achieved from the Algebra Linear and Algebra Structure subjects was carried out. Assessment was also addressed on the concepts and learning theory for Linear Algebra and Algebra Structure subjects.

Design Stage

At this stage, prototyping or modeling a teaching materials was undertaken. There are three methods that can be taken into account in arranging teaching materials design (Tiarani, 2011), namely (1) start from scratch (2) information repackaging or text transformation), and (3) compilation or wrap around text). At this point of stage, teaching materials, number of chapters / material, organization of the teaching materials, number of meeting required or school hours was also drafted in the book.

This study used compilation or wrap around text due to the low economic cost and less time needed. Teachers should collect textbooks by browsing literature in libraries, selecting material in bookstores, and actual information in newspapers, magazines, journals and others (Ekawarna, 2007: 43).

Furthermore, materials needed are collected, copied, and then selected and arranged according to standard competence and the subjects order in syllabus and Linear Algebra and Algebra Structure subject. In addition, the teaching materials was also equipped with concept maps. Summary and simplification of topics were delivered in graphical representation which aims to ease students in understanding and recalling the topic. The steps of making concept maps are sorting, constructing concepts start from the most inclusive concept to the least

inclusive concept, connecting the concepts associated with the linking lines, and then adding linking words on each of the linking lines. Here is one example of Linear Algebra's concept map of "Linear Independence" material.

Figure 1. Concept Maps of Linear Independence Material

While on the Algebra Structure is the material "Group "as shown below

Figure 2. Concept Maps of Group Material

Develop Stage

At this stage, a draft of teaching materials was made which further submitted to the experts to be reviewed. The draft will be judged in terms of content properness, presentation properness, language properness, and graphic properness. The criteria of properness of teaching materials can be seen in Table 1

Table 1 Properness Conversion Criteria

Percentage (%)	Qualification	Decision
90-100	Very Good	Products ready to be utilized in real field for learning activities not revisions
80-90	Good	The product is ready to be utilized in real field for learning activities without revision
70-79	Pretty good	The product can be continued by adding some parts that are lacking, doing certain considerations, the addition is not too large and not fundamental
60-69	Not Good	Revise by thoroughly re-examining and looking for product flaws to improve
0-59	Not very good	The product failed, revised large and basic about the content of the product

Furthermore, a test on designed teaching materials was conducted on a limited basis. After the limited test had conducted, the teaching materials were revised for further taken into classical test through real learning implementation stage by using concept-based teaching materials.

Disseminate Stage

At this stage, the reviewed teaching materials was implemented in the teaching and learning process. This stage is the last stage of development to promote teaching materials to be used by either individual, group, system, or program.

Research Findings

Based on expert validation at the develop stage of teaching materials obtained results as follows

Table 2 Validation Results on the Content Properness Aspect

Content Properness	Teaching Materials	
	Linear Algebra	Algebra Structure
The truth of material content	4,33	4,33
Material accuracy	4,00	3,67
Material coverage	4,00	4,00
Clarity of concept maps	4,00	4,00
Grouping materials	4,00	4,00
Compatibility with the syllabus	4,00	4,00
Total Score	24,33	24,00
Average Score	4,06	4,00
Presentage	81,11%	80,00%
Average	80,56%	

Maximum Score =5

In Table 2. obtained by validation percentage of Linear Algebra and Algebra Structure based concept maps got 80,56% is in good qualification (80-89) so that the content properness aspect of product is declared viable without revision.

Table 3 Validation Results on the Presentation Properness Aspect

Presentation Properness	Teaching Materials	
	Linear Algebra	Algebra Structure
Clarity of indicators to be achieved	3,33	3,33
Clarity of material distribution	3,67	3,67
Giving stimulus and response for students	3,33	3,67
The information is complete	4,00	4,00
Ordered material presentation	3,67	3,67
Motivate students and have appeal	3,33	3,33
Total Score	21,33	21,67
Average Score	3,56	3,61
Presentage	71,11	72,22
Average	71,67	

Maximum Score =5

In Table 3 obtained by validation percentage of Linear Algebra and Algebra Structure based concept maps got 71,67% is in pretty good qualification (70-79) so that the presentation properness aspect of product is declared viable but there are some revisions to the two teaching materials, namely clarity of indicators to be achieved. giving stimulus and response for students and motivate students and have appeal.

Table 4 Validation Results on the Language Properness Aspect

Language Properness	Teaching Materials	
	Linear Algebra	Algebra Structure
The truth of grammar	3,67	4,00
Clarity of information	4,00	3,67
Using good language rules and right	4,00	4,33
Using effective and efficient language (clear and concise)	4,33	4,00
Conformity of sentences with the level of student development	4,33	4,00
The simplicity of sentence structure	3,67	3,67
The communicative nature of the language used	4,33	3,67
Total Score	24,67	23,33
Average Score	4,11	3,89
Percentage	82,22	77,78
Average	80,00	

Maximum Score =5

In Table 4. obtained by validation percentage of Linear Algebra and Algebra Structure based concept maps got 80,00% is in good qualification (80% -89%) so that the language properness. aspect of product is declared viable without revision.

Table 5. Validation Results on the Graphic Properness Aspect

Graphic Properness	Teaching Materials	
	Linear Algebra	Algebra Structure
Clear number system	3,67	4,00
Space organization/layout	3,00	4,00
Accuracy in paragraph arrangement	3,33	3,67
Clear writing or typewriting	3,67	4,00
Appropriate type and size of font	4,00	3,33
Physical size of the teaching materials with the student	3,67	4,00
Total Score	21,34	23,00
Average Score	3,56	3,83
Percentage	71,13	76,67
Average	73,90	

Maximum Score =5

In Table 5. obtained by validation percentage of Linear Algebra and Algebra Structure based concept maps got 73,90% is in pretty good qualification (70% -79%) so that the Graphic Properness aspect of product is declared viable but there are some revisions to the two teaching materials, namely accuracy in paragraph arrangement

Based on the results of the calculation of properness in terms of content, presentation, language, and graphic then the average properness of teaching materials as a whole can be seen in Table 6.

Table 6. Properness of Material Teaching

Properness Aspect	Percentage
Content Properness	80,56
Presentation Properness	71,67
Language properness	80,00
Graphic Properness	73,90
Average	76,53

In Table 6, the average of the properness of teaching materials is 76,53 including good enough category (70% - 79%). After the draft of the teaching materials was revised based on expert input, limited test was conducted on some students. Based on the limited test, the result shows that the students find the teaching materials based concept maps to be very helpful, especially in terms of mathematical proof. In addition, the product is ready to be utilized in the real situation (classical test) for learning activity without revision. This activity is real learning situation where teaching materials based concept maps are used.

At the last meeting of the classical test, students were given a test containing the problem of mathematical proof with results 75% of students got a score higher than 70. This means that has achieved mastery learning in classical. Furthermore, the teaching materials were taken to disseminate stage as the final stage of material development where the teaching materials were used outside the research subject.

Conclusion

Based on the result of development and the test on teaching materials (limited test and classical test) on the students of Mathematics Eductaion UIN Sunan Gunung Djati Bandung, Indonesia it can be concluded that teaching materials of Linear Algebra and Algebra Structure based concept maps using Four-D model have met the standards to be used in learning process,

The properness result of the teaching materials which is used is gained from the expert in term of content properness got 80,56%, presentation properness got 71,67%, language properness got 80,00%, and graphic properness got 73,90%. Therefore the average result is 76,53.% with very appropriate criterion.

Based on the criteria of classical mastery learning, class using Linear Algebra and Algebra Structure is said to be complete, this is obtained from the average value of students, i.e 75% of the number of students got a score above 70. So, that Algebra Linear and Algebra Structures teaching materials based concept maps can be effectively used to overcome student difficulties in mathematical proof.

Acknowledgements

We would like to thank the Dean of the Faculty of Education and Teacher Training, State Islamic University (UIN) Sunan Gunung Djati Bandung Indonesia, for supporting us.

References

- Ekawarna. 2007. Mengembangkan Bahan Ajar Mata Kuliah Permodalan Koperasi untuk Meningkatkan Motivasi dan Hasil Belajar Mahasiswa. Sistem Informasi Jurnal Ilmiah UI.Jurnal Makara, Sosial Humaniora, Volume 11, No,1, Juni 2007, 42-43 .
- Grob, C., Heinze, A., Reiss, K. M. & Renki, A. 2008. Reasoning and proof in geometry: effects of a learning environment based on heuristic worked-out examples. The International Journal on Mathematics Educations, 40(3), pp. 455-467.
- Hernadi, J. 2008. Metoda Pembuktian Dalam Matematika. Jurnal Pendidikan Matematika, Volume 2, No. 1, Januari 2008. Retrieved from http://eprints.unsri.ac.id/794/1/URUT_1_Hal_GANJIL_YULAN.pdf pp 2-13

- Iona Stoica, Silvia Moraru, Cristina Miron . 2011. Concept Map, a Must for The Modern Teaching-Learning Process. Romanian Reports in Physics, Vol. 63, No. 2, pp 567–576, 2011
- Knapp, Jessica. 2006 A Framework to Examine Definition use in Proof. Proceeding of the North American Chapter of the International Group for the PME. pp vol.2-15 – 22
- National Centre For Competency Based Training. (2007). Pengertian Bahan Ajar. Retrieved from <http://www.kajianteorit.com/2014/02/pengertian-bahan-ajar-menurut-ahli.html>
- National Council of Teachers of Mathematics. 2000. Principles and Standards for School Mathematics. Reston, V.A: National Council of Teachers of Mathematics
- Novak and Cañas .2009. Concept Mapping in Mathematics. Research into Practice. Editor by Karolina Afamasaga-Fuata'i. University of New England Australia . Chapter 1. The Development and Evolution of the Concept Mapping Tool Leading to a New Model for Mathematics Education (Springer Science+Business Media, LLC 2009) pp. 3-7
- Radix , Cathy-Ann & Abdool Azim. 2013. Using mind maps for the measurement and improvement of learning quality. Caribbean Teaching Scholar Vol. 3, No. 1, April 2013, 3–21. EDUCATIONAL RESEARCH ASSOCIATION
- Recio, A.M. & Godino, J.D. 2001. Institutional and Personal Meanings of Mathematical Proof. In Educational Studies in Mathematics ,Volume 48: 83-99. Retrieved from <http://www.jstor.org>
- Sudjana. Nana, 2005. Penilaian Hasil Proses Belajar Mengajar. Bandung : PT. Remaja Rosdakarya.
- Tall, D. 1999. Advanced Mathematical Thinking. Boston: Kluwer Academics Publisher.pp 30-32
- Thiagarajan, S., Semmel, D. S & Semmel, M. I. 1974. *Instructional Development for Training Teachers of Exceptional Children*. Minneapolis, Minnesota: Leadership Training Institute/Special Education, University of Minnesota. <https://files.eric.ed.gov/fulltext/ED090725.pdf> pp 5-9
- Tiarani, V.A. (2011). Teknik Pengembangan Bahan Ajar Dwi Bahasa untuk Kelas Internasional. Universitas Negeri Yogyakarta. <http://staff.uny.ac.id/sites/default/files/tmp/>
- Weber, K. (2001). Student Difficulty in Constructing Proofs: The Need for Strategic Knowledge. Dalam Educational Studies in Mathematics , Volume 48: 101-119. Retrieved from <http://www.jstor.org>
- Students' Difficulties with Proof. Retrieved from http://www.maa.org/t_and_l/rs_8.html